ALNOJOUM ACADEMY

أكاديمية النجوم

Chapter 3 Vectors

Written solutions to the textbook problems

PHYS 105 | Prince Sultan University | 9 solved problems

What this chapter covers

3.1 Vectors and their Components

3.2 Unit Vectors and Adding by Components

3.3 Multiplying Vectors

All key formulas are on the next page.

Watch out

- Find the quadrant from the signs of the components before using $\tan^{-1}$.
- The calculator's $\tan^{-1}$ only returns angles between −90° and +90°.
- Use cos with the angle measured from the x axis; if the angle is from the y axis, cos and sin swap.
- "Greater component" compares signed values: 0 is greater than −113.
- Add vectors component by component — never add magnitudes directly.

Problems in this file

1 Ranking the vectors of Figure 3-31

2 Driving up an inclined road

3 Components of a position vector

4 Distance from a lighthouse

5 Direction and magnitude from components

6 Driving east then north

7 Comparing components of two vectors

8 A diving whale

9 Adding and subtracting vectors graphically

Every problem has a QR code that opens its full video solution.

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Chapter 3

Key formulas

Organized by the sections of the Chapter 3 slides (3.1 – 3.3).

3.1 Vectors and their Components

Components	$A_x = A\cos\theta, \quad A_y = A\sin\theta$	θ measured from the +x axis
Magnitude	$A = \sqrt{A_x^2 + A_y^2}$	
Direction	$\theta = \tan^{-1}\dfrac{A_y}{A_x}$	Add 180° if $A_x < 0$
Vector subtraction	$\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$	−b has the same length, opposite direction

3.2 Unit Vectors and Adding by Components

Unit-vector notation	$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$	$\lvert\hat{i}\rvert = \lvert\hat{j}\rvert = \lvert\hat{k}\rvert = 1$
Adding vectors	$r_x = a_x + b_x, \quad r_y = a_y + b_y$	Then find r and θ
Subtracting vectors	$d_x = a_x - b_x, \quad d_y = a_y - b_y$	

3.3 Multiplying Vectors

Scalar × vector	$c\vec{A} = cA_x\hat{i} + cA_y\hat{j}$	c < 0 reverses the direction
Dot (scalar) product	$\vec{a}\cdot\vec{b} = ab\cos\varphi = a_xb_x + a_yb_y + a_zb_z$	Result is a scalar
Cross (vector) product	$\lvert\vec{a}\times\vec{b}\rvert = ab\sin\varphi$	⊥ to both; right-hand rule
Unit-vector products	$\hat{i}\cdot\hat{i} = 1, \quad \hat{i}\cdot\hat{j} = 0, \quad \hat{i}\times\hat{j} = \hat{k}$	**a × b = −b × a**

◆ Signs of the components and the direction angle

	A_X	A_Y	DIRECTION ANGLE
Quadrant I	+	+	$\theta = \tan^{-1}(A_y/A_x)$
Quadrant II	−	+	$\theta = \tan^{-1}(\ldots) + 180°$
Quadrant III	−	−	$\theta = \tan^{-1}(\ldots) + 180°$
Quadrant IV	+	−	$\theta = \tan^{-1}(\ldots)$ (negative) or + 360°

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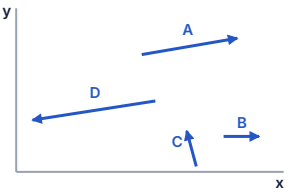
PROBLEM 1

Ranking the vectors of Figure 3-31

Magnitude | Components

Watch the video

Rank the vectors in Figure 3-31 in order of increasing (a) magnitude, (b) x component, (c) y component. (A points right and slightly up, B right, C mostly up and slightly left, D left and slightly down.)



FIND rankings

SOLUTION

(a) Compare the lengths of the arrows.

$$|B| < |C| < |A| < |D|$$

(b) Components can be negative: D points left (large negative x), C slightly left (small negative x).

$$D_x < C_x < B_x < A_x$$

(c) D points slightly down (negative y), B is horizontal (zero), A slightly up, C mostly up.

$$D_y < B_y < A_y < C_y$$

Remember: "increasing value" means from the most negative to the most positive, not by size.

