ALNOJOUM ACADEMY

أكاديمية النجوم

Chapter 7
Work and Kinetic Energy

Written solutions to the textbook problems

PHYS 105 | Prince Sultan University | 15 solved problems

Key formulas

Work by a constant force

$$W = Fd\cos\theta$$

Work as a dot product

$$W = \vec{F}\cdot\vec{d} = F_x d_x + F_y d_y$$

Kinetic energy

$$K = \tfrac{1}{2}mv^2$$

Work–energy theorem

$$W_{\text{net}} = \Delta K = K_f - K_i$$

Kinetic friction

$$f_k = \mu_k N$$

Unit of work and energy

$$1\ \text{J} = 1\ \text{N}\cdot\text{m}$$

Sign of work

$0^\circ \le \theta < 90^\circ$	$W > 0$	Force helps the motion
$\theta = 90^\circ$	$W = 0$	Force ⊥ displacement
$90^\circ < \theta < 180^\circ$	$W < 0$	Force opposes the motion

Problems in this file

1 Pushing a bale of hay
2 Sliding a suitcase against friction
3 Lifting, holding and lowering a paint can
4 Pulling a wagon by an inclined handle
5 Pulling a crate with a rope
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9 Pulling a dog on a leash
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13 Skylab film vault
14 Kinetic energy of a bullet
15 Falling object
16 Car crossing a sandy stretch

Every problem has a QR code that opens its full video solution.

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PROBLEM 1

Pushing a bale of hay

Work by a constant force

Watch the video

A 26-kg bale of hay is pushed 3.9 m across a level barn floor by a horizontal force of 88 N. Find the work done by the pushing force.

GIVEN $m = 26\ \text{kg}$ $F = 88\ \text{N}$ $d = 3.9\ \text{m}$ $\theta = 0^\circ$ **FIND** W

SOLUTION

The force is horizontal and in the same direction as the motion, so θ = 0° and cos 0° = 1.

$$W = Fd\cos\theta = (88)(3.9)\cos 0^\circ$$

The mass is not needed here.

ANSWER $W = 343\ \text{J}\ \ (3.43 \times 10^2\ \text{J})$

PROBLEM 2

Sliding a suitcase against friction

Kinetic friction · Work by friction

Watch the video

A 71.5-kg suitcase rests on a level floor where the coefficient of kinetic friction is 0.272. How far can it be pushed (at constant speed) if 642 J of work is done?

GIVEN $\mu_k = 0.272$ $m = 71.5\ \text{kg}$ $W = 642\ \text{J}$ **FIND** d

SOLUTION

On a level floor N = mg, so the kinetic friction force is:

$$f_k = \mu_k N = \mu_k mg = (0.272)(71.5)(9.81) = 191\ \text{N}$$

At constant speed the pushing force equals f_k, so the work done against friction equals the applied work:

$$W = f_k\, d \ \Rightarrow\ d = \frac{W}{f_k} = \frac{642}{191}$$

ANSWER $d \approx 3.37\ \text{m}$

PROBLEM 3

Lifting, holding and lowering a paint can

Work against gravity · Zero displacement

Watch the video

A 3.4-kg can of paint is lifted from the ground to a height of 1.8 m. (a) Work done by you while lifting? (b) Work done while holding it still for 30 s? (c) Work done by you while lowering it back to the ground?

GIVEN $m = 3.4\ \text{kg}$ $h = 1.8\ \text{m}$ $t = 30\ \text{s}$ FIND $W_a,\ W_b,\ W_c$

SOLUTION

(a) Lifting slowly: your force F = mg points up, displacement points up (θ = 0°).

$$W_a = mgh = (3.4)(9.81)(1.8)$$

(b) Holding the can still: the displacement is zero, so no work is done regardless of time.

$$W_b = F\,(0) = 0$$

(c) Lowering: your force still points up, but the displacement points down (θ = 180°).

$$W_c = mgh\cos 180^\circ = -(3.4)(9.81)(1.8)$$

ANSWER **(a)** $W_a = 60\ \text{J}$ **(b)** $W_b = 0$ **(c)** $W_c = -60\ \text{J}$

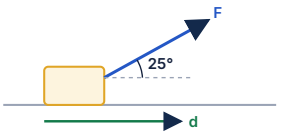
PROBLEM 4

Pulling a wagon by an inclined handle

Force at an angle

Watch the video

A child pulls a wagon at constant speed with a force of 16 N for 10.0 m. The handle makes 25° with the horizontal. Find the work done by the child.



