ALNOJOUM ACADEMY
أكاديمية النجوم

Chapter 9
Linear Momentum and Collisions

Written solutions to the textbook problems

PHYS 105 · Prince Sultan University · 12 solved problems

What this chapter covers

9.1 Center of Mass
9.3 Linear Momentum
9.4 Collision and Impulse
9.5 Conservation of Linear Momentum
9.6 Momentum and Kinetic Energy in Collisions
9.7 Elastic Collisions in One Dimension

All key formulas are on the next page.

Watch out

- Momentum and impulse are vectors — choose a positive direction and keep the signs.
- Convert g → kg and ms → s before substituting.
- In 2D problems, conserve momentum in x and in y separately.
- Momentum is conserved in every collision of an isolated system; kinetic energy only in elastic ones.
- Collision + motion after it: use momentum for the collision, then energy only after the collision ends.

Problems in this file

1 Dog, cat and owner
2 Dropped baseball
4 Kicking a soccer ball
6 Croquet ball and mallet
7 Spiking a volleyball
8 Bounce pass
10 Heading a soccer ball
11 Two ice skaters push off
12 Two hockey players collide
13 Putty hits a hanging block
14 Putty hits a block on a spring
15 Hammer strikes a nail

Every problem has a QR code that opens its full video solution.
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Chapter 9

Key formulas

Organized by the sections of the Chapter 9 slides (9.1 – 9.7).

9.1 Center of Mass

Center of mass (n particles)	$x_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i, \quad M = \sum m_i$	Same form for y_{com} and z_{com}

9.3 Linear Momentum

Linear momentum	$\vec{p} = m\vec{v} \quad (p_x = mv_x,\ p_y = mv_y)$	Vector, same direction as v; unit kg·m/s
Change in momentum	$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$	Rebound with same speed: $\lvert\Delta p\rvert$ = 2mv
Newton's second law	$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$	Equivalent to $F_{\text{net}} = ma$

9.4 Collision and Impulse

Impulse (definition)	$\vec{J} = \int_{t_i}^{t_f} \vec{F}\, dt = \vec{F}_{\text{avg}}\, \Delta t$	Vector, same direction as F_{avg}; unit N·s = kg·m/s
Impulse–momentum theorem	$\vec{J} = \Delta\vec{p} = m(\vec{v}_f - \vec{v}_i) \quad (J_x = \Delta p_x)$	J = net impulse; keep the signs
Average force	$\vec{F}_{\text{avg}} = \frac{\Delta\vec{p}}{\Delta t}$	Area under F–t curve = J

9.5 Conservation of Linear Momentum

Closed, isolated system	$\vec{F}_{\text{ext,net}} = 0 \ \Rightarrow\ \vec{P} = \text{constant},\ \ \vec{P}_i = \vec{P}_f$	No net external force, no mass in or out
During a collision	$\vec{J}_{\text{ext}} = \sum \vec{F}_{\text{ext}}\, \Delta t \approx 0 \ \Rightarrow\ \vec{P}_i \approx \vec{P}_f$	External impulse zero or negligible (very short Δt)
Two objects	$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$	In 2D apply to x and y separately

9.6 Momentum and Kinetic Energy in Collisions

Completely inelastic (stick together)	$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2)\, V$	Greatest loss of K (to heat, sound, deformation)
Special case $v_{2i} = 0$	$V = \frac{m_1}{m_1 + m_2}\, v_{1i}$	

9.7 Elastic Collisions in One Dimension

Stationary target ($v_{2i} = 0$)	$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}\, v_{1i}$	
	$v_{2f} = \frac{2m_1}{m_1 + m_2}\, v_{1i}$	
Equal masses	$v_{1f} = 0, \quad v_{2f} = v_{1i}$	Velocities are exchanged

◆ Types of collisions (isolated system)

	ELASTIC	INELASTIC	PERFECTLY INELASTIC
Momentum	Conserved	Conserved	Conserved
Kinetic energy	Conserved	Not conserved	Not conserved (largest loss)
After the collision	Objects separate	Objects separate	Objects stick together

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PROBLEM 1

Dog, cat and owner

Momentum vector · Components

Watch the video

A 26.2-kg dog runs north at 2.70 m/s while a 5.30-kg cat runs east at 3.04 m/s. Their 74.0-kg owner has the same total momentum as the two pets together. Find the magnitude and direction of the owner's velocity.

