ALNOJOUM ACADEMY
أكاديمية النجوم

Chapter 8
Potential Energy and Conservation of Energy

Written solutions to the textbook problems

PHYS 105 | Prince Sultan University | 6 solved problems

What this chapter covers

8.1 Potential Energy

8.2 Conservation of Mechanical Energy

8.4 Work Done on a System by an External Force

8.5 Conservation of Energy and Power

All key formulas are on the next page.

Watch out

- $\Delta U = U_f - U_i$ is negative when the object moves down or the spring relaxes.
- The value of U depends on where you put y = 0, but ΔU does not.
- In $U = \frac{1}{2}kx^2$, convert x to meters (cm → m, mm → m) before squaring.
- Use $K_2 + U_2 = K_1 + U_1$ only when nonconservative forces do no work; otherwise use $W_{nc} = \Delta E_{mec}$.

Problems in this file

1 Cliff diver

2 Standing on Mt. Everest

3 The wing of a hawkmoth

4 Soap dispenser spring

5 Stretching a spring

6 Pendulum swinging from A to B

Every problem has a QR code that opens its full video solution.

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Chapter 8

Key formulas

8.1 Potential Energy

Work by a conservative force	$W_c = -\Delta U$	Gravity and the spring force are conservative
Change in gravitational PE	$\Delta U = mg\,(y_f - y_i)$	y is measured upward
Gravitational PE	$U(y) = mgy$	Relative to the chosen reference level y = 0
Elastic (spring) PE	$U(x) = \frac{1}{2}kx^2$	x from the relaxed length; same U for stretch or compression
Hooke's law	$F_s = -kx \quad (F = kx)$	Spring force opposes the displacement

8.2 Conservation of Mechanical Energy

Mechanical energy	$E_{\text{mec}} = K + U$	
Energy exchange	$\Delta K = -\Delta U$	Only conservative forces do work
Conservation of E_{mec}	$K_2 + U_2 = K_1 + U_1$	Isolated system, only conservative forces
Equivalent form	$\Delta E_{\text{mec}} = \Delta K + \Delta U = 0$	

8.4 Work Done on a System by an External Force

No friction	$W = \Delta E_{\text{mec}} = \Delta K + \Delta U$	W = work by the external force
Work done by kinetic friction	$W_f = -f_k\,d, \quad f_k = \mu_k N$	Always negative
With nonconservative forces	$W_{\text{nc}} = \Delta E_{\text{mec}} = \Delta K + \Delta U$	$W_{nc} < 0$ means mechanical energy is lost
With friction (thermal energy)	$W = \Delta E_{\text{mec}} + \Delta E_{\text{th}}, \quad \Delta E_{\text{th}} = f_k d$	Same idea, written with thermal energy

8.5 Conservation of Energy and Power

Total energy of a system	$W = \Delta E = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}}$	
Isolated system	$\Delta E = 0$	Total energy cannot change
Average power	$P_{\text{avg}} = \dfrac{\Delta E}{\Delta t}$	1 W = 1 J/s
Instantaneous power	$P = \dfrac{dE}{dt} = \vec{F} \cdot \vec{v} = Fv\cos\phi$	φ = angle between F and v

Conservative vs. nonconservative forces

	CONSERVATIVE	NONCONSERVATIVE
Examples	Gravity, spring force	Kinetic friction, air drag
Work depends on	Initial and final positions only	The path taken
Work around a closed path	Zero	Not zero (negative for friction)
Potential energy	Has a potential energy U	No U — energy goes to thermal energy

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PROBLEM 1

Cliff diver

Gravitational potential energy

Watch the video

A cliff diver drops 46 m to the water, and his gravitational potential energy decreases by 25,000 J. Find his weight in newtons.

GIVEN $h = 46\ \text{m}$ $\Delta U = -25{,}000\ \text{J}$ **FIND** mg

SOLUTION

Take the water level as y = 0, so U_i = mgh and U_f = 0.

$$\Delta U = U_f - U_i = 0 - mgh$$

Substitute and solve for the weight mg:

$$-25{,}000 = -\,mg\,(46) \;\Rightarrow\; mg = \frac{25{,}000}{46}$$

ANSWER $mg \approx 543\ \text{N}$

PROBLEM 2

Standing on Mt. Everest

Gravitational potential energy · Reference level

Watch the video

Find the gravitational potential energy of an 88-kg person standing on top of Mt. Everest, at an altitude of 8848 m. Take sea level as y = 0.

GIVEN $m = 88\ \text{kg}$ $y = 8848\ \text{m}$ $y_0 = 0\ (\text{sea level})$ **FIND** U

SOLUTION

$$U = mgy = (88)(9.81)(8848) = 7{,}638{,}301\ \text{J}$$

ANSWER $U \approx 7.64 \times 10^6\ \text{J}$

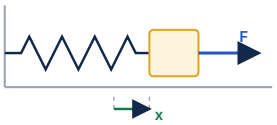
PROBLEM 3

The wing of a hawkmoth

Hooke's law · Spring potential energy

Watch the video

Experiments on the wing of a hawkmoth show that it deflects by x = 4.8 mm when a force F = 3.0 mN is applied at its tip. Treating the wing as an ideal spring, find (a) its force constant, (b) the energy stored in the deflected wing, and (c) the force needed at the tip to store twice that energy.



