ALNOJOUM ACADEMY

أكاديمية النجوم

Chapter 10 Rotation

Written solutions to the textbook problems

PHYS 105 | Prince Sultan University | 7 solved problems

What this chapter covers

10.1 Rotational Variables

10.2 Rotation with Constant Angular Acceleration

10.3 Relating the Linear and Angular Variables

10.6 Torque and Static Equilibrium

All key formulas are on the next page.

Watch out

- $s = r\theta$, $v = \omega r$ and $a_t = \alpha r$ work only with angles in radians.
- Convert rev/min to rad/s: multiply by $2\pi/60$.
- Angular speed is a magnitude (always +); angular velocity ω_z is − for clockwise rotation.
- All points of a rigid body share the same ω and α, but v and a depend on r.
- In $\tau = rF \sin\varphi$, φ is the angle between r and F — not always the angle given in the figure.

Problems in this file

1 Degrees to radians

2 Radians to degrees

3 The hands of Big Ben

5 Ranking angular speeds

6 Paint spot on a bicycle tire

7 Earth orbiting the Sun

8 Earth spinning on its axis

Every problem has a QR code that opens its full video solution.

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Chapter 10

Key formulas

Organized by the sections of the Chapter 10 slides (10.1 – 10.6).

10.1 Rotational Variables

Angular position (radian measure)	$\theta = \dfrac{s}{r} \quad (s = r\theta)$	θ must be in radians
Revolution	$1\ \text{rev} = 2\pi\ \text{rad} = 360^\circ$	1 rad ≈ 57.3°
Angular displacement	$\Delta\theta = \theta_2 - \theta_1$	Counterclockwise +, clockwise −
Angular velocity	$\omega_{\text{avg}} = \dfrac{\Delta\theta}{\Delta t}, \qquad \omega = \dfrac{d\theta}{dt}$	rad/s; rpm × 2π/60 → rad/s
Angular acceleration	$\alpha_{\text{avg}} = \dfrac{\Delta\omega}{\Delta t}, \qquad \alpha = \dfrac{d\omega}{dt}$	rad/s²

10.2 Rotation with Constant Angular Acceleration

Missing θ − θ_0	$\omega = \omega_0 + \alpha t$	Valid only if α is constant
Missing ω	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$	
Missing t	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	
Missing α	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)\, t$	

10.3 Relating the Linear and Angular Variables

Speed	$v = \omega r$	ω in rad/s; r from the axis
Period	$T = \dfrac{2\pi}{\omega} = \dfrac{2\pi r}{v}$	
Tangential acceleration	$a_t = \alpha r$	Changes the speed
Radial (centripetal) acceleration	$a_r = \dfrac{v^2}{r} = \omega^2 r$	Points toward the axis
Total acceleration	$a = \sqrt{a_t^2 + a_r^2}$	$a_t \perp a_r$

10.6 Torque and Static Equilibrium

Torque	$\tau = rF\sin\phi = rF_t$	φ = angle between r and F; unit N·m
Special cases	$\phi = 0^\circ : \tau = 0, \qquad \phi = 90^\circ : \tau_{\max} = rF$	Counterclockwise +, clockwise −
Static equilibrium	$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum \tau = 0$	Any axis may be chosen

◆ Linear ↔ angular quantities

	LINEAR	ANGULAR	LINK
Position	x	θ	s = rθ
Velocity	v	ω	v = ωr
Acceleration	a	α	a_t = αr
Cause of acceleration	F	τ	τ = rF sin φ

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PROBLEM 1

Degrees to radians

Angular position · Unit conversion

Watch the video

Convert the following angles from degrees to radians: 30°, 45°, 90°, 180°.

