ALNOJOUM ACADEMY

أكاديمية النجوم

Chapter 6
Force and Motion — II

Written solutions to the textbook problems

PHYS 105 | Prince Sultan University | 17 solved problems

What this chapter covers

6.1 Friction

6.2 Uniform Circular Motion

+ Springs and Equilibrium (book problems)

All key formulas are on the next page.

Watch out

- Free-body diagram: draw only the forces acting **on** the body (W, N, T, f, applied F) — never ma or v; N is perpendicular to the surface.
- $f_s = \mu_s N$ only at the point of slipping; otherwise static friction equals the applied force.
- N is not always mg: a push down adds F sin θ, a pull up subtracts it, and on an incline N = mg cos θ.
- Friction always opposes sliding (or the tendency to slide).
- In a spring problem, x is the stretch or compression from the natural length — not the total length.
- For equilibrium with angled ropes, write $\Sigma F_x = 0$ and $\Sigma F_y = 0$ separately.

Problems in this file

1 Sliding into third base

2 A child on a playground slide

3 Accelerating a Porsche

4 Sliding a book on a table

5 Pushing a crate at 21° below horizontal

6 A crate on a tilting ramp

7 A sprinter's start

8 Lifting a bucket from a well

9 A mass on a vertical spring

10 A car's rear suspension

11 Tension in a hammock

12 Stretching and compressing a spring

13 A spring holding a block against a wall

14 A picture hanging from two strings

15 Two blocks tied to a wall on an incline

16 Balancing a block on a smooth incline

17 Three connected masses

Every problem has a QR code that opens its full video solution.

Mr. Abdullah Abdelfattah · alnojoumacademy.com · @Chem31Phys

Chapter 6

Key formulas

Organized by the sections of the Chapter 6 slides (6.1 – 6.2), plus springs and equilibrium from the book problems.

6.1 Friction

Static friction	$f_s \le \mu_s N$	Only as large as needed to prevent sliding
Maximum static friction	$f_{s,max} = \mu_s N$	Object about to slide
Kinetic friction	$f_k = \mu_k N$	Opposes the sliding; $\mu_k < \mu_s$
Sliding to a stop (flat)	$a = -\mu_k g$	
Incline: about to slip	$\mu_s = \tan\theta_{slip}$	
Incline: sliding down	$a = g(\sin\theta - \mu_k \cos\theta)$	N = mg cos θ

6.2 Uniform Circular Motion

Centripetal acceleration	$a_c = \dfrac{v^2}{r}$	Directed toward the center
Centripetal force	$F_c = \dfrac{mv^2}{r}$	The net force toward the center

+ Springs and Equilibrium (book problems)

Hooke's law	$F = -kx$	Restoring force, opposite to x
Hanging/resting mass	$kx = mg$	
Equilibrium	$\Sigma F_x = 0, \quad \Sigma F_y = 0$	Resolve each tension into components

◆ Which friction acts?

	FRICTION	RESULT
Object at rest, no push	0	—
Push below the limit	f_s = F (static)	Stays at rest
Push = $\mu_s N$	$f_{s,max}$	About to move
Object sliding	$f_k = \mu_k N$	Moves (a may be ≠ 0)

PROBLEM 1

Sliding into third base

Kinetic friction · Kinematics

Watch the video

A baseball player slides into third base with an initial speed of 4.0 m/s. If the coefficient of kinetic friction between the player and the ground is 0.46, how far does the player slide before coming to rest?

GIVEN $v_0 = 4.0\ \text{m/s}$ $v = 0$ $\mu_k = 0.46$ FIND Δx

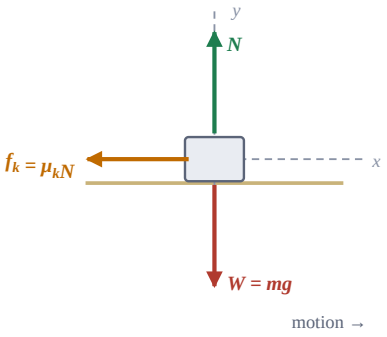
SOLUTION

N = mg, and friction is the only horizontal force:

$$-\mu_k mg = ma \rightarrow a = -\mu_k g = -0.46(9.81) = -4.51\ \text{m/s}^2$$

$$\Delta x = \frac{0 - 4.0^2}{2(-4.51)} = 1.77\ \text{m}$$

FREE-BODY DIAGRAM



ANSWER $1.77\ \text{m}$

PROBLEM 2

A child on a playground slide

Kinetic friction · Inclined plane

Watch the video

A child goes down a playground slide with an acceleration of 1.26 m/s². Find the coefficient of kinetic friction between the child and the slide if the slide is inclined at an angle of 33.0° below the horizontal.