GIVEN $m_d = 26.2\ \text{kg},\ v_d = 2.70\ \text{m/s (N)}\quad m_c = 5.30\ \text{kg},\ v_c = 3.04\ \text{m/s (E)}\quad M = 74.0\ \text{kg}$

FIND $\vec{v}_{\text{owner}}$

SOLUTION

Take east = +x and north = +y. Momentum of each pet:

$$\vec{p}_d = (26.2)(2.70)\,\hat{y} = 70.7\,\hat{y}\ \text{kg·m/s}, \qquad \vec{p}_c = (5.30)(3.04)\,\hat{x} = 16.1\,\hat{x}\ \text{kg·m/s}$$

Momentum is a vector, so the pets' total momentum is the **vector sum** (not 70.7 + 16.1):

$$\vec{P} = \vec{p}_d + \vec{p}_c = (16.1\,\hat{x} + 70.7\,\hat{y})\ \text{kg·m/s}$$

The owner has this total momentum, so divide by his mass:

$$\vec{v} = \frac{\vec{P}}{M} = \frac{16.1\,\hat{x} + 70.7\,\hat{y}}{74.0} = (0.218\,\hat{x} + 0.956\,\hat{y})\ \text{m/s}$$

Magnitude and direction:

$$|\vec{v}| = \sqrt{0.218^2 + 0.956^2} = 0.980\ \text{m/s}, \qquad \theta = \tan^{-1}\left(\frac{0.956}{0.218}\right) = 77.2^\circ$$

ANSWER $v = 0.980\ \text{m/s},\ 77.2^\circ$ north of east

PROBLEM 2

Dropped baseball

Momentum · Free fall

Watch the video

A 0.150-kg baseball is dropped from rest. Just before it hits the ground, the magnitude of its momentum is 0.780 kg·m/s. From what height was it dropped?

GIVEN $m = 0.150\ \text{kg}\quad p = 0.780\ \text{kg·m/s}\quad v_i = 0$ **FIND** h

SOLUTION

Speed just before impact:

$$v = \frac{p}{m} = \frac{0.780}{0.150} = 5.20\ \text{m/s}$$

Free fall from rest, $v^2 = 2gh$:

$$h = \frac{v^2}{2g} = \frac{(5.20)^2}{2(9.81)}$$

ANSWER $h \approx 1.38\ \text{m}$

PROBLEM 4

Kicking a soccer ball

Impulse

Watch the video

A player kicks a soccer ball with a force of 1250 N, and the foot stays in contact with the ball for 5.95×10^{-3} s. Find the magnitude of the impulse delivered to the ball.

GIVEN $F_{\text{av}} = 1250\ \text{N}$ $\Delta t = 5.95 \times 10^{-3}\ \text{s}$ FIND J

SOLUTION

The force of a kick rises and falls during contact, so 1250 N is treated as the **average** force over Δt.

$$J = F_{\text{av}}\,\Delta t = (1250)(5.95 \times 10^{-3})$$

ANSWER $J \approx 7.44\ \text{N·s}$

PROBLEM 6

Croquet ball and mallet

Impulse–momentum theorem

Watch the video

A 0.50-kg croquet ball is at rest on the grass. A mallet strikes it with an average force of 230 N, and the ball leaves at 3.2 m/s. How long was the mallet in contact with the ball?

GIVEN $m = 0.50\ \text{kg}$ $F_{\text{av}} = 230\ \text{N}$ $v_i = 0,\ v_f = 3.2\ \text{m/s}$ FIND Δt

SOLUTION

Impulse–momentum theorem:

$$F_{\text{av}}\,\Delta t = \Delta p = m(v_f - v_i)$$

$$\Delta t = \frac{m(v_f - v_i)}{F_{\text{av}}} = \frac{(0.50)(3.2 - 0)}{230} = 6.96 \times 10^{-3}\ \text{s}$$

ANSWER $\Delta t \approx 7.0 \times 10^{-3}\ \text{s} = 7.0\ \text{ms}$

PROBLEM 7

Spiking a volleyball

Impulse–momentum theorem

Watch the video

When spiking a volleyball, a player changes its velocity from 4.2 m/s to −24 m/s along a certain direction. If the impulse delivered by the player is −9.3 kg·m/s, what is the mass of the volleyball?