GIVEN $x = 4.8 \times 10^{-3}\ \text{m}$ $F = 3.0 \times 10^{-3}\ \text{N}$ **FIND** $k,\ U,\ F_{2U}$

SOLUTION

(a) Hooke's law, F = kx:

$$k = \frac{F}{x} = \frac{3.0 \times 10^{-3}}{4.8 \times 10^{-3}} = 0.625\ \text{N/m}$$

(b)

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(0.625)(4.8 \times 10^{-3})^2 = 7.2 \times 10^{-6}\ \text{J}$$

(c) Find the deflection that stores 2U = 1.44 × 10^{-5} J, then the force:

$$x' = \sqrt{\frac{2(2U)}{k}} = \sqrt{\frac{2(1.44 \times 10^{-5})}{0.625}} = 6.8 \times 10^{-3}\ \text{m}$$

$$F' = kx' = (0.625)(6.8 \times 10^{-3}) = 4.2 \times 10^{-3}\ \text{N}$$

Shortcut: U ∝ x², so doubling U multiplies x (and F) by √2 → F' = √2 (3.0 mN).

ANSWER **(a)** $k = 0.625\ \text{N/m}$ **(b)** $U = 7.2 \times 10^{-6}\ \text{J}$ **(c)** $F \approx 4.2 \times 10^{-3}\ \text{N}$

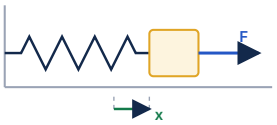
PROBLEM 4

Soap dispenser spring

Spring potential energy

Watch the video

Pushing the pump of a soap dispenser compresses a small spring. A compression of 0.50 cm stores 0.0025 J of potential energy. (a) Find the force constant. (b) What compression is needed to store 0.0084 J?



GIVEN $x = 0.50\ \text{cm} = 5.0 \times 10^{-3}\ \text{m}$ $U = 0.0025\ \text{J}$ $U' = 0.0084\ \text{J}$ FIND $k,\ x'$

SOLUTION

(a) Solve U = ½kx² for k (convert cm → m first):

$$k = \frac{2U}{x^2} = \frac{2(0.0025)}{(5.0 \times 10^{-3})^2} = 200\ \text{N/m}$$

(b) Solve for x:

$$x' = \sqrt{\frac{2U'}{k}} = \sqrt{\frac{2(0.0084)}{200}} = 9.2 \times 10^{-3}\ \text{m}$$

ANSWER **(a)** $k = 200\ \text{N/m}$ **(b)** $x' \approx 0.92\ \text{cm}$

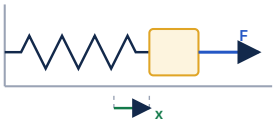
PROBLEM 5

Stretching a spring

Hooke's law · Spring potential energy

Watch the video

A force of 4.1 N stretches a spring by 1.4 cm. How far must it be stretched for its potential energy to be (a) 0.020 J, (b) 0.080 J?



GIVEN $F = 4.1\ \text{N}$ $x_0 = 1.4 \times 10^{-2}\ \text{m}$ FIND $x_a,\ x_b$

SOLUTION

First find k from Hooke's law:

$$k = \frac{F}{x_0} = \frac{4.1}{1.4 \times 10^{-2}} = 293\ \text{N/m}$$

(a)

$$x_a = \sqrt{\frac{2U}{k}} = \sqrt{\frac{2(0.020)}{293}} = 1.17 \times 10^{-2}\ \text{m}$$

(b) The energy is 4 times larger, so the stretch is √4 = 2 times larger:

$$x_b = \sqrt{\frac{2(0.080)}{293}} = 2.34 \times 10^{-2}\ \text{m}$$

ANSWER **(a)** $x \approx 1.2\ \text{cm}$ **(b)** $x \approx 2.3\ \text{cm}$

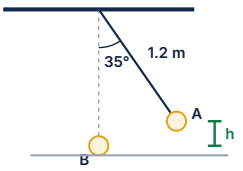
PROBLEM 6

Pendulum swinging from A to B

Gravitational potential energy | Geometry of the pendulum

Watch the video

A 0.33-kg pendulum bob hangs on a 1.2-m string. Point A is where the string makes 35° with the vertical, and point B is the lowest point. Find the change in gravitational potential energy as the bob swings from A to B.



GIVEN $m = 0.33\ \text{kg}$ $L = 1.2\ \text{m}$ $\theta = 35°$ **FIND** $\Delta U_{A\to B}$

SOLUTION

The vertical distance from the pivot to A is L cos θ, so the height of A above B is:

$$h = L - L\cos\theta = 1.2 - 1.2\cos 35° = 1.2 - 0.98 = 0.22\ \text{m}$$

Take y = 0 at B:

$$U_A = mgh = (0.33)(9.81)(0.22) = 0.70\ \text{J}, \qquad U_B = 0$$

$$\Delta U = U_B - U_A = 0 - 0.70$$

ANSWER $\Delta U \approx -0.70\ \text{J}$

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Answer summary

Problem	Topic	Final answer(s)
1	Cliff diver	$mg \approx 543\ \text{N}$
2	Standing on Mt. Everest	$U \approx 7.64 \times 10^{6}\ \text{J}$
3	The wing of a hawkmoth	**(a)** $k = 0.625\ \text{N/m}$ **(b)** $U = 7.2 \times 10^{-6}\ \text{J}$ **(c)** $F \approx 4.2 \times 10^{-3}\ \text{N}$
4	Soap dispenser spring	**(a)** $k = 200\ \text{N/m}$ **(b)** $x' \approx 0.92\ \text{cm}$
5	Stretching a spring	**(a)** $x \approx 1.2\ \text{cm}$ **(b)** $x \approx 2.3\ \text{cm}$
6	Pendulum swinging from A to B	$\Delta U \approx -0.70\ \text{J}$

g = 9.81 m/s² is used throughout. Convert cm and mm to meters before using U = ½kx². Answers are rounded to the significant figures given in each problem.

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