GIVEN $F = 16\ \text{N}$ $d = 10.0\ \text{m}$ $\theta = 25^\circ$ FIND W

SOLUTION

Only the horizontal component F cos θ does work along the displacement.

$$W = Fd\cos\theta = (16)(10.0)\cos 25^\circ$$

ANSWER $W = 145\ \text{J}$

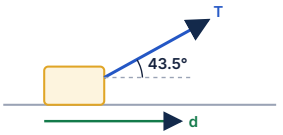
PROBLEM 5

Pulling a crate with a rope

Force at an angle

Watch the video

A 51-kg crate is pulled at constant speed across a rough floor by a rope at 43.5° above the horizontal. The rope tension is 115 N. Find the work done by the rope over 8.0 m.



GIVEN $T = 115\ \text{N}$ $d = 8.0\ \text{m}$ $\theta = 43.5°$ FIND W_T

SOLUTION

The work done by the tension depends only on T, d and the angle — the mass and the friction are not needed.

$$W_T = Td\cos\theta = (115)(8.0)\cos 43.5°$$

Note: because the speed is constant, friction does an equal amount of negative work, so the net work is zero.

ANSWER $W_T = 667\ \text{J}$

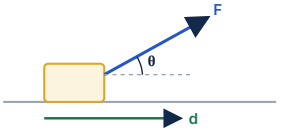
PROBLEM 6

Pushing a mop handle

Force at an angle · Effect of angle

Watch the video

A janitor pushes a mop handle with a force of 50.0 N. (a) If the handle makes 55° with the horizontal, find the work done in moving the mop 0.50 m. (b) If the angle is increased to 65°, does the work increase, decrease, or stay the same?



GIVEN $F = 50.0\ \text{N}$ $d = 0.50\ \text{m}$ $\theta_1 = 55°,\ \theta_2 = 65°$ FIND W

SOLUTION

(a)

$$W_1 = Fd\cos\theta_1 = (50.0)(0.50)\cos 55° = 14.3\ \text{J}$$

(b) A larger angle means a smaller horizontal component (cos 65° < cos 55°).

$$W_2 = (50.0)(0.50)\cos 65° = 10.6\ \text{J}$$

ANSWER **(a)** $W = 14.3\ \text{J}$ **(b)** Work decreases $(10.6\ \text{J})$

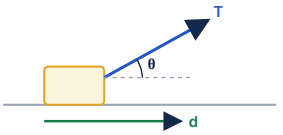
PROBLEM 7

Plane towing a glider

Finding the angle from work

Watch the video

A small plane tows a glider at constant speed and altitude. The plane does 2.00 × 10^5 J of work over 145 m, and the tension in the tow rope is 2560 N. Find the angle between the rope and the horizontal.



GIVEN $W = 2.00 \times 10^5\ \text{J}$ $T = 2560\ \text{N}$ $d = 145\ \text{m}$ FIND θ

SOLUTION

Solve the work equation for cos θ:

$$W = Td\cos\theta \ \Rightarrow\ \cos\theta = \frac{W}{Td} = \frac{2.00 \times 10^5}{(2560)(145)} = 0.539$$

$$\theta = \cos^{-1}(0.539)$$

ANSWER $\theta \approx 57.4^\circ$

PROBLEM 8

Skateboarder pulled by a bicycle

Dot product · Work by a vector force

Watch the video

A skateboarder is pulled by a rope tied to a bicycle. Her velocity is v = (4.1 m/s) x̂ and the rope force on her is F = (17 N) x̂ + (12 N) ŷ. (a) Find the work done on the skateboarder by the rope in 25 s. (b) If the bicycle moves with the same velocity, find the work done by the rope on the bicycle in 25 s.

GIVEN $\vec{v} = (4.1\ \text{m/s})\,\hat{x}$ $\vec{F} = (17\ \text{N})\hat{x} + (12\ \text{N})\hat{y}$ $t = 25\ \text{s}$ FIND W_{skater}, W_{bike}

SOLUTION

Constant velocity, so the displacement is:

$$\vec{d} = \vec{v}\,t = (4.1)(25)\,\hat{x} = (102.5\ \text{m})\,\hat{x}$$

(a) Use the dot product; only the x-components multiply because the displacement has no y-component.

$$W = \vec{F}\cdot\vec{d} = (17)(102.5) + (12)(0) = 1742.5\ \text{J}$$

(b) By Newton's third law the rope pulls the bicycle with the opposite force, −F, and the bicycle has the same displacement.

$$W_{\text{bike}} = (-\vec{F})\cdot\vec{d} = -1742.5\ \text{J}$$

ANSWER **(a)** $W \approx 1.74 \times 10^3\ \text{J}$ **(b)** $W \approx -1.74 \times 10^3\ \text{J}$

PROBLEM 9

Pulling a dog on a leash

Dot product · Unit vectors

Watch the video

Susan pulls on her dog's leash with a constant force F = (12.2 N) $\hat{x}$ + (1.1 N) $\hat{y}$. Find the work she does on the dog if its displacement is (a) (0.25 m) $\hat{x}$, (b) (0.25 m) $\hat{y}$, (c) (−0.50 m) $\hat{x}$ + (−0.50 m) $\hat{y}$.