ANSWER **(a)** B, C, A, D **(b)** D, C, B, A **(c)** D, B, A, C

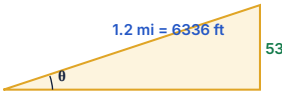
PROBLEM 2

Driving up an inclined road

Trigonometry | Components

Watch the video

After driving 1.2 mi up a straight inclined road, your elevation has increased by 530 ft. (a) What is the angle of the road above the horizontal? (b) How far do you have to drive to gain an additional 150 ft of elevation?



GIVEN $L = 1.2\ \text{mi} = 6336\ \text{ft}$ $h = 530\ \text{ft}$ FIND $\theta,\ \Delta L$

SOLUTION

(a) The elevation is the side opposite θ; the road is the hypotenuse.

$$\sin\theta = \frac{530}{6336} \rightarrow \theta = 4.80^\circ$$

(b)

$$\Delta L = \frac{150}{\sin 4.80^\circ} = 1.79 \times 10^3\ \text{ft} = 0.34\ \text{mi}$$

ANSWER **(a)** 4.8° **(b)** $1.8 \times 10^3\ \text{ft} \approx 0.34\ \text{mi}$

PROBLEM 3

Components of a position vector

Components

Watch the video

Find the x and y components of a position vector r of magnitude 75 m if its angle relative to the x axis is (a) 35.0° and (b) 65.0°.

GIVEN $r = 75\ \text{m}$ FIND $r_x,\ r_y$

SOLUTION

(a)

$$r_x = 75\cos 35.0^\circ = 61.4\ \text{m}, \qquad r_y = 75\sin 35.0^\circ = 43.0\ \text{m}$$

(b)

$$r_x = 75\cos 65.0^\circ = 31.7\ \text{m}, \qquad r_y = 75\sin 65.0^\circ = 68.0\ \text{m}$$

ANSWER **(a)** $61\ \text{m},\ 43\ \text{m}$ **(b)** $32\ \text{m},\ 68\ \text{m}$

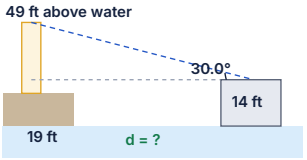
PROBLEM 4

Distance from a lighthouse

Trigonometry

Watch the video

A lighthouse whose top is 49 ft above the water sits on a rocky cliff that extends 19 ft from its base. A sailor whose eyes are 14 ft above the water sees the top of the lighthouse at 30.0° above the horizontal. How far is the ship from the rocks?



GIVEN $49\ \text{ft} - 14\ \text{ft} = 35\ \text{ft}$ $\theta = 30.0^\circ$ FIND d

SOLUTION

Height of the top above the sailor's eyes = 35 ft (opposite side).

$$D = \frac{35}{\tan 30.0^\circ} = 60.6\ \text{ft}\ (\text{to the lighthouse})$$

Subtract the 19 ft of rocks:

$$d = 60.6 - 19 = 41.6\ \text{ft}$$

ANSWER $41.6\ \text{ft}$

PROBLEM 5

Direction and magnitude from components

Magnitude · Direction angle

Watch the video

The components of a vector r are r_x = 14 m and r_y = −9.5 m. Find (a) its direction and (b) its magnitude. (c) If both r_x and r_y are doubled, how do the answers change?

GIVEN $r_x = 14\ \text{m}$ $r_y = -9.5\ \text{m}$ FIND $\theta,\ r$

SOLUTION

(a) (+, −) → 4th quadrant:

$$\theta = \tan^{-1}\left(\frac{-9.5}{14}\right) = -34.2^\circ\ (= 325.8^\circ)$$

(b)

$$r = \sqrt{14^2 + (-9.5)^2} = 16.9\ \text{m}$$

(c) Doubling both components keeps the ratio r_y/r_x:

$$\theta \text{ unchanged}, \quad r = \sqrt{28^2 + 19^2} = 33.8\ \text{m (doubled)}$$

ANSWER **(a)** -34° (below +x) **(b)** 16.9 m **(c)** same angle, 33.8 m

PROBLEM 6

Driving east then north

Vector addition · Direction

Watch the video

You drive 680 ft east, then 340 ft north. (a) What is the magnitude of your displacement? (b) Sketch it and estimate its direction. (c) Verify part (b) with a calculation.