GIVEN $360^\circ = 2\pi \text{ rad}$ FIND $\theta \text{ in rad}$

SOLUTION

Multiply by the conversion factor 2π rad / 360° (= π rad / 180°):

$$\theta_{\text{rad}} = \theta_{\text{deg}} \times \frac{\pi \text{ rad}}{180^\circ}$$

$$30^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{6}, \quad 45^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{4}, \quad 90^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{2}, \quad 180^\circ \times \frac{\pi}{180^\circ} = \pi$$

ANSWER $\frac{\pi}{6} = 0.524,\ \frac{\pi}{4} = 0.785,\ \frac{\pi}{2} = 1.57,\ \pi = 3.14 \text{ rad}$

PROBLEM 2

Radians to degrees

Angular position · Unit conversion

Watch the video

Convert the following angles from radians to degrees: π/6, 0.70π, 1.5π, 5π.

GIVEN $2\pi \text{ rad} = 360^\circ$ FIND $\theta \text{ in degrees}$

SOLUTION

Multiply by 180° / π rad (1 rad ≈ 57.3°):

$$\theta_{\text{deg}} = \theta_{\text{rad}} \times \frac{180^\circ}{\pi \text{ rad}}$$

$$\frac{\pi}{6} \to 30^\circ, \quad 0.70\pi \to 126^\circ, \quad 1.5\pi \to 270^\circ, \quad 5\pi \to 900^\circ$$

5π rad = 900° is 2.5 revolutions — an angular position can be larger than 360°.

ANSWER $30^\circ,\ 126^\circ,\ 270^\circ,\ 900^\circ$

PROBLEM 3

The hands of Big Ben

Angular speed · ω = Δθ/Δt

Watch the video

Find the angular speed of (a) the minute hand and (b) the hour hand of the clock that rings Big Ben in London.

GIVEN $\Delta\theta = 1\text{ rev} = 2\pi\text{ rad}$ $T_{\text{min}} = 60\text{ min}$ $T_{\text{hour}} = 12\text{ h}$ FIND ω

SOLUTION

Each hand turns one revolution (2π rad) in its period. Convert the time to seconds.

(a) Minute hand:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{60 \times 60\text{ s}} = 1.75 \times 10^{-3}\text{ rad/s}$$

(b) Hour hand:

$$\omega = \frac{2\pi}{12 \times 3600\text{ s}} = 1.45 \times 10^{-4}\text{ rad/s}$$

Angular **speed** is a magnitude, so it is positive. Clock hands turn clockwise, so the angular **velocity** is ω_z = −1.75 × 10^{-3} rad/s and −1.45 × 10^{-4} rad/s.

ANSWER **(a)** $1.75 \times 10^{-3}\text{ rad/s}$ **(b)** $1.45 \times 10^{-4}\text{ rad/s}$

PROBLEM 5

Ranking angular speeds

Unit conversion · rev/min → rad/s

Watch the video

Rank in order of increasing angular speed: a car tire rotating at 2.00 × 10^3 deg/s, an electric drill rotating at 400.0 rev/min, and an airplane propeller rotating at 40.0 rad/s.

GIVEN $\omega_{\text{tire}} = 2.00 \times 10^3\text{ deg/s}$ $\omega_{\text{drill}} = 400.0\text{ rev/min}$ $\omega_{\text{prop}} = 40.0\text{ rad/s}$ FIND ranking

SOLUTION

Convert everything to rad/s before comparing.

Tire:

$$2.00 \times 10^3\ \frac{\text{deg}}{\text{s}} \times \frac{2\pi\text{ rad}}{360\text{ deg}} = 34.9\text{ rad/s}$$

Drill:

$$400.0\ \frac{\text{rev}}{\text{min}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} \times \frac{1\text{ min}}{60\text{ s}} = 41.9\text{ rad/s}$$

Propeller: already 40.0 rad/s.

ANSWER tire (34.9) < propeller (40.0) < drill (41.9) rad/s

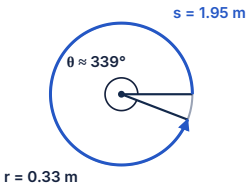
PROBLEM 6

Paint spot on a bicycle tire

Arc length · s = rθ

Watch the video

A spot of paint on a bicycle tire moves on a circle of radius 0.33 m. When the spot has traveled a linear distance of 1.95 m, through what angle has the tire rotated? Give your answer in radians.



