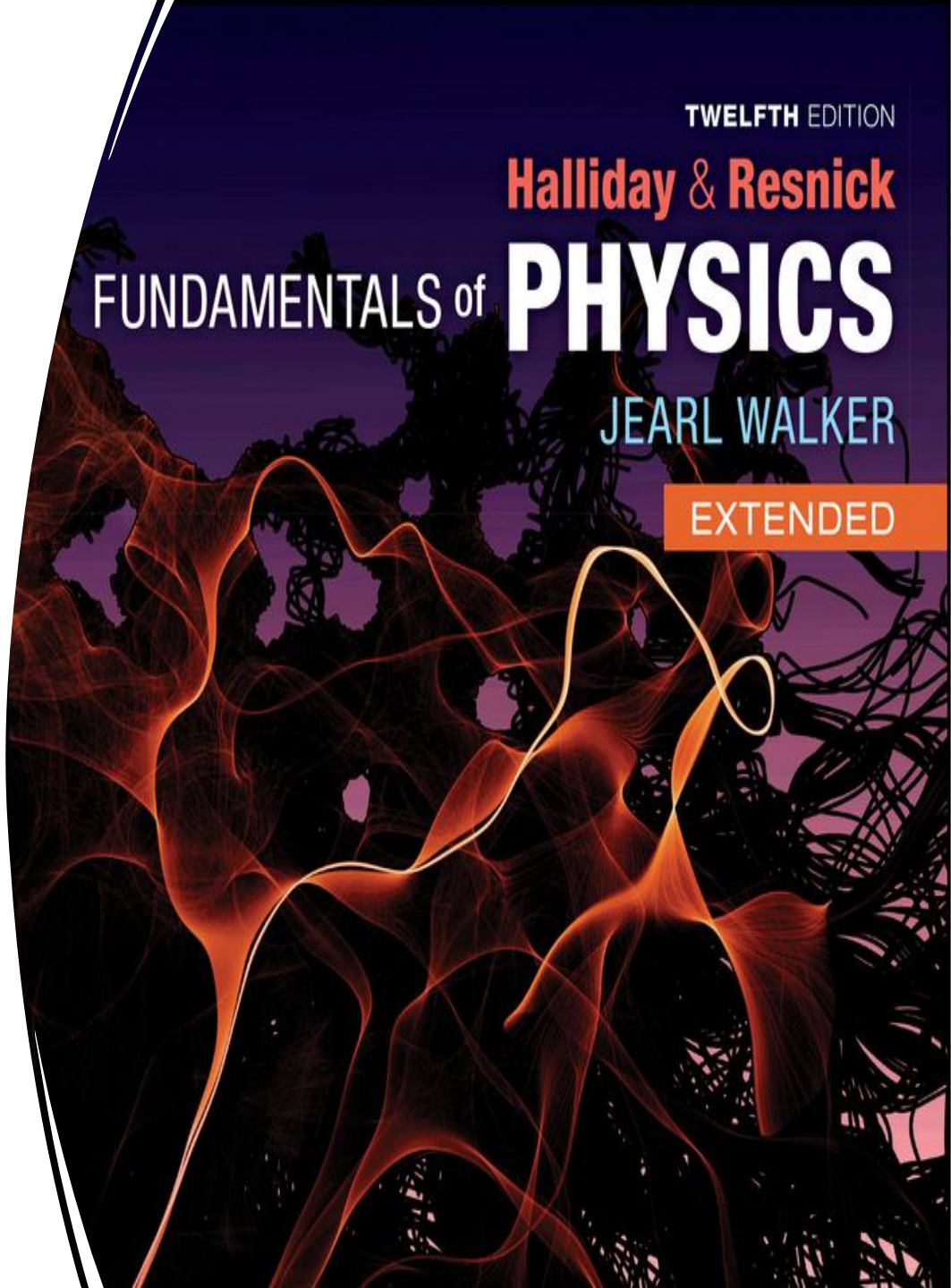
Chapter 9

Center of Mass and Linear Momentum



Units of Chapter 9

9.1. Center of Mass

9.3. Linear Momentum

9.4. Collision and Impulse

9.5. Conservation of Linear Momentum

9.6. Momentum and kinetic Energy in Collision

9.7. Elastic Collision in one dimension

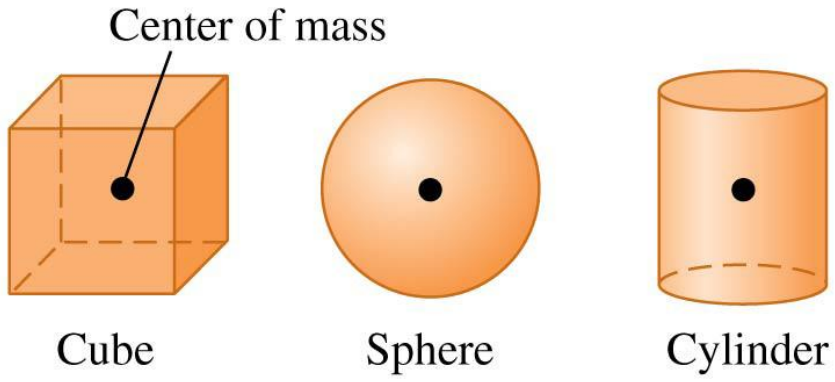
9.1 Center of Mass

- The motion of rotating objects can be complicated (imagine flipping a baseball bat into the air)
- But there is a special point on the object for which the motion is simple
- The center of mass of the bat traces out a parabola, just as a tossed ball does
- All other points rotate around this point



(*b*)

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If a homogeneous object has a geometric center, that is where the center of mass is located.