GIVEN $\vec{r} = (680\,\hat{x} + 340\,\hat{y})\ \text{ft}$ FIND $r,\ \theta$

SOLUTION

(a)

$$r = \sqrt{680^2 + 340^2} = 760\ \text{ft}$$

(c)

$$\theta = \tan^{-1}\left(\frac{340}{680}\right) = 26.6^\circ \text{ north of east}$$

ANSWER **(a)** 760 ft **(c)** 26.6° N of E

PROBLEM 7

Comparing components of two vectors

Components · Signs

Watch the video

Vector A has magnitude 50 units and points in the +x direction. Vector B has magnitude 120 units and points 70° below the x axis. Which vector has (a) the greater x component and (b) the greater y component?

GIVEN $\vec{A} = 50\,\hat{x}$ $B = 120,\ 70^\circ$ below $+x$ FIND compare

SOLUTION

Components of B:

$$B_x = 120\cos 70^\circ = 41, \qquad B_y = -120\sin 70^\circ = -113$$

(a)

$$A_x = 50 > B_x = 41$$

(b) "Greater" compares the signed values:

$$A_y = 0 > B_y = -113$$

B has the larger *magnitude* of y component (113), but its y component is negative, so A's is greater.

ANSWER **(a)** A **(b)** $A\ (0 > -113)$

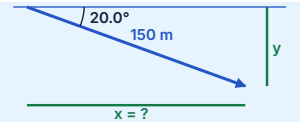
PROBLEM 8

A diving whale

Components

Watch the video

A whale dives at 20.0° below the horizontal and swims 150 m in a straight line. (a) How deep is it? (b) How far has it traveled horizontally?



GIVEN $L = 150\ \text{m}$ $\theta = 20.0^\circ$ FIND $y,\ x$

SOLUTION

(a)

$$y = 150\sin 20.0^\circ = 51.3\ \text{m deep}$$

(b)

$$x = 150\cos 20.0^\circ = 141\ \text{m}$$

ANSWER **(a)** $51\ \text{m}$ **(b)** $141\ \text{m}$

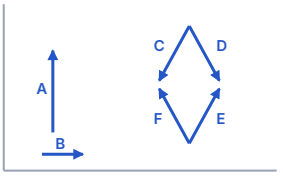
PROBLEM 9

Adding and subtracting vectors graphically

Graphical addition | Vector subtraction

Watch the video

Using vectors A and B in Figure 3-36, which of the vectors C, D, E, F best represents the direction of (a) A + B, (b) A − B, (c) B − A? (d) Is |A + D| greater than, less than, or equal to |A + E|? (e) Is |A + E| greater than, less than, or equal to |A + F|?



FIND directions

SOLUTION

(a) A (up) + B (right) → up and to the right.

$$A + B \parallel E$$

(b) A − B = A + (−B) → up and to the left.

$$A - B \parallel F$$

(c) B − A = −(A − B) → down and to the right.

$$B - A \parallel D$$

(d) D points partly against A, E points partly along A.

$$|A + D| < |A + E|$$

(e) E and F make the same angle with A on opposite sides.

$$|A + E| = |A + F|$$

ANSWER **(a)** E **(b)** F **(c)** D **(d)** less **(e)** equal

Answer summary

Problem	Topic	Key physics idea	Final answer(s)
1	Ranking the vectors of Figure 3-31	**Signed components: most negative first**	**(a)** $B,\ C,\ A,\ D$ **(b)** $D,\ C,\ B,\ A$ **(c)** $D,\ B,\ A,\ C$
2	Driving up an inclined road	**sin θ = opposite / hypotenuse**	**(a)** 4.8° **(b)** 1.8×10^3 ft ≈ 0.34 mi
3	Components of a position vector	**A_x = A cos θ, A_y = A sin θ**	**(a)** 61 m, 43 m **(b)** 32 m, 68 m
4	Distance from a lighthouse	**tan θ = opposite / adjacent**	41.6 ft
5	Direction and magnitude from components	**Quadrant from the signs**	**(a)** -34° (below +x) **(b)** 16.9 m **(c)** same angle, 33.8 m
6	Driving east then north	**Pythagoras + $\tan^{-1}$**	**(a)** 760 ft **(c)** 26.6° N of E
7	Comparing components of two vectors	**Compare signed values**	**(a)** A **(b)** A $(0 > -113)$
8	A diving whale	**Resolve into components**	**(a)** 51 m **(b)** 141 m
9	Adding and subtracting vectors graphically	**A − B = A + (−B)**	**(a)** E **(b)** F **(c)** D **(d)** less **(e)** equal

Angles are measured counterclockwise from the +x axis unless stated otherwise. Answers are rounded to the significant figures given in each problem.

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