ALNOJOUM ACADEMY

أكاديمية النجوم

Chapter 4
Motion in Two and Three Dimensions

Written solutions to the textbook problems

PHYS 105 | Prince Sultan University | 11 solved problems

What this chapter covers

4.1 Position and Displacement

4.2 Average and Instantaneous Velocity

4.3 Average and Instantaneous Acceleration

4.4 Projectile Motion

All key formulas are on the next page.

Watch out

- Time is the only quantity shared by the x and y motions — find it from one direction and use it in the other.
- The horizontal velocity never changes in projectile motion ($a_x = 0$).
- At the highest point $v_y = 0$, but the speed is not zero: $v = v_{0x}$.
- For a horizontal launch, $v_{0y} = 0$ — a faster horizontal speed does not change the fall time.
- Use the range and flight-time formulas only when the projectile lands at its launch height.

Problems in this file

1 A sailboat moving north of west

2 Walking to class

3 A car accelerating up a hill

4 An arrow shot horizontally

5 Gravity on planet Zircon

6 A diver leaving a diving board

7 Shortstop throwing to second base

8 A crow drops a clam

9 Throw from second to first base

10 A champagne cork

11 Time of flight of a soccer ball

Every problem has a QR code that opens its full video solution.

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Chapter 4

Key formulas

Organized by the sections of the Chapter 4 slides (4.1 – 4.4).

4.1 Position and Displacement

Position vector	$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$	
Displacement	$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k}$	

4.2 Average and Instantaneous Velocity

Average velocity	$\vec{v}_{avg} = \dfrac{\Delta\vec{r}}{\Delta t}$	Same direction as Δr
Instantaneous velocity	$\vec{v} = \dfrac{d\vec{r}}{dt} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$	Tangent to the path

4.3 Average and Instantaneous Acceleration

Average acceleration	$\vec{a}_{avg} = \dfrac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \dfrac{\Delta\vec{v}}{\Delta t}$	Same direction as Δv
Instantaneous acceleration	$\vec{a} = \dfrac{d\vec{v}}{dt}$	

4.4 Projectile Motion

Launch components	$v_{0x} = v_0\cos\theta_0, \quad v_{0y} = v_0\sin\theta_0$	$a_x = 0$, $a_y = -g$
Horizontal motion	$x - x_0 = v_{0x}t, \quad v_x = v_{0x}$	Constant velocity
Vertical motion	$y - y_0 = v_{0y}t - \frac{1}{2}gt^2, \quad v_y = v_{0y} - gt$	$v_y^2 = v_{0y}^2 - 2g\Delta y$
Maximum height	$H = \dfrac{(v_0\sin\theta_0)^2}{2g}$	$v_y = 0$ at the top
Time of flight (same level)	$t = \dfrac{2v_0\sin\theta_0}{g}$	
Range (same level)	$R = \dfrac{v_0^2\sin 2\theta_0}{g}$	Maximum at 45°

◆ Projectile motion: horizontal vs. vertical

	HORIZONTAL (X)	VERTICAL (Y)
Acceleration	0	$-g = -9.81\ m/s^2$
Velocity	$v_{0x} = v_0\cos\theta$ (constant)	$v_y = v_0\sin\theta - gt$
Position	$x = v_{0x}\,t$	$y = v_{0y}\,t - \frac{1}{2}gt^2$
Horizontal launch	$v_{0x} = v_0$	$v_{0y} = 0$

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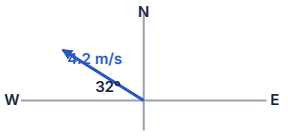
PROBLEM 1

A sailboat moving north of west

Velocity components · Constant velocity

Watch the video

A sailboat runs before the wind with a constant speed of 4.2 m/s in a direction 32° north of west. How far (a) west and (b) north has the sailboat traveled in 25 min?