GIVEN $v_i = 4.2\ \text{m/s}$ $v_f = -24\ \text{m/s}$ $J = -9.3\ \text{kg·m/s}$ FIND m

SOLUTION

Impulse equals the change in momentum (keep the signs):

$$J = \Delta p = m(v_f - v_i)$$

$$m = \frac{J}{v_f - v_i} = \frac{-9.3}{-24 - 4.2} = \frac{-9.3}{-28.2}$$

ANSWER $m \approx 0.33\ \text{kg}$

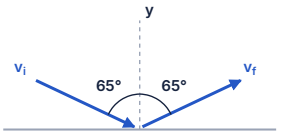
PROBLEM 8

Bounce pass

Impulse in 2D · Components

Watch the video

A player throws a 0.60-kg basketball at the floor. It hits the floor at 5.4 m/s, at 65° to the vertical, and rebounds with the same speed and angle. What impulse does the floor deliver to the ball?



GIVEN $m = 0.60\ \text{kg}$ $v = 5.4\ \text{m/s}$ $\theta = 65^\circ$ from the vertical FIND $\vec{J}$

SOLUTION

The angle is measured from the **vertical**, so the vertical component uses cos and the horizontal uses sin:

$$v_y = v\cos 65^\circ = 2.28\ \text{m/s}, \qquad v_x = v\sin 65^\circ = 4.89\ \text{m/s}$$

Horizontal: the ball moves in +x before **and** after the bounce, with the same value 4.89 m/s, so J_x = m(4.89 − 4.89) = 0.

Vertical: the component reverses, from −2.28 m/s (down) to +2.28 m/s (up):

$$J_y = m(v_{fy} - v_{iy}) = (0.60)\big[2.28 - (-2.28)\big]$$

ANSWER $\vec{J} \approx 2.7\ \text{N·s, straight up}$

PROBLEM 10

Heading a soccer ball

Impulse in 2D | Vector direction

Watch the video

A player heads a 0.43-kg soccer ball, changing its velocity from v_i = (8.8 m/s) $\hat{x}$ + (−2.3 m/s) $\hat{y}$ to v_f = (5.2 m/s) $\hat{x}$ + (3.7 m/s) $\hat{y}$. The contact time is 6.7 ms. Find (a) the direction and (b) the magnitude of the impulse delivered to the ball.

GIVEN $m = 0.43\ \text{kg}$ $\vec{v}_i = (8.8\,\hat{x} - 2.3\,\hat{y})\ \text{m/s}$ $\vec{v}_f = (5.2\,\hat{x} + 3.7\,\hat{y})\ \text{m/s}$ **FIND** $\vec{J}$

SOLUTION

Change in velocity:

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i = (-3.6\,\hat{x} + 6.0\,\hat{y})\ \text{m/s}$$

Impulse = change in momentum:

$$\vec{J} = m\,\Delta\vec{v} = (0.43)(-3.6\,\hat{x} + 6.0\,\hat{y}) = (-1.55\,\hat{x} + 2.58\,\hat{y})\ \text{N·s}$$

(a) Find the quadrant from the signs first: J_x < 0 (left) and J_y > 0 (up) → **2nd quadrant**, so the angle must be between 90° and 180°.

The calculator returns −59.0°, a 4th-quadrant angle, because $\tan^{-1}$ cannot tell (−,+) from (+,−). Add 180° to move to the 2nd quadrant:

$$\theta = \tan^{-1}\left(\frac{2.58}{-1.55}\right) + 180^\circ = -59.0^\circ + 180^\circ = 121^\circ$$

Check: 121° from +x is the same as 59.0° above the −x axis.

(b)

$$|\vec{J}| = \sqrt{(-1.55)^2 + (2.58)^2} = 3.0\ \text{N·s}$$

The contact time is only needed for the average force: F_{av} = J/Δt ≈ 450 N.