9.1 Center of Mass

- The **center of mass** (com) of a system of particles:

  The center of mass of a system of particles is the point that moves as though (1) all of the system's mass were concentrated there and (2) all external forces were applied there.

For many particles, we can generalize the equation, where $M = m_1 + m_2 + \ldots + m_n$:

$$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots + m_n x_n}{M}$$

$$= \frac{1}{M}\sum_{i=1}^{n} m_i x_i .$$

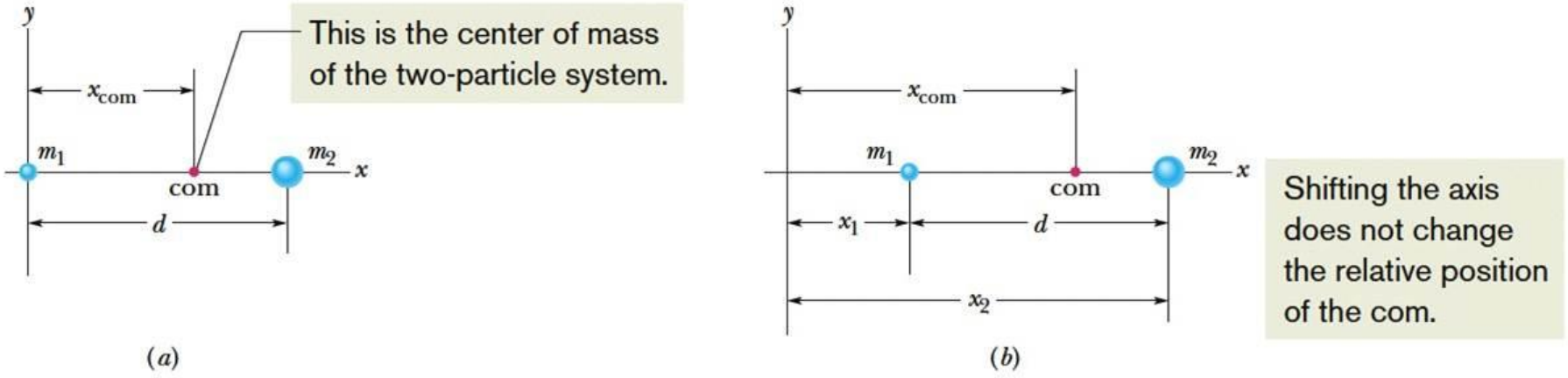
In three dimensions, we find the center of mass along each axis separately:

$$x_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i , \quad y_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i , \quad z_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i .$$

- For two particles, for an arbitrary choice of origin:

$$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}.$$

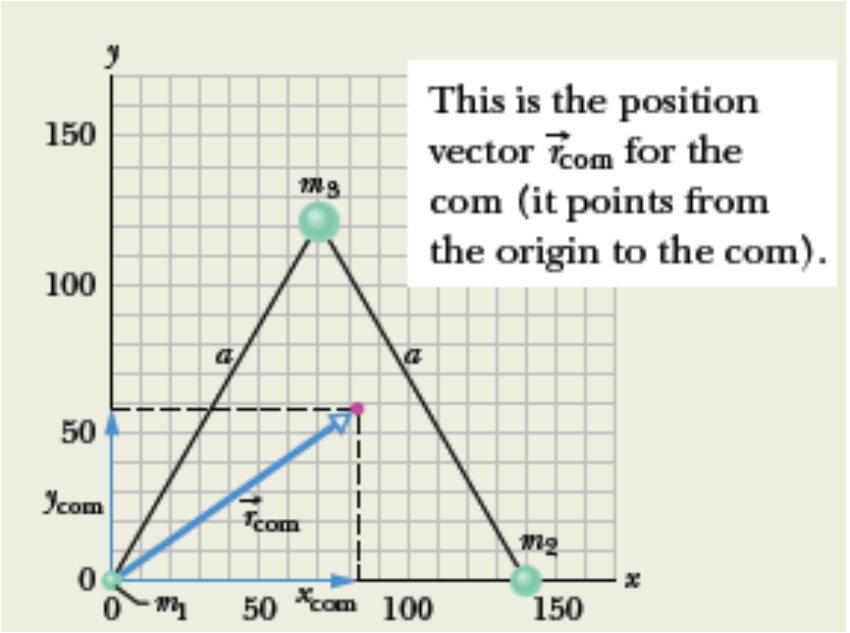
- The center of mass is in the same location regardless of the coordinate system used
- It is a property of the <u>particles</u>, not the coordinates



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Example 1: Three particles of masses $m_1 = 1.2$ kg, $m_2 = 2.5$ kg, and $m_3 = 3.4$ kg form an equilateral triangle of edge length $a = 140$ cm. Where is the center of mass of this system?

Answer: $x_{com} = 83\ cm, y_{com} = 58\ cm$



The three particles then have the following coordinates:

Particle	Mass (kg)	x (cm)	y (cm)
1	1.2	0	0
2	2.5	140	0
3	3.4	70	120

$$x_{com} = \frac{1}{M}\sum_{i=1}^{3} m_i x_i = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{M}$$

$$= \frac{(1.2\text{ kg})(0) + (2.5\text{ kg})(140\text{ cm}) + (3.4\text{ kg})(70\text{ m})}{7.1\text{ kg}}$$

$$= 83\text{ cm} \qquad \text{(Answer)}$$

$$\text{and} \quad y_{com} = \frac{1}{M}\sum_{i=1}^{3} m_i y_i = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{M}$$

$$= \frac{(1.2\text{ kg})(0) + (2.5\text{ kg})(0) + (3.4\text{ kg})(120\text{ m})}{7.1\text{ kg}}$$

$$= 58\text{ cm.} \qquad \text{(Answer)}$$

9.3 Linear Momentum

- And, of course, sports teams or individual players can gain or lose momentum.
- What one implies with these statements is that the objects said to gain momentum are now harder to stop.
- Physically, the more momentum an object has, the more force has to be applied to stop it in a given time.

Imagine coasting down a hill of uniform slope on your bike without pedaling or using the brakes. Because of the force of gravity, you will accelerate; that is, your velocity will increase with time. This idea is often expressed by saying that you are "picking up speed" or "gathering momentum."

The faster you move, **the more momentum you have** and **the more difficult it is to come to a stop**.



9.3 Linear Momentum

- The **linear momentum** is defined as:

$$\vec{p} = m\vec{v}$$

  - Where: $\vec{p}$ = linear momentum, m = mass, and $\vec{v}$ = velocity
- The momentum:
  - Points in the same direction as the velocity
  - Can only be changed by a net external force
- SI unit of momentum: $kg \cdot m/s$

Change in Momentum

A beanbag bear and a rubber ball, with the same mass *m* and the same downward speed *v*, hit the floor. **(a)** The beanbag bear comes to rest on hitting the floor.
(b) The rubber ball bounces upward with a speed *v*.

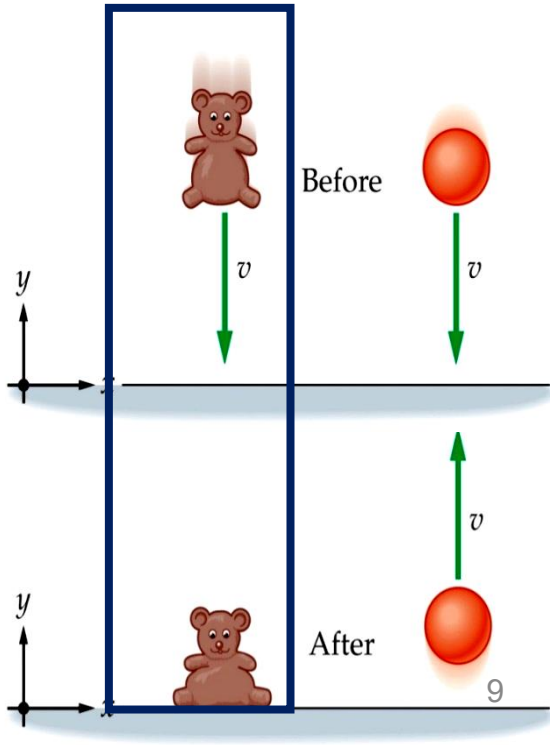
$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