GIVEN $v = 4.2\ \text{m/s}$ $\theta = 32^\circ\ \text{N of W}$ $t = 25\ \text{min} = 1500\ \text{s}$ FIND $\Delta x,\ \Delta y$

SOLUTION

Velocity components (constant velocity, so a = 0):

$$v_x = 4.2\cos 32^\circ = 3.56\ \text{m/s (west)}, \quad v_y = 4.2\sin 32^\circ = 2.23\ \text{m/s (north)}$$

(a)

$$\Delta x = v_x t = 3.56(1500) = 5.34 \times 10^3\ \text{m west}$$

(b)

$$\Delta y = v_y t = 2.23(1500) = 3.34 \times 10^3\ \text{m north}$$

ANSWER **(a)** $5.34\ \text{km}$ west **(b)** $3.34\ \text{km}$ north

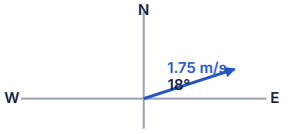
PROBLEM 2

Walking to class

Velocity components

Watch the video

As you walk to class with a constant speed of 1.75 m/s, you are moving in a direction 18.0° north of east. How much time does it take to change your displacement by (a) 20.0 m east or (b) 30.0 m north?



GIVEN $v = 1.75\ \text{m/s}$ $\theta = 18.0^\circ\ \text{N of E}$ FIND t

SOLUTION

Components:

$$v_x = 1.75\cos 18.0^\circ = 1.66\ \text{m/s}, \quad v_y = 1.75\sin 18.0^\circ = 0.541\ \text{m/s}$$

(a) Only the east component changes the east displacement:

$$t = \frac{\Delta x}{v_x} = \frac{20.0}{1.66} = 12.0\ \text{s}$$

(b)

$$t = \frac{\Delta y}{v_y} = \frac{30.0}{0.541} = 55.5\ \text{s}$$

ANSWER **(a)** $12.0\ \text{s}$ **(b)** $55.5\ \text{s}$

PROBLEM 3

A car accelerating up a hill

Acceleration components | Constant acceleration

Watch the video

Starting from rest, a car accelerates at 2.0 m/s² up a hill that is inclined 5.5° above the horizontal. How far (a) horizontally and (b) vertically has the car traveled in 12 s?



GIVEN $v_0 = 0 \quad a = 2.0\ \text{m/s}^2 \quad \theta = 5.5^\circ \quad t = 12\ \text{s}$ **FIND** $\Delta x,\ \Delta y$

SOLUTION

Components of the acceleration:

$$a_x = 2.0\cos 5.5^\circ = 1.99\ \text{m/s}^2, \quad a_y = 2.0\sin 5.5^\circ = 0.192\ \text{m/s}^2$$

(a)

$$\Delta x = \tfrac{1}{2}a_x t^2 = \tfrac{1}{2}(1.99)(12)^2 = 143\ \text{m}$$

(b)

$$\Delta y = \tfrac{1}{2}a_y t^2 = \tfrac{1}{2}(0.192)(12)^2 = 13.8\ \text{m}$$

ANSWER **(a)** 143 m **(b)** 13.8 m

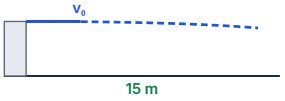
PROBLEM 4

An arrow shot horizontally

Horizontal launch

Watch the video

An archer shoots an arrow horizontally at a target 15 m away. The arrow is aimed directly at the center of the target, but it hits 52 cm lower. What was the initial speed of the arrow?



GIVEN $\Delta x = 15\ \text{m} \quad \Delta y = -0.52\ \text{m} \quad v_{0y} = 0$ **FIND** v_0

SOLUTION

Time from the vertical drop:

$$-0.52 = -\tfrac{1}{2}(9.81)t^2 \rightarrow t = 0.326\ \text{s}$$

Horizontal motion has constant velocity:

$$v_0 = \frac{\Delta x}{t} = \frac{15}{0.326} = 46.1\ \text{m/s}$$

ANSWER 46.1 m/s

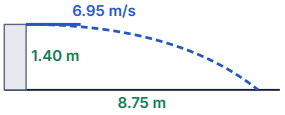
PROBLEM 5

Gravity on planet Zircon

Horizontal launch

Watch the video

An astronaut on the planet Zircon tosses a rock horizontally with a speed of 6.95 m/s. The rock falls through a vertical distance of 1.40 m and lands a horizontal distance of 8.75 m from the astronaut. What is the acceleration of gravity on Zircon?