GIVEN $a = 1.26\ \text{m/s}^2$ $\theta = 33.0^\circ$ FIND μ_k

SOLUTION

Perpendicular to the slide:

$$N = mg\cos\theta$$

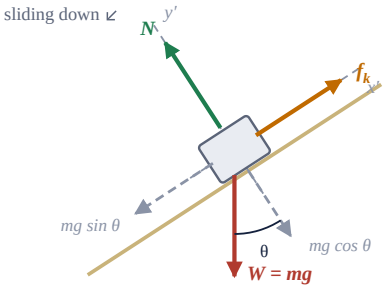
Along the slide (down is +):

$$mg\sin\theta - \mu_k mg\cos\theta = ma$$

The mass cancels:

$$\mu_k = \frac{g\sin\theta - a}{g\cos\theta} = \frac{9.81\sin 33.0^\circ - 1.26}{9.81\cos 33.0^\circ} = 0.50$$

FREE-BODY DIAGRAM



ANSWER 0.50

PROBLEM 3

Accelerating a Porsche

Static friction

Watch the video

Hopping into your Porsche, you floor it and accelerate at 12 m/s² without spinning the tires. Determine the minimum coefficient of static friction between the tires and the road needed to make this possible.

GIVEN $a = 12\ \text{m/s}^2$ $N = mg$ **FIND** $\mu_{s,min}$

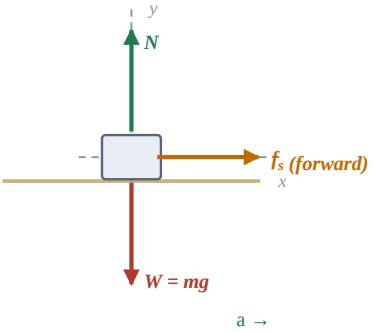
SOLUTION

The tires do not slip, so static friction pushes the car forward:

$$\mu_s mg = ma$$

$$\mu_s = \frac{a}{g} = \frac{12}{9.81} = 1.22$$

FREE-BODY DIAGRAM



ANSWER 1.22

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PROBLEM 4

Sliding a book on a table

Static friction | Kinetic friction

Watch the video

When you push a 1.80-kg book resting on a tabletop, it takes 2.25 N to start the book sliding. Once it is sliding, it takes only 1.50 N to keep it moving with constant speed. (a) What are the coefficients of static and kinetic friction between the book and the tabletop? (b) What is the frictional force on the book when you push on it with a force of 0.75 N?

GIVEN $m = 1.80\ \text{kg}$ $F_{start} = 2.25\ \text{N}$ $F_{slide} = 1.50\ \text{N}$ **FIND** $\mu_s,\ \mu_k$

SOLUTION

N = mg = 1.80(9.81) = 17.7 N

$$\mu_s = \frac{2.25}{17.7} = 0.127$$

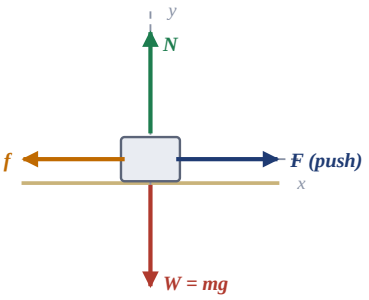
Constant speed → F = f_k:

$$\mu_k = \frac{1.50}{17.7} = 0.085$$

(b) 0.75 N is less than $f_{s,max}$ = 2.25 N, so the book stays at rest and static friction just balances the push:

$$f_s = 0.75\ \text{N}$$

FREE-BODY DIAGRAM



f = f_s (at rest) or f_k (sliding)

ANSWER **(a)** $\mu_s = 0.127,\ \mu_k = 0.085$ **(b)** $0.75\ \text{N}$

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PROBLEM 5

Pushing a crate at 21° below horizontal

Static friction · Kinetic friction

Watch the video

To move a large crate across a rough floor, you push on it with a force F at an angle of 21° below the horizontal (Figure 6-18). Find the force necessary to start the crate moving, given that its mass is 32 kg and μ_s = 0.57. Then find the acceleration of the crate if the applied force is 330 N and μ_k = 0.45.