- Teddy Bear: $\Delta p = 0 - (-mv) = mv$

$$= (0.10\ \text{kg})(4.0\ \text{m/s}) = 0.40\ \text{kg} \cdot \text{m/s}$$

- Bouncing Ball: $\Delta p = mv - (-\text{mv}) = 2mv$

$$= 2(0.10\ \text{kg})(4.0\ \text{m/s}) = 0.80\ \text{kg} \cdot \text{m/s}$$



Momentum Form of Newton's Second Law

The time rate of change of the momentum of a particle is equal to the net force acting on the particle and is in the direction of that force.

We can write Newton's second law thus:

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}.$$

the relations $\vec{F}_{net} = \frac{d\vec{P}}{dt}$ and $\vec{F}_{net} = m\vec{a}$ are equivalent expressions of Newton's second law of motion for a particle.

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}.$$

Example 2:

1. A 1180 kg car drives along a city street at 13.4 m/s. What is the magnitude of the car's momentum?

2. A major-league pitcher can give a 0.142 kg baseball a speed of 45.1 m/s. Find the magnitude of the baseball's momentum?

Answer: (1) 15,800 kg.m/s, (2) 6.40 kg.m/s

SOLUTION

a. Using $p = mv$, we find

$$p_c = m_c v_c = (1180\ \text{kg})(13.4\ \text{m/s}) = 15{,}800\ \text{kg} \cdot \text{m/s}$$

b. Similarly,

$$p_b = m_b v_b = (0.142\ \text{kg})(45.1\ \text{m/s}) = 6.40\ \text{kg} \cdot \text{m/s}$$

Example 3: A 0.70 kg ball moving horizontally at 5.0 m/s strikes a vertical wall and rebounds with speed 2.0 m/s. What is the magnitude of the change in its linear momentum?

Answer: $\Delta p = 4.9\ kg.m/s$

18. The magnitude of the ball's momentum change is

$$\Delta p = m\left|v_i - v_f\right| = (0.70\ \text{kg})\left|(5.0\ \text{m/s}) - (-2.0\ \text{m/s})\right| = 4.9\ \text{kg}\cdot\text{m/s}.$$

Example 4: A 0.15 kg baseball is dropped from rest. If the magnitude of the baseball's momentum is 0.780 kg.m/s just before it lands on the ground, from what height was it dropped?

Answer: h = 1.38 m

Solution: 1. Use equation 9-1 to find the speed of the ball when it lands:

$$v = \frac{p}{m} = \frac{0.780\ \text{kg}\cdot\text{m/s}}{0.150\ \text{kg}} = \underline{\underline{5.20\ \text{m/s}}}$$

2. (b) Use equation 2-12 to solve for y_0. Let $y = 0$ and $v_0 = 0$:

$$v^2 = v_0^2 - 2g(y - y_0)$$

$$y_0 = \frac{v^2}{2g} = \frac{(5.20\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = \boxed{1.38\ \text{m}}$$

Example 5: A 2100 kg truck traveling north at 41 km/h turns east and accelerates to 51 km/h.
(a) What is the change in the truck's kinetic energy?
(b) What are the magnitude and direction of the change in its momentum?
Answer: $\Delta K = 7.5 \times 10^4\ J, (b)\ \Delta v = 18.2 \frac{m}{s}, \theta = -39°$

19. (a) The change in kinetic energy is

$$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}(2100\ \text{kg})\left((14.2\ \text{m/s})^2 - (11.4\ \text{m/s})^2\right)$$