GIVEN $\vec{F} = (12.2\ \text{N})\hat{x} + (1.1\ \text{N})\hat{y}$ FIND $W = \vec{F} \cdot \vec{d}$

SOLUTION

Use $\hat{x}\cdot\hat{x} = \hat{y}\cdot\hat{y} = 1$ and $\hat{x}\cdot\hat{y} = 0$, so $W = F_x d_x + F_y d_y$.

(a)

$$W = (12.2)(0.25) + (1.1)(0) = 3.05\ \text{J}$$

(b)

$$W = (12.2)(0) + (1.1)(0.25) = 0.275\ \text{J}$$

(c)

$$W = (12.2)(-0.50) + (1.1)(-0.50) = -6.10 - 0.55$$

ANSWER **(a)** $W = 3.05\ \text{J}$ **(b)** $W = 0.275\ \text{J}$ **(c)** $W = -6.65\ \text{J}$

PROBLEM 10

Pitcher and catcher

Work–energy theorem · Conceptual

Watch the video

A pitcher throws a ball at 90 mi/h and the catcher stops it in her glove. (a) Is the work done on the ball by the pitcher positive, negative, or zero? (b) What about the work done by the catcher?

FIND Sign of W

SOLUTION

Use the work–energy theorem:

$$W = \Delta K = K_f - K_i$$

(a) The ball starts at rest ($K_i = 0$) and leaves the hand moving ($K_f > 0$), so $\Delta K > 0$. The pitcher's force is in the direction of motion.

(b) The ball arrives moving ($K_i > 0$) and ends at rest ($K_f = 0$), so $\Delta K < 0$. The catcher's force opposes the motion.

ANSWER **(a)** Positive **(b)** Negative

PROBLEM 12

Accelerating a runner

Work–energy theorem

Watch the video

How much work is required for a 73-kg runner to accelerate from rest to 7.7 m/s?

GIVEN $m = 73\ \text{kg}$ $v_i = 0$ $v_f = 7.7\ \text{m/s}$ **FIND** W

SOLUTION

Initial and final kinetic energies:

$$K_i = 0, \qquad K_f = \tfrac{1}{2}mv_f^2 = \tfrac{1}{2}(73)(7.7)^2 = 2.16 \times 10^3\ \text{J}$$

Work–energy theorem:

$$W = \Delta K = K_f - K_i$$

ANSWER $W \approx 2.16 \times 10^3\ \text{J}$

PROBLEM 13

Skylab film vault

Kinetic energy

Watch the video

When Skylab re-entered the atmosphere in 1979, one fragment — a 1770-kg lead-lined film vault — landed at an estimated 120 m/s. Find its kinetic energy on landing.

GIVEN $m = 1770\ \text{kg}$ $v = 120\ \text{m/s}$ **FIND** K

SOLUTION

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1770)(120)^2 = 1.27 \times 10^7\ \text{J}$$

ANSWER $K \approx 1.27 \times 10^7\ \text{J}$

PROBLEM 14

Kinetic energy of a bullet

Kinetic energy · K ∝ v²

Watch the video

A 9.50-g bullet moves at 1.30 km/s. (a) Find its kinetic energy. (b) Find the kinetic energy if its speed is halved. (c) If its speed is doubled?

GIVEN $m = 9.50 \times 10^{-3}\ \text{kg}$ $v = 1.30 \times 10^{3}\ \text{m/s}$ FIND K

SOLUTION

(a) Convert to SI first (g → kg, km/s → m/s):

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(9.50 \times 10^{-3})(1.30 \times 10^{3})^2 = 8.03 \times 10^{3}\ \text{J}$$

(b) K ∝ v², so halving v divides K by 4:

$$K_2 = \frac{K}{4} = \frac{8.03 \times 10^{3}}{4} = 2.01 \times 10^{3}\ \text{J}$$

(c) Doubling v multiplies K by 4:

$$K_3 = 4K = 3.21 \times 10^{4}\ \text{J}$$

ANSWER **(a)** $8.03 \times 10^{3}\ \text{J}$ **(b)** $2.01 \times 10^{3}\ \text{J}$ **(c)** $3.21 \times 10^{4}\ \text{J}$

PROBLEM 15

Falling object

Kinetic energy · Work–energy theorem

Watch the video

At t = 1.0 s a 0.40-kg object is falling at 6.0 m/s. At t = 2.0 s its kinetic energy is 25 J. (a) Kinetic energy at t = 1.0 s? (b) Speed at t = 2.0 s? (c) Work done on the object between t = 1.0 s and t = 2.0 s?