GIVEN $m = 32\ \text{kg}$ $\theta = 21^\circ$ $\mu_s = 0.57$ $\mu_k = 0.45$ **FIND** $F,\ a$

SOLUTION

Pushing down increases the normal force:

$$N = mg + F\sin 21^\circ$$

(a) Just about to move:

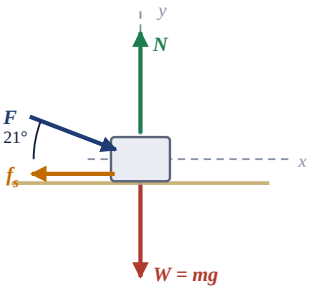
$$F\cos 21^\circ = 0.57(F\sin 21^\circ + 313.9) \rightarrow F = 245\ \text{N}$$

(b) With F = 330 N:

$$N = 330\sin 21^\circ + 313.9 = 432\ \text{N}, \quad f_k = 0.45(432) = 194.5\ \text{N}$$

$$a = \frac{330\cos 21^\circ - 194.5}{32} = 3.55\ \text{m/s}^2$$

FREE-BODY DIAGRAM



ANSWER **(a)** 245 N **(b)** $3.55\ \text{m/s}^2$

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PROBLEM 6

A crate on a tilting ramp

Static friction · Inclined plane

Watch the video

A 48-kg crate is placed on an inclined ramp. When the angle the ramp makes with the horizontal is increased to 26°, the crate begins to slide downward. (a) What is the coefficient of static friction between the crate and the ramp? (b) At what angle does the crate begin to slide if its mass is doubled?

GIVEN $m = 48\ \text{kg}$ $\theta_{slip} = 26^\circ$ **FIND** μ_s

SOLUTION

At the slipping angle, f_s is at its maximum:

$$mg\sin\theta = \mu_s mg\cos\theta$$

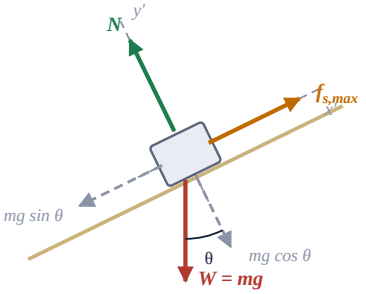
(a)

$$\mu_s = \tan 26^\circ = 0.49$$

(b) The mass cancels, so the angle does not change:

$$\theta = 26^\circ$$

FREE-BODY DIAGRAM



ANSWER **(a)** 0.49 **(b)** 26° (unchanged)

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PROBLEM 7

A sprinter's start

Static friction · Kinematics

Watch the video

A 97-kg sprinter wishes to accelerate from rest to a speed of 13 m/s in a distance of 22 m. What coefficient of static friction is required between the sprinter's shoes and the track?

GIVEN $m = 97\ \text{kg}$ $v = 13\ \text{m/s}$ $\Delta x = 22\ \text{m}$ **FIND** μ_s

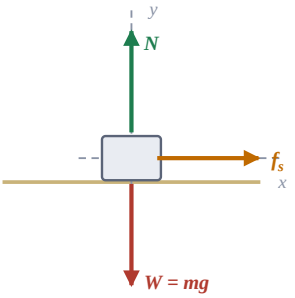
SOLUTION

$$a = \frac{13^2}{2(22)} = 3.84\ \text{m/s}^2$$

Static friction provides the push:

$$\mu_s mg = ma \rightarrow \mu_s = \frac{3.84}{9.81} = 0.39$$

FREE-BODY DIAGRAM



ANSWER 0.39

PROBLEM 8

Lifting a bucket from a well

Tension · Second law

Watch the video

Pulling up on a rope, you lift a 4.35-kg bucket of water from a well with an acceleration of 1.78 m/s². What is the tension in the rope?