$$= 7.5\times10^4\ \text{J}.$$

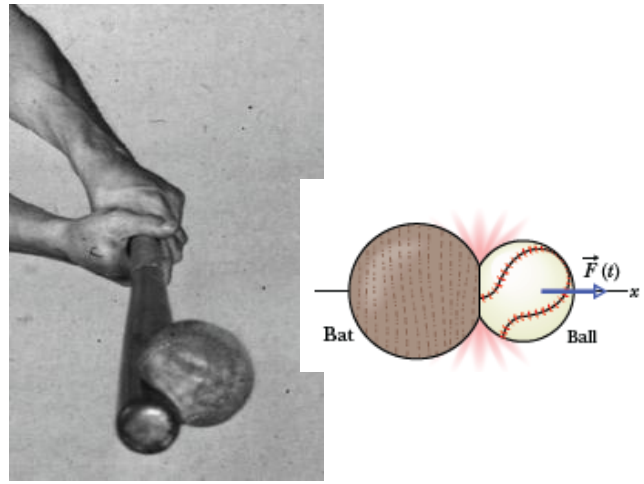
9.4 Collision and Impulse

An impulse is a **force** exerted over a finite period of **time.** Impulse is a vector, in the same direction as the average force. We also define the **impulse *J*** acting during a collision

$$\vec{J} = \vec{F}_{avg}\Delta t$$

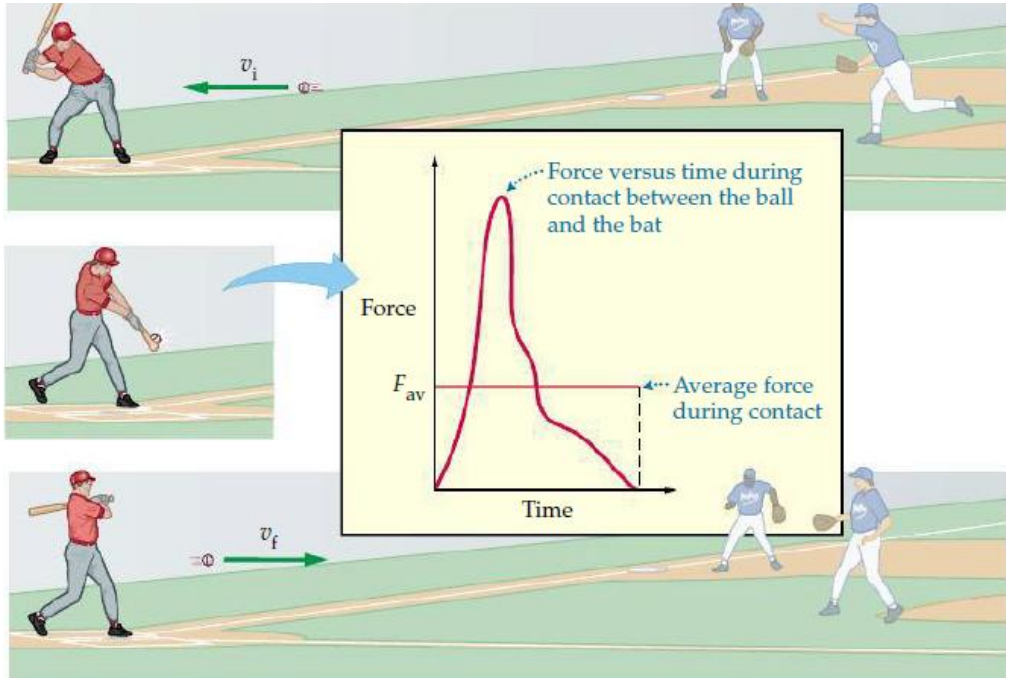
$\vec{F}_{avg}$ the average force and the Δt duration of the collision.

SI unit of impulse: $N \cdot s$



The collision of a ball with a bat collapses part of the ball.

The curve shows the magnitude of the time-varying force *F*(*t*) that acts on the ball in the collision. The area under the curve is equal to the magnitude of the impulse on the ball in the collision.
when a bat hits a baseball, rises rapidly to very large values, then drops again to zero in a matter of milliseconds.
We focus on its average value, F_{avg}, Note that the area under the F_{avg} rectangle is the same as the area under the actual force curve.



Momentum–Impulse Theorem

An **impulse** will **change the momentum** of an object in accordance with Newton's second law

We see that the average force times is simply the change in momentum of the ball due to the bat:

$$\vec{F}_{avg} = \frac{\Delta\vec{P}}{\Delta t} \quad \Rightarrow \quad \Delta\vec{P} = \vec{F}_{avg}\Delta t$$

the change in an object's momentum is equal to the impulse on the object;