ANSWER **(a)** 121° from the $+$x axis **(b)** $J \approx 3.0\ \text{N·s}$

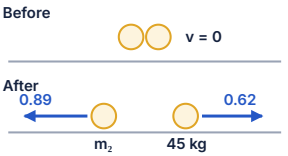
PROBLEM 11

Two ice skaters push off

Conservation of momentum

Watch the video

Two ice skaters stand at rest at the center of a rink. When they push off against each other, the 45-kg skater moves at 0.62 m/s. If the other skater moves at 0.89 m/s, what is that skater's mass?



GIVEN $m_1 = 45\ \text{kg}$ $v_1 = 0.62\ \text{m/s}$ $v_2 = 0.89\ \text{m/s (opposite)}$ **FIND** m_2

SOLUTION

No external horizontal force, and the total momentum starts at zero:

$$0 = m_1 v_1 + m_2 v_2$$

The skaters move in opposite directions (v_2 = −0.89 m/s):

$$m_2 = -\frac{m_1 v_1}{v_2} = -\frac{(45)(0.62)}{-0.89}$$

ANSWER $m_2 \approx 31\ \text{kg}$

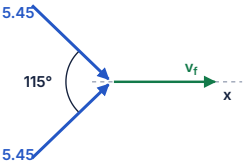
PROBLEM 12

Two hockey players collide

Perfectly inelastic · 2D momentum

Watch the video

Two 72.0-kg hockey players, each skating at 5.45 m/s, collide and stick together. The angle between their initial directions is 115°. What is their speed after the collision?



GIVEN $m_1 = m_2 = 72.0\ \text{kg}$ $v_1 = v_2 = 5.45\ \text{m/s}$ $\text{angle} = 115^\circ$ **FIND** v_f

SOLUTION

Choose the x axis along the bisector, so each player moves at 115°/2 = 57.5° to it. The y-components are equal and opposite, so they cancel.

Conserve the x-component of momentum:

$$mv\cos 57.5^\circ + mv\cos 57.5^\circ = (2m)\,v_f$$

The mass cancels:

$$v_f = v\cos 57.5^\circ = (5.45)\cos 57.5^\circ$$

ANSWER $v_f \approx 2.93\ \text{m/s}$

PROBLEM 13

Putty hits a hanging block

Perfectly inelastic | Then energy conservation

Watch the video

A 0.420-kg block hangs from the ceiling by a string. A 0.0750-kg wad of putty is thrown straight up, hits the bottom of the block at 5.74 m/s and sticks. (a) Is the mechanical energy of the system conserved? (b) How high does the putty–block system rise above the block's original position?

GIVEN $M = 0.420\ \text{kg}$ $m = 0.0750\ \text{kg}$ $v = 5.74\ \text{m/s}$ FIND h

SOLUTION

(a) The putty sticks, so the collision is perfectly inelastic. Mechanical energy is **not** conserved during the collision:

$$K_i = \tfrac{1}{2}(0.0750)(5.74)^2 = 1.24\ \text{J} \longrightarrow K_f = \tfrac{1}{2}(0.495)(0.870)^2 = 0.187\ \text{J}$$

The missing 1.05 J is not destroyed: it becomes thermal energy, sound and the permanent deformation of the putty (total energy is conserved).

(b) Split the motion into two stages, each with its own conservation law:

Stage 1 — during the collision (very short): momentum is conserved, because the impulse of gravity in that short time is negligible. Kinetic energy is not conserved.

$$mv = (m+M)\,v_f \;\Rightarrow\; v_f = \frac{(0.0750)(5.74)}{0.495} = 0.870\ \text{m/s}$$

Stage 2 — after the collision (rising): no more sticking or deforming, and only gravity does work, so mechanical energy is conserved. Momentum is not conserved here — gravity slows the system down.

$$\tfrac{1}{2}(m+M)v_f^2 = (m+M)\,g\,h \;\Rightarrow\; h = \frac{v_f^2}{2g} = \frac{(0.870)^2}{2(9.81)}$$

ANSWER **(a)** No **(b)** $h \approx 3.9\ \text{cm}$

PROBLEM 14

Putty hits a block on a spring

Perfectly inelastic · Spring energy

Watch the video

A 0.430-kg block is attached to a horizontal spring (k = 20.0 N/m) at its equilibrium length, on a frictionless surface. A 0.0500-kg wad of putty hits the block horizontally at 2.30 m/s and sticks. How far does the putty–block system compress the spring?