GIVEN $m = 0.40\ \text{kg}$ $v_1 = 6.0\ \text{m/s}$ $K_2 = 25\ \text{J}$ FIND $K_1,\ v_2,\ W$

SOLUTION

(a)

$$K_1 = \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}(0.40)(6.0)^2 = 7.2\ \text{J}$$

(b) Solve K = ½mv² for v:

$$v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(25)}{0.40}} = 11.2\ \text{m/s}$$

(c)

$$W = \Delta K = K_2 - K_1 = 25 - 7.2 = 17.8\ \text{J}$$

ANSWER **(a)** $7.2\ \text{J}$ **(b)** $11\ \text{m/s}\ (11.2)$ **(c)** $17.8\ \text{J}$

PROBLEM 16

Car crossing a sandy stretch

Work–energy theorem · Average force

Watch the video

A 1100-kg car coasts on a level road at 19 m/s. After crossing a 32-m sandy stretch its speed drops to 12 m/s. (a) Was the net work on the car positive, negative, or zero? (b) Find the magnitude of the average net force in the sandy section.

GIVEN $m = 1100\ \text{kg}$ $v_i = 19\ \text{m/s}$ $v_f = 12\ \text{m/s}$ $d = 32\ \text{m}$ **FIND** $W_{\text{net}},\ F_{\text{av}}$

SOLUTION

(a) The speed decreased, so the kinetic energy decreased (ΔK < 0) — the net work is **negative**.

$$W_{\text{net}} = \Delta K = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) = \tfrac{1}{2}(1100)\left(12^2 - 19^2\right) = -1.19 \times 10^5\ \text{J}$$

(b) The net force opposes the motion (θ = 180°):

$$|W_{\text{net}}| = F_{\text{av}}\, d \ \Rightarrow\ F_{\text{av}} = \frac{1.19 \times 10^5}{32}$$

ANSWER **(a)** Negative, $W_{\text{net}} = -1.19 \times 10^5\ \text{J}$ **(b)** $F_{\text{av}} \approx 3.73 \times 10^3\ \text{N}$

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Answer summary

Problem	Topic	Final answer(s)
1	Pushing a bale of hay	$W = 343\ \text{J}\ \ (3.43 \times 10^2\ \text{J})$
2	Sliding a suitcase against friction	$d \approx 3.37\ \text{m}$
3	Lifting, holding and lowering a paint can	**(a)** $W_a = 60\ \text{J}$ **(b)** $W_b = 0$ **(c)** $W_c = -60\ \text{J}$
4	Pulling a wagon by an inclined handle	$W = 145\ \text{J}$
5	Pulling a crate with a rope	$W_T = 667\ \text{J}$
6	Pushing a mop handle	**(a)** $W = 14.3\ \text{J}$ **(b)** $\text{Work decreases}\ (10.6\ \text{J})$
7	Plane towing a glider	$\theta \approx 57.4^\circ$
8	Skateboarder pulled by a bicycle	**(a)** $W \approx 1.74 \times 10^3\ \text{J}$ **(b)** $W \approx -1.74 \times 10^3\ \text{J}$
9	Pulling a dog on a leash	**(a)** $W = 3.05\ \text{J}$ **(b)** $W = 0.275\ \text{J}$ **(c)** $W = -6.65\ \text{J}$
10	Pitcher and catcher	**(a)** Positive **(b)** Negative
12	Accelerating a runner	$W \approx 2.16 \times 10^3\ \text{J}$
13	Skylab film vault	$K \approx 1.27 \times 10^7\ \text{J}$
14	Kinetic energy of a bullet	**(a)** $8.03 \times 10^3\ \text{J}$ **(b)** $2.01 \times 10^3\ \text{J}$ **(c)** $3.21 \times 10^4\ \text{J}$
15	Falling object	**(a)** $7.2\ \text{J}$ **(b)** $11\ \text{m/s}\ (11.2)$ **(c)** $17.8\ \text{J}$
16	Car crossing a sandy stretch	**(a)** $\text{Negative},\ W_{\text{net}} = -1.19 \times 10^5\ \text{J}$ **(b)** $F_{\text{av}} \approx 3.73 \times 10^3\ \text{N}$

g = 9.81 m/s² is used throughout. Answers are rounded to the number of significant figures given in each problem.