$$\Delta\vec{P} = \vec{F}_{avg}\Delta t = \vec{J}$$

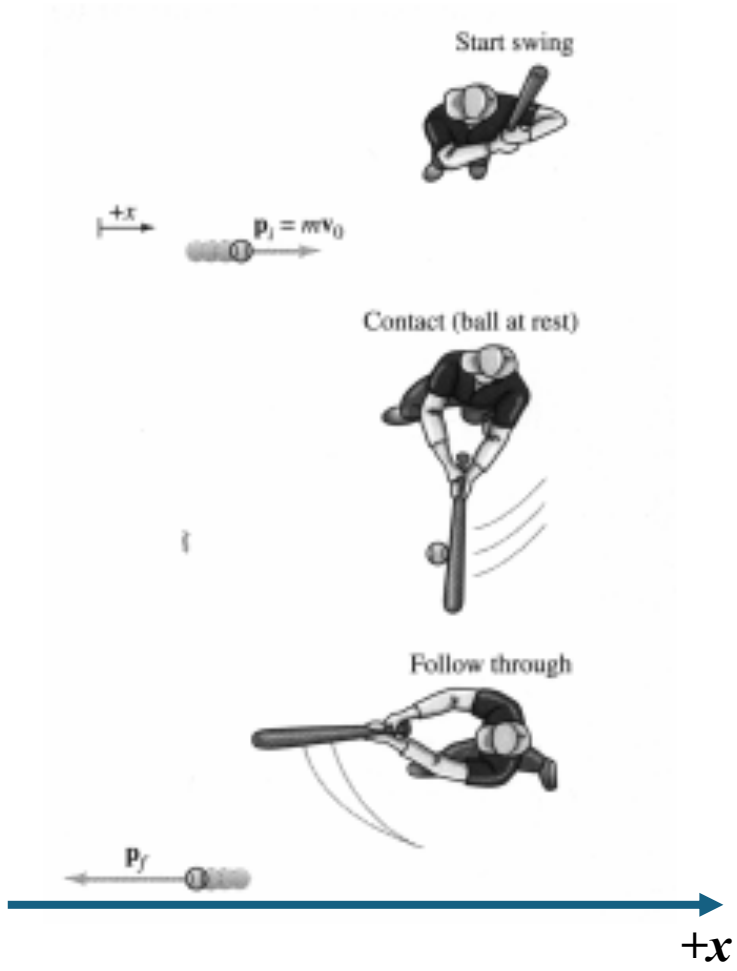
linear momentum–impulse theorem

Example 6: A 0.2 kg baseball traveling at 40 m/s is hit and returned at 50 m/s in the opposite direction. If the ball and bat are in contact for 0.002 s, determine the **average force on the ball?**

Answer: $F = -9000\ N$

$$I = F\Delta t = \Delta p$$

$$F = \frac{\Delta p}{\Delta t} = \frac{mv_f - mv_i}{\Delta t} = \frac{(0.2kg)\left(-50\frac{m}{s}\right) - (0.2kg)(40\frac{m}{s})}{0.002s} = -9000N$$



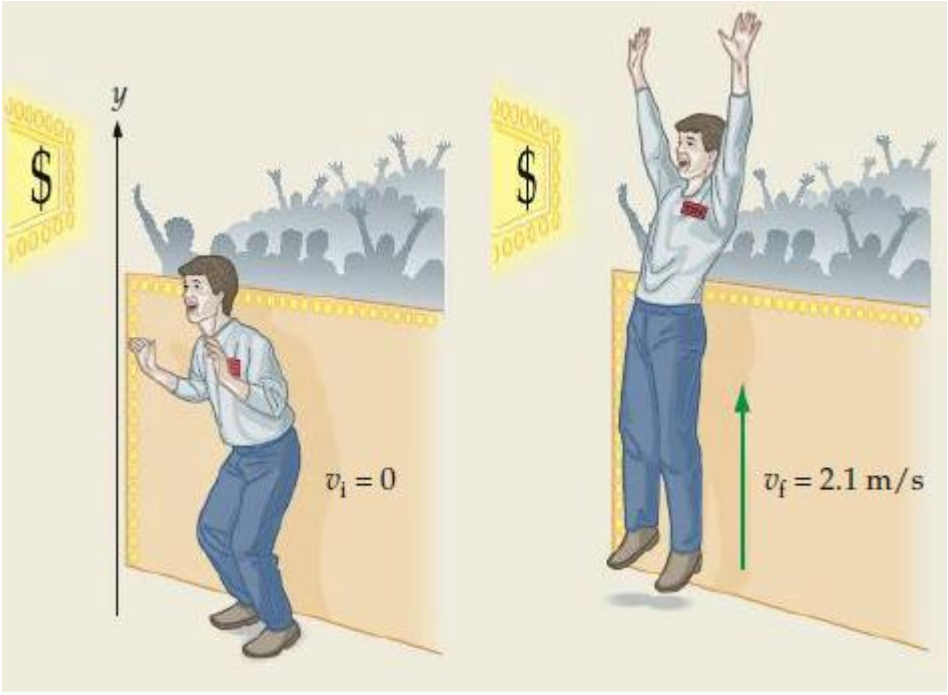
Example 7: After wining a prize on a game shown, a 72 kg contestant jumps for joy.