GIVEN $r = 0.33\ \text{m}$ $s = 1.95\ \text{m}$ **FIND** θ

SOLUTION

The distance along the circle is the arc length s = rθ, with θ in radians:

$$\theta = \frac{s}{r} = \frac{1.95}{0.33} = 5.91\ \text{rad}$$

In degrees (for a feel of the size — slightly less than one full turn):

$$5.91\ \text{rad} \times \frac{180^\circ}{\pi} = 339^\circ$$

ANSWER $\theta \approx 5.9\ \text{rad}\ (\approx 339^\circ)$

PROBLEM 7

Earth orbiting the Sun

Angular speed · Unit conversion

Watch the video

What is the angular speed (in rev/min) of the Earth as it orbits the Sun?

GIVEN $\Delta\theta = 1\ \text{rev}$ $T = 365.25\ \text{days}$ **FIND** $\omega\ (\text{rev/min})$

SOLUTION

Convert one year to minutes:

$$T = 365.25 \times 24 \times 60 = 5.26 \times 10^5\ \text{min}$$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{1\ \text{rev}}{5.26 \times 10^5\ \text{min}} = 1.90 \times 10^{-6}\ \text{rev/min}$$

In SI units:

$$\omega = \frac{2\pi\ \text{rad}}{3.16 \times 10^7\ \text{s}} = 1.99 \times 10^{-7}\ \text{rad/s}$$

ANSWER $\omega \approx 1.9 \times 10^{-6}\ \text{rev/min}\ (1.99 \times 10^{-7}\ \text{rad/s})$

PROBLEM 8

Earth spinning on its axis

Angular speed

Watch the video

Find the angular speed of the Earth as it spins about its axis. Give your result in rad/s.

GIVEN $\Delta\theta = 2\pi\ \text{rad}$ $T = 24\ \text{h}$ **FIND** $\omega\ (\text{rad/s})$

SOLUTION

$$T = 24 \times 60 \times 60 = 86\,400\ \text{s}$$

$$\omega = \frac{2\pi\ \text{rad}}{86\,400\ \text{s}} = 7.27 \times 10^{-5}\ \text{rad/s}$$

Note: one turn relative to the distant stars (sidereal day) takes 23.93 h, which gives 7.29 × 10^{-5} rad/s. Using 24 h is the standard textbook approximation.

ANSWER $\omega \approx 7.27 \times 10^{-5}\ \text{rad/s}$

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Answer summary

Problem	Topic	Key physics idea	Final answer(s)
1	Degrees to radians	**Multiply by π/180**	$\frac{\pi}{6} = 0.524,\ \frac{\pi}{4} = 0.785,\ \frac{\pi}{2} = 1.57,\ \pi = 3.14\ \text{rad}$
2	Radians to degrees	**Multiply by 180/π**	$30^\circ,\ 126^\circ,\ 270^\circ,\ 900^\circ$
3	The hands of Big Ben	**ω = 2π / period**	**(a)** $1.75 \times 10^{-3}\ \text{rad/s}$ **(b)** $1.45 \times 10^{-4}\ \text{rad/s}$
5	Ranking angular speeds	**Compare only in the same unit (rad/s)**	$\text{tire}\ (34.9) < \text{propeller}\ (40.0) < \text{drill}\ (41.9)\ \text{rad/s}$
6	Paint spot on a bicycle tire	**Arc length s = rθ (θ in rad)**	$\theta \approx 5.9\ \text{rad}\ (\approx 339^\circ)$
7	Earth orbiting the Sun	**ω = 1 rev / period**	$\omega \approx 1.9 \times 10^{-6}\ \text{rev/min}\ (1.99 \times 10^{-7}\ \text{rad/s})$
8	Earth spinning on its axis	**ω = 2π / period**	$\omega \approx 7.27 \times 10^{-5}\ \text{rad/s}$

Angles in s = rθ and v = ωr are in radians. Answers are rounded to the significant figures given in each problem.

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