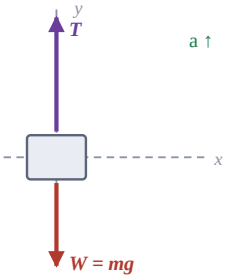
GIVEN $m = 4.35\ \text{kg}$ $a = 1.78\ \text{m/s}^2\ (\text{up})$ **FIND** T

SOLUTION

$$T - mg = ma$$

$$T = m(g + a) = 4.35(9.81 + 1.78) = 50.4\ \text{N}$$

FREE-BODY DIAGRAM



ANSWER $50.4\ \text{N}$

PROBLEM 9

A mass on a vertical spring

Spring force

Watch the video

When a 9.09-kg mass is placed on top of a vertical spring, the spring compresses 4.18 cm. Find the force constant of the spring.

GIVEN $m = 9.09\ \text{kg}$ $x = 0.0418\ \text{m}$ FIND k

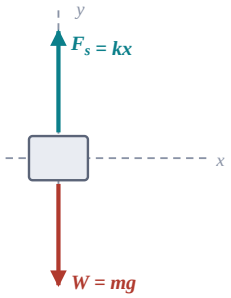
SOLUTION

Equilibrium:

$$kx = mg$$

$$k = \frac{9.09(9.81)}{0.0418} = 2.13 \times 10^3\ \text{N/m}$$

FREE-BODY DIAGRAM



ANSWER $2.13 \times 10^3\ \text{N/m}$

PROBLEM 10

A car's rear suspension

Spring force

Watch the video

A 110-kg box is loaded into the trunk of a car. If the height of the car's bumper decreases by 13 cm, what is the force constant of its rear suspension?

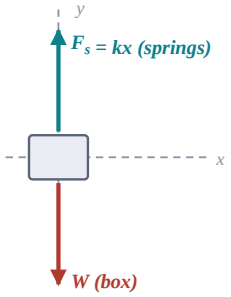
GIVEN $m = 110\ \text{kg}$ $x = 0.13\ \text{m}$ FIND k

SOLUTION

$$F = mg = 110(9.81) = 1079\ \text{N}$$

$$k = \frac{F}{x} = \frac{1079}{0.13} = 8.30 \times 10^3\ \text{N/m}$$

FREE-BODY DIAGRAM



ANSWER $8.30 \times 10^3\ \text{N/m}$

PROBLEM 11

Tension in a hammock

Equilibrium · Tension

Watch the video

A 50.0-kg person takes a nap in a backyard hammock. Both ropes supporting the hammock are at an angle of 15.0° above the horizontal. Find the tension in the ropes.

GIVEN $m = 50.0\ \text{kg}$ $\theta = 15.0^\circ$ **FIND** T

SOLUTION

By symmetry both tensions are equal; the horizontal parts cancel.

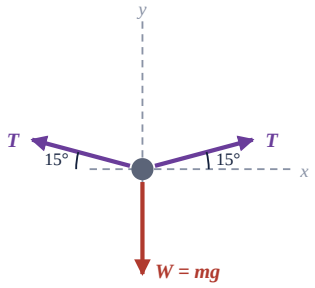
Vertical equilibrium:

$$2T\sin 15.0^\circ = mg$$

$$T = \frac{50.0(9.81)}{2\sin 15.0^\circ} = 948\ \text{N}$$

The small angle makes T almost twice the person's weight.

FREE-BODY DIAGRAM



ANSWER $948\ \text{N}$

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PROBLEM 12

Stretching and compressing a spring

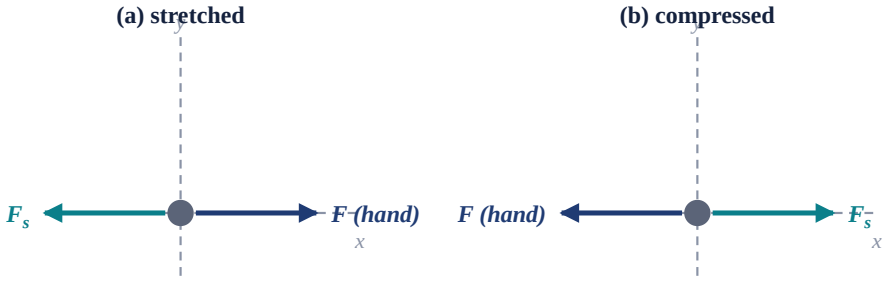
Spring force

Watch the video

The equilibrium length of a certain spring with a force constant of k = 250 N/m is 0.18 m. (a) What is the magnitude of the force required to hold this spring at twice its equilibrium length? (b) Is the force required to keep the spring compressed to half its equilibrium length greater than, less than, or equal to the force in part (a)? Explain.