a) If the jump results in an upward speed of 2.1 m/s, what is the impulse experienced by the contestant?

b) Before the jump, the floor exerts an upward force of (mass×g) on the contestant. What additional average upward force does the floor exert if the contestant pushes down on it for 0.36 s during the jump?

Given: $m = 72kg, \quad v_f = 2.1m/s\,, \quad t = 0.36s$

Answer: $F = (420\hat{y})\ N$



SOLUTION

Part (a)

1. Write an expression for the impulse, noting that $\vec{v}_i = 0$: $\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = m\vec{v}_f$
2. Substitute numerical values: $\vec{I} = m\vec{v}_f = (72\ \text{kg})(2.1\ \text{m/s})\hat{y} = (150\ \text{kg}\cdot\text{m/s})\hat{y}$

Part (b)

3. Express the average force in terms of the impulse $\vec{I}$ and the time interval Δt: $\vec{F}_{av} = \dfrac{\vec{I}}{\Delta t} = \dfrac{(150\ \text{kg}\cdot\text{m/s})\hat{y}}{0.36\ \text{s}} = (420\ \text{kg}\cdot\text{m/s}^2)\hat{y} = (420\ \text{N})\hat{y}$

Example 8: A force in the negative direction of an *x* axis is applied for 27 ms to a 0.40 kg ball initially moving at 14 m/s in the positive direction of the axis. The force varies in magnitude, and the impulse has magnitude 32.4 N · s.

What are the ball's (a) speed and (b) direction of travel just after the force is applied?

Answer: $Favg = 1.20 \times 10^3\ N, v_f = -67\frac{m}{s}$

The initial direction of motion is in the $+x$ direction. The magnitude of the average force F_{avg} is given by

$$F_{avg} = \frac{J}{\Delta t} = \frac{32.4\ \text{N}\cdot\text{s}}{2.70\times10^{-2}\ \text{s}} = 1.20\times10^{3}\ \text{N}.$$

The force is in the negative direction. Using the linear momentum–impulse theorem stated we have

$$-F_{avg}\Delta t = J = \Delta p = m(v_f - v_i),$$

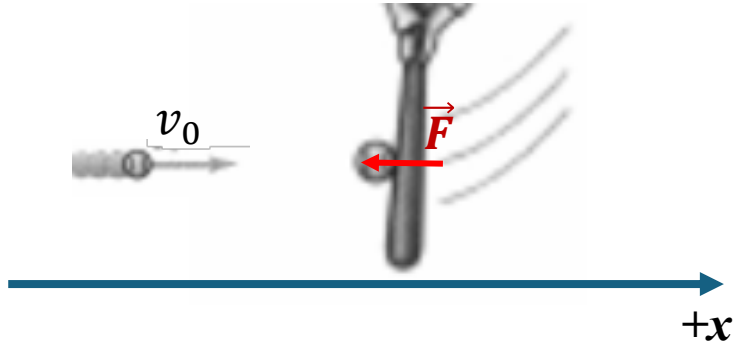
(a) Using the two expressions, we find

$$v_f = \frac{mv_i - F_{avg}\Delta t}{m}$$

$$= \frac{(0.40\,\text{kg})(14\,\text{m/s}) - (1200\,\text{N})(27\times10^{-3}\,\text{s})}{0.40\,\text{kg}} = -67\,\text{m/s}.$$

The final speed of the ball is $|v_f| = 67$ m/s.

(b) The negative sign for v_f indicates that the velocity is in the $-x$ direction, which is opposite the initial direction of travel.