GIVEN $v_{0x} = 6.95\ \text{m/s}$ $\Delta x = 8.75\ \text{m}$ $\Delta y = -1.40\ \text{m}$ **FIND** g_{Zircon}

SOLUTION

Time from the horizontal motion:

$$t = \frac{8.75}{6.95} = 1.26\ \text{s}$$

Vertical motion ($v_{0y} = 0$):

$$-1.40 = -\tfrac{1}{2}g(1.26)^2 \rightarrow g = 1.77\ \text{m/s}^2$$

ANSWER $1.77\ \text{m/s}^2$

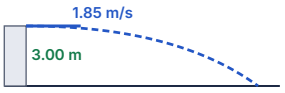
PROBLEM 6

A diver leaving a diving board

Horizontal launch · Final speed

Watch the video

A diver runs horizontally off the end of a diving board with an initial speed of 1.85 m/s. If the diving board is 3.00 m above the water, what is the diver's speed just before she enters the water?



GIVEN $v_{0x} = 1.85\ \text{m/s}$ $v_{0y} = 0$ $\Delta y = -3.00\ \text{m}$ **FIND** v

SOLUTION

v_x stays the same; find v_y:

$$v_y^2 = 0 + 2(9.81)(3.00) \rightarrow v_y = -7.67\ \text{m/s}$$

Speed and direction:

$$v = \sqrt{1.85^2 + 7.67^2} = 7.89\ \text{m/s}, \quad \theta = \tan^{-1}\frac{7.67}{1.85} = 76.4^\circ\ \text{below horizontal}$$

ANSWER $7.89\ \text{m/s}\ (76.4^\circ\ \text{below horizontal})$

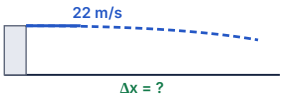
PROBLEM 7

Shortstop throwing to second base

Horizontal launch

Watch the video

Playing shortstop, you pick up a ground ball and throw it horizontally with a speed of 22 m/s toward point A (Figure 4-13). The second baseman catches it at point B, 0.45 s later. (a) How far were you from the second baseman? (b) What is the vertical drop AB?



GIVEN $v_{0x} = 22\ \text{m/s}$ $v_{0y} = 0$ $t = 0.45\ \text{s}$ FIND $\Delta x,\ AB$

SOLUTION

(a)

$$\Delta x = v_{0x}t = 22(0.45) = 9.9\ \text{m}$$

(b)

$$AB = \tfrac{1}{2}gt^2 = \tfrac{1}{2}(9.81)(0.45)^2 = 0.99\ \text{m}$$

Extra (board homework): velocity at B:

$$v_y = -9.81(0.45) = -4.41\ \text{m/s} \rightarrow v = 22.4\ \text{m/s},\ 11.3^\circ\ \text{below horizontal}$$

ANSWER **(a)** $9.9\ \text{m}$ **(b)** $0.99\ \text{m}$

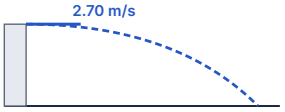
PROBLEM 8

A crow drops a clam

Horizontal launch · Velocity components

Watch the video

A crow is flying horizontally with a constant speed of 2.70 m/s when it releases a clam from its beak (Figure 4-14). The clam lands 2.10 s later. Just before it lands, what is (a) its horizontal component of velocity, and (b) its vertical component of velocity? (c) How would your answers change if the speed of the crow were increased?



GIVEN $v_{0x} = 2.70\ \text{m/s}$ $v_{0y} = 0$ $t = 2.10\ \text{s}$ FIND $v_x,\ v_y$

SOLUTION

(a) No horizontal acceleration:

$$v_x = 2.70\ \text{m/s}$$

(b)

$$v_y = -gt = -9.81(2.10) = -20.6\ \text{m/s}$$

(c) A faster crow increases v_x only; v_y depends only on the fall time, so it does not change.

ANSWER **(a)** $2.70\ \text{m/s}$ **(b)** $-20.6\ \text{m/s}$ **(c)** $v_x \uparrow,\ v_y$ same

PROBLEM 9

Throw from second to first base

Projectile at an angle · Symmetry

Watch the video

A second baseman tosses the ball to the first baseman, who catches it at the same level from which it was thrown. The throw is made with an initial speed of 18.0 m/s at an angle of 37.5° above the horizontal. (a) What is the horizontal component of the ball's velocity just before it is caught? (b) How long is the ball in the air? (c) What are the y component of the velocity and its direction of motion just before it is caught?