GIVEN $k = 250\ \text{N/m}$ $L_0 = 0.18\ \text{m}$ **FIND** F

FREE-BODY DIAGRAM



SOLUTION

(a) Twice the length → stretched by x = 0.18 m:

$$F = kx = 250(0.18) = 45\ \text{N}$$

(b) Half the length → compressed by only x = 0.09 m:

$$F = 250(0.09) = 22.5\ \text{N (less)}$$

ANSWER **(a)** 45 N **(b)** less (22.5 N)

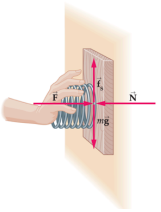
PROBLEM 13

A spring holding a block against a wall

Spring force · Static friction

Watch the video

A spring with a force constant of 120 N/m is used to push a 0.27-kg block of wood against a wall (Figure 6-20). (a) Find the minimum compression of the spring needed to keep the block from falling, given that $\mu_s = 0.46$ between the block and the wall. (b) Does your answer to part (a) change if the mass of the block is doubled? Explain.



GIVEN $k = 120\ \text{N/m}$ $m = 0.27\ \text{kg}$ $\mu_s = 0.46$ **FIND** x_{min}

SOLUTION

The spring force is the normal force: N = kx. Static friction holds the weight:

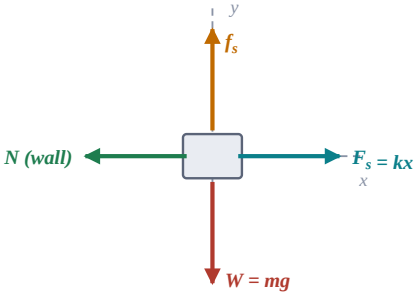
$$\mu_s k x = mg$$

(a)

$$x = \frac{mg}{\mu_s k} = \frac{0.27(9.81)}{0.46(120)} = 0.048\ \text{m} = 4.8\ \text{cm}$$

(b) Yes: x is proportional to m, so doubling the mass doubles the compression (9.6 cm).

FREE-BODY DIAGRAM



spring pushes →, wall pushes ←

ANSWER **(a)** 4.8 cm **(b)** doubles (9.6 cm)

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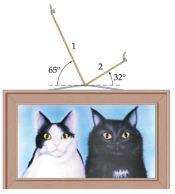
PROBLEM 14

A picture hanging from two strings

Equilibrium · Tension

Watch the video

A picture hangs on the wall suspended by two strings, as shown in Figure 6-21 (string 1 at 65° and string 2 at 32° above the horizontal). The tension in string 1 is 1.7 N. (a) Is the tension in string 2 greater than, less than, or equal to 1.7 N? Explain. (b) Verify your answer by calculating the tension in string 2. (c) What is the weight of the picture?



GIVEN $T_1 = 1.7\ \text{N}$ at 65° T_2 at 32° **FIND** $T_2,\ W$

SOLUTION

(a) The horizontal parts must cancel. String 2 is closer to horizontal, so a smaller tension gives the same horizontal pull → $T_2 < 1.7$ N.

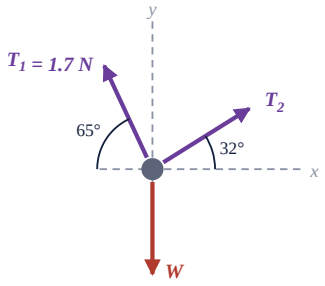
(b)

$$T_2 \cos 32^\circ = T_1 \cos 65^\circ \rightarrow T_2 = \frac{1.7 \cos 65^\circ}{\cos 32^\circ} = 0.85\ \text{N}$$

(c) The vertical parts hold the weight:

$$W = 1.7 \sin 65^\circ + 0.85 \sin 32^\circ = 2.0\ \text{N}$$

FREE-BODY DIAGRAM



ANSWER **(a)** less **(b)** 0.85 N **(c)** 2.0 N

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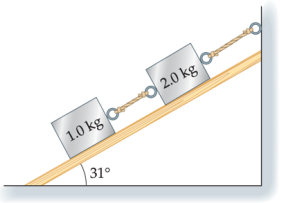
PROBLEM 15