GIVEN $M = 0.430\ \text{kg}$ $m = 0.0500\ \text{kg}$ $v = 2.30\ \text{m/s}$ $k = 20.0\ \text{N/m}$ FIND $x_{\max}$

SOLUTION

Stage 1 — during the collision: the collision is so short that the spring has not yet compressed, so it exerts no force and horizontal momentum is conserved. Kinetic energy is not (the putty sticks).

$$mv = (m+M)\,v_f \;\Rightarrow\; v_f = \frac{(0.0500)(2.30)}{0.480} = 0.240\ \text{m/s}$$

Stage 2 — after the collision ends: energy conservation starts here, with the combined speed v_f (not the putty's 2.30 m/s). The surface is frictionless, so all the kinetic energy becomes spring energy:

$$\tfrac{1}{2}(m+M)v_f^2 = \tfrac{1}{2}kx^2 \;\Rightarrow\; x = v_f\sqrt{\frac{m+M}{k}} = (0.240)\sqrt{\frac{0.480}{20.0}}$$

ANSWER $x \approx 3.71\ \text{cm}$

PROBLEM 15

Hammer strikes a nail

Elastic collision · 1D

Watch the video

The collision between a hammer and a nail is approximately elastic. Find the kinetic energy acquired by a 12-g nail when it is struck by a 550-g hammer moving at 4.5 m/s.

GIVEN $m_1 = 0.550\ \text{kg (hammer)}$ $m_2 = 0.012\ \text{kg (nail)}$ $v_{1i} = 4.5\ \text{m/s},\ v_{2i} = 0$ FIND K_{2f}

SOLUTION

Conservation of momentum and of kinetic energy, solved together, give for a target at rest:

$$v_{2f} = \frac{2m_1}{m_1+m_2}\,v_{1i} = \frac{2(0.550)}{0.562}(4.5) = 8.81\ \text{m/s}$$

Kinetic energy of the nail:

$$K_{2f} = \tfrac{1}{2}m_2 v_{2f}^2 = \tfrac{1}{2}(0.012)(8.81)^2$$

Note: "elastic" is an approximation. A real hammer blow loses some energy to sound, heat and deforming the nail and wood, so 0.47 J is an upper estimate.

ANSWER $K \approx 0.47\ \text{J}$

Answer summary

Problem	Topic	Key physics idea	Final answer(s)
1	Dog, cat and owner	Momentum is a vector — add components	$v = 0.980\ \text{m/s},\ 77.2^\circ$ north of east
2	Dropped baseball	p = mv, then free-fall kinematics	$h \approx 1.38\ \text{m}$
4	Kicking a soccer ball	Impulse = average force × time	$J \approx 7.44\ \text{N·s}$
6	Croquet ball and mallet	Impulse–momentum theorem	$\Delta t \approx 7.0 \times 10^{-3}\ \text{s} = 7.0\ \text{ms}$
7	Spiking a volleyball	Impulse–momentum theorem (signs)	$m \approx 0.33\ \text{kg}$
8	Bounce pass	Only the momentum component that changes gets an impulse	$\vec{J} \approx 2.7\ \text{N·s}$, straight up
10	Heading a soccer ball	2D impulse — find the quadrant first	**(a)** 121° from the +x axis **(b)** $J \approx 3.0\ \text{N·s}$
11	Two ice skaters push off	Momentum conserved from rest (P = 0)	$m_2 \approx 31\ \text{kg}$
12	Two hockey players collide	2D perfectly inelastic collision	$v_f \approx 2.93\ \text{m/s}$
13	Putty hits a hanging block	Momentum in the collision, then energy after it	**(a)** No **(b)** $h \approx 3.9\ \text{cm}$
14	Putty hits a block on a spring	Momentum in the collision, then spring energy	$x \approx 3.71\ \text{cm}$
15	Hammer strikes a nail	1D elastic collision, target at rest	$K \approx 0.47\ \text{J}$

g = 9.81 m/s² is used throughout. East = +x and north = +y unless stated otherwise. Answers are rounded to the significant figures given in each problem.