GIVEN $v_0 = 18.0\ \text{m/s}$ $\theta = 37.5^\circ$ FIND $v_x,\ t,\ v_y$

SOLUTION

Components:

$$v_{0x} = 18.0\cos 37.5^\circ = 14.3\ \text{m/s}, \quad v_{0y} = 18.0\sin 37.5^\circ = 11.0\ \text{m/s}$$

(a)

$$v_x = v_{0x} = 14.3\ \text{m/s}$$

(b) Same level → time up = time down:

$$t = \frac{2v_{0y}}{g} = \frac{2(11.0)}{9.81} = 2.23\ \text{s}$$

(c) By symmetry:

$$v_y = -11.0\ \text{m/s}, \quad \theta = 37.5^\circ \text{ below horizontal}$$

ANSWER **(a)** $14.3\ \text{m/s}$ **(b)** $2.23\ \text{s}$ **(c)** $-11.0\ \text{m/s},\ 37.5^\circ$ below

PROBLEM 10

A champagne cork

Projectile at an angle

Watch the video

A cork shoots out of a champagne bottle at an angle of 35.0° above the horizontal. If the cork travels a horizontal distance of 1.30 m in 1.25 s, what was its initial speed?



GIVEN $\theta = 35.0^\circ$ $\Delta x = 1.30\ \text{m}$ $t = 1.25\ \text{s}$ FIND v_0

SOLUTION

Horizontal velocity is constant:

$$v_{0x} = \frac{1.30}{1.25} = 1.04\ \text{m/s}$$

$$v_0 = \frac{v_{0x}}{\cos 35.0^\circ} = \frac{1.04}{\cos 35.0^\circ} = 1.27\ \text{m/s}$$

ANSWER $1.27\ \text{m/s}$

PROBLEM 11

Time of flight of a soccer ball

Projectile at an angle | Time of flight

Watch the video

A soccer ball is kicked with a speed of 9.85 m/s at an angle of 35.0° above the horizontal. If the ball lands at the same level from which it was kicked, how long was it in the air?



GIVEN $v_0 = 9.85\ \text{m/s}$ $\theta = 35.0^\circ$ **FIND** t

SOLUTION

$$v_{0y} = 9.85 \sin 35.0^\circ = 5.65\ \text{m/s}$$

At the top $v_y = 0$:

$$0 = 5.65 - 9.81 t_{up} \rightarrow t_{up} = 0.576\ \text{s}$$

Same level → double it:

$$t = 2t_{up} = 1.15\ \text{s}$$

ANSWER $1.15\ \text{s}$

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Answer summary

Problem	Topic	Key physics idea	Final answer(s)
1	A sailboat moving north of west	v = const → $\Delta x = v_x t$	**(a)** $5.34\ \text{km}$ west **(b)** $3.34\ \text{km}$ north
2	Walking to class	Only v_x changes x	**(a)** $12.0\ \text{s}$ **(b)** $55.5\ \text{s}$
3	A car accelerating up a hill	Resolve a along x and y	**(a)** $143\ \text{m}$ **(b)** $13.8\ \text{m}$
4	An arrow shot horizontally	Fall time sets the flight time	$46.1\ \text{m/s}$
5	Gravity on planet Zircon	Same t for x and y	$1.77\ \text{m/s}^2$
6	A diver leaving a diving board	v_x constant, v_y grows	$7.89\ \text{m/s}$ (76.4° below horizontal)
7	Shortstop throwing to second base	x and y are independent	**(a)** $9.9\ \text{m}$ **(b)** $0.99\ \text{m}$
8	A crow drops a clam	v_y does not depend on v_x	**(a)** $2.70\ \text{m/s}$ **(b)** $-20.6\ \text{m/s}$ **(c)** $v_x \uparrow$, v_y same
9	Throw from second to first base	Symmetric path	**(a)** $14.3\ \text{m/s}$ **(b)** $2.23\ \text{s}$ **(c)** $-11.0\ \text{m/s},\ 37.5^\circ$ below
10	A champagne cork	$v_{0x} = \Delta x / t$	$1.27\ \text{m/s}$
11	Time of flight of a soccer ball	$t = 2v_0 \sin\theta / g$	$1.15\ \text{s}$

$g = 9.81\ \text{m/s}^2$. The $+y$ axis points up, so $a_y = -g$. Answers are rounded to the significant figures given in each problem.