Two blocks tied to a wall on an incline

Equilibrium · Tension · Inclined plane

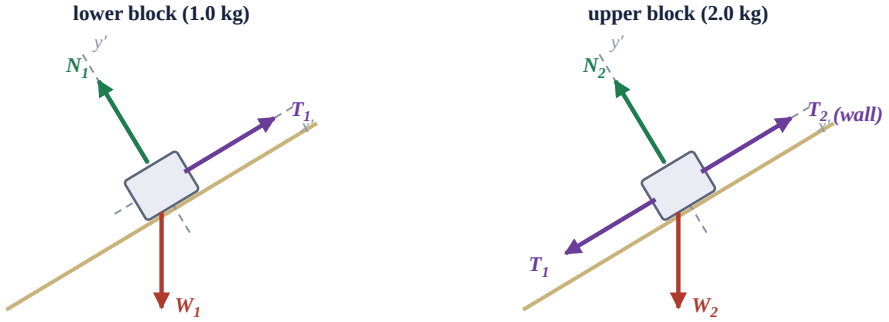
Watch the video

Two blocks on a frictionless 31° incline are connected by a string and tied to the wall at the top. The lower block has a mass of 1.0 kg and the upper block 2.0 kg. Find the tension in (a) the string connecting the two blocks and (b) the string tied to the wall.



GIVEN $m_1 = 1.0\ \text{kg (lower)}$ $m_2 = 2.0\ \text{kg (upper)}$ $\theta = 31^\circ$ **FIND** $T_1,\ T_2$

FREE-BODY DIAGRAM



SOLUTION

(a) The connecting string holds only the lower block along the incline:

$$T_1 = m_1 g \sin 31^\circ = 1.0(9.81)(0.515) = 5.1\ \text{N}$$

(b) The wall string holds both blocks:

$$T_2 = (m_1 + m_2) g \sin 31^\circ = 3.0(9.81)(0.515) = 15\ \text{N}$$

ANSWER **(a)** $5.1\ \text{N}$ **(b)** $15\ \text{N}$

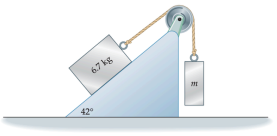
PROBLEM 16

Balancing a block on a smooth incline

Equilibrium · Inclined plane

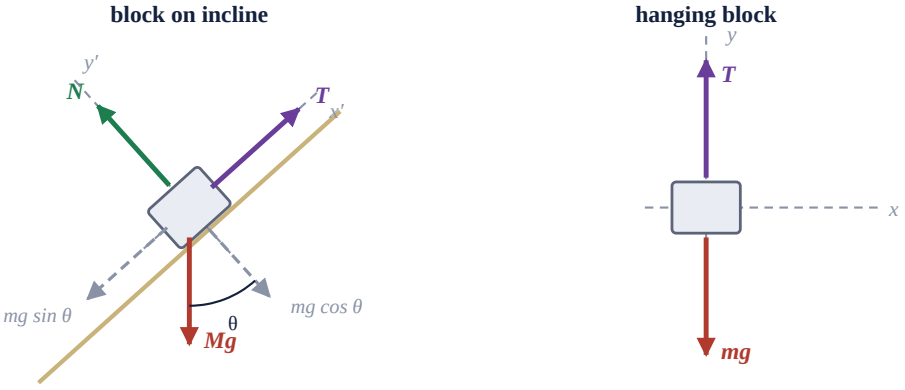
Watch the video

Two blocks are connected by a string over an ideal pulley (Figure 6-25). The smooth inclined surface makes an angle of 42° with the horizontal, and the block on the incline has a mass of 6.7 kg. Find the mass of the hanging block that keeps the system in equilibrium.



GIVEN $M = 6.7\ \text{kg}$ $\theta = 42^\circ$ smooth incline **FIND** m

FREE-BODY DIAGRAM



SOLUTION

Hanging block: T = mg. Block on the incline: T = Mg sin θ.

$$mg = Mg\sin 42^\circ \rightarrow m = 6.7\sin 42^\circ = 4.5\ \text{kg}$$

ANSWER $4.5\ \text{kg}$

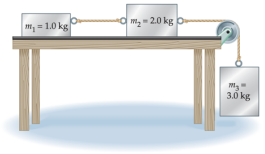
PROBLEM 17

Three connected masses

Connected bodies · Second law

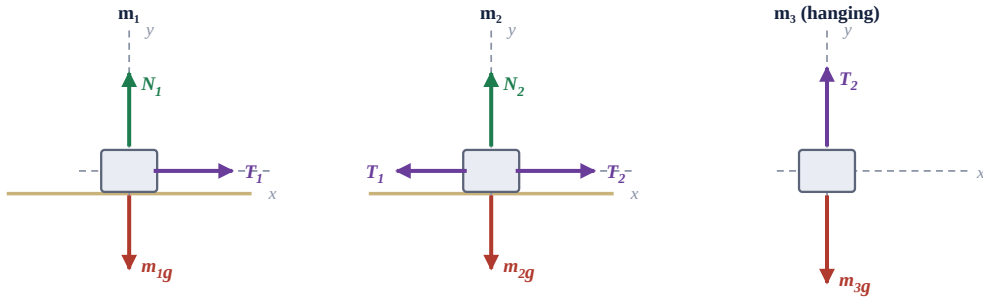
Watch the video

Find the acceleration of the masses shown in Figure 6-30, given that m_1 = 1.0 kg, m_2 = 2.0 kg, and m_3 = 3.0 kg. Assume the table is frictionless and the masses move freely. (m_1 and m_2 are on the table; m_3 hangs.)



GIVEN $m_1 = 1.0\ \text{kg}$ $m_2 = 2.0\ \text{kg}$ $m_3 = 3.0\ \text{kg (hanging)}$ **FIND** a

FREE-BODY DIAGRAM



SOLUTION

Only the weight of the hanging mass drives the whole system (the tensions are internal):

$$m_3 g = (m_1 + m_2 + m_3)a$$

$$a = \frac{3.0(9.81)}{6.0} = 4.9\ \text{m/s}^2$$

ANSWER $4.9\ \text{m/s}^2$

Answer summary

Problem	Topic	Key physics idea	Final answer(s)
1	Sliding into third base	$a = -\mu_k g$	$1.77\ \text{m}$
2	A child on a playground slide	$\mu_k = (g \sin\theta - a)/(g\cos\theta)$	0.50
3	Accelerating a Porsche	Static friction drives the car	1.22
4	Sliding a book on a table	f_s only as large as needed	**(a)** $\mu_s = 0.127,\ \mu_k = 0.085$ **(b)** $0.75\ \text{N}$
5	Pushing a crate at 21° below horizontal	Push down → larger N	**(a)** $245\ \text{N}$ **(b)** $3.55\ \text{m/s}^2$
6	A crate on a tilting ramp	μ_s = tan θ_slip	**(a)** 0.49 **(b)** 26° (unchanged)
7	A sprinter's start	μ_s = a/g	0.39
8	Lifting a bucket from a well	T = m(g + a)	$50.4\ \text{N}$
9	A mass on a vertical spring	kx = mg	$2.13 \times 10^3\ \text{N/m}$
10	A car's rear suspension	k = F/x	$8.30 \times 10^3\ \text{N/m}$
11	Tension in a hammock	Small angle → large T	$948\ \text{N}$
12	Stretching and compressing a spring	F depends on x, not on length	**(a)** $45\ \text{N}$ **(b)** less ($22.5\ \text{N}$)
13	A spring holding a block against a wall	Spring force = normal force	**(a)** $4.8\ \text{cm}$ **(b)** doubles ($9.6\ \text{cm}$)
14	A picture hanging from two strings	Horizontal parts cancel	**(a)** less **(b)** $0.85\ \text{N}$ **(c)** $2.0\ \text{N}$
15	Two blocks tied to a wall on an incline	Each string holds what is below it	**(a)** $5.1\ \text{N}$ **(b)** $15\ \text{N}$
16	Balancing a block on a smooth incline	mg = Mg sin θ	$4.5\ \text{kg}$
17	Three connected masses	Hanging weight drives all masses	$4.9\ \text{m/s}^2$

g = 9.81 m/s². Problems 11, 12, 13, 15, 16 and 17 are explained in the videos; their written solutions were added here.