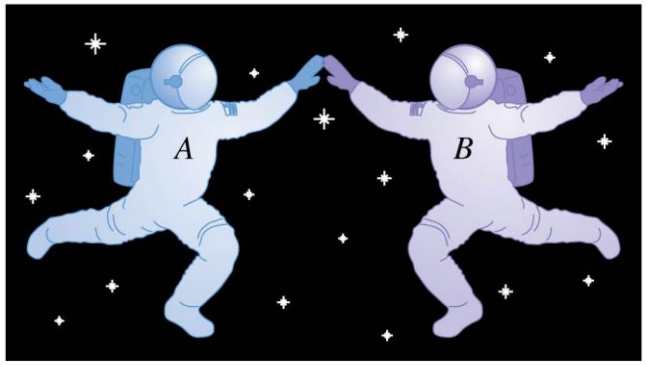
9.5 Conservation of Linear Momentum

Suppose that the net external force $\vec{F}_{net}$ (and thus, the net impulse $\vec{J}$) acting on a system of particles is zero (the system is isolated) and that no particles leave or enter the system (the system is closed)

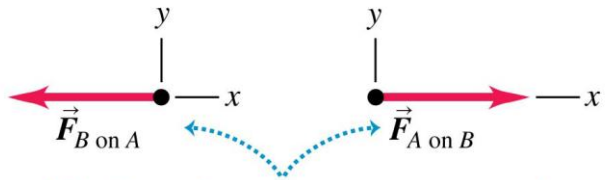
$$\vec{P} = \text{constant} \quad (\text{closed, isolated system}).$$

Which says that:

If no net external force acts on a system of particles, the total linear momentum $\vec{P}$ of the system cannot change.



No external forces act on the two-astronaut system, so its total momentum is conserved.



The forces the astronauts exert on each other form an action–reaction pair.

This result is called the **law of conservation of linear momentum** and is an extremely powerful tool in solving problems. In the homework we usually write the law as

$$\vec{P}_i = \vec{P}_f$$

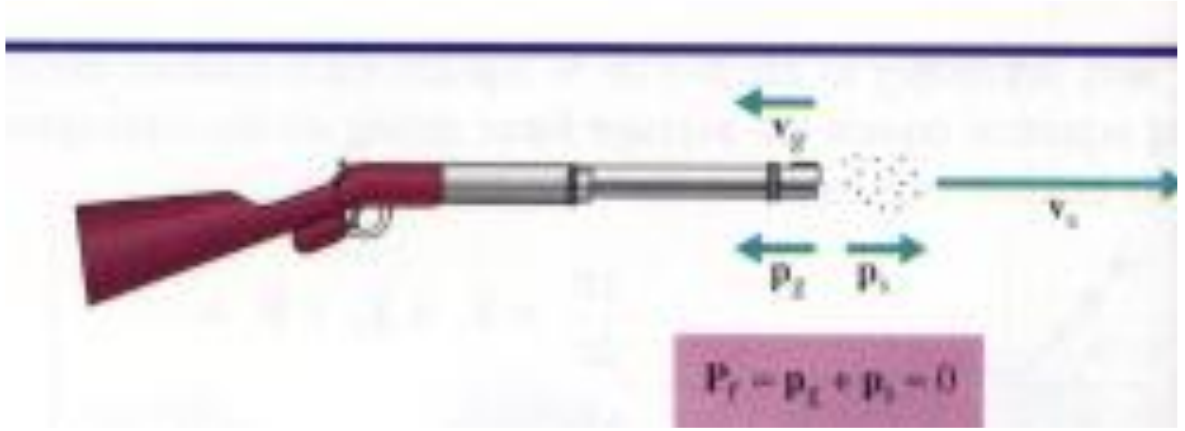
In words, this equation says that, for a closed, isolated system,

$$\begin{pmatrix}\text{total linear momentum}\\ \text{at some initial time } t_i\end{pmatrix} = \begin{pmatrix}\text{total linear momentum}\\ \text{at some later time } t_f\end{pmatrix}.$$

Example 9: A shotgun with a mass of 3.0 kg fires a shot with a total mass of 0.05 kg at a muzzle velocity of 525 m/s. What is the recoil velocity of the shotgun?

Answer: $v_g = -9$ m/s

$$\vec{P}_f = \vec{P}_i$$
$$(m_g + m_s)v_i = m_g v_g + m_s v_s$$



Example 10: A 5.20 g bullet moving at 672 m/s strikes a 700 g wooden block at rest on a frictionless surface. The bullet emerges, traveling in the same direction with its speed reduced to 428 m/s. What is the resulting speed of the block?

Answer: $v_2 = 1.81$ m/s

$$m_{\text{bullet}}\vec{v}_i = m_{\text{bullet}}\vec{v}_1 + m_{\text{block}}\vec{v}_2$$
$$(5.2\text{ g})(672\text{ m/s}) = (5.2\text{ g})(428\text{ m/s}) + (700\text{ g})\vec{v}_2$$

which yields $v_2 = 1.81$ m/s.

9.6 Momentum and Kinetic Energy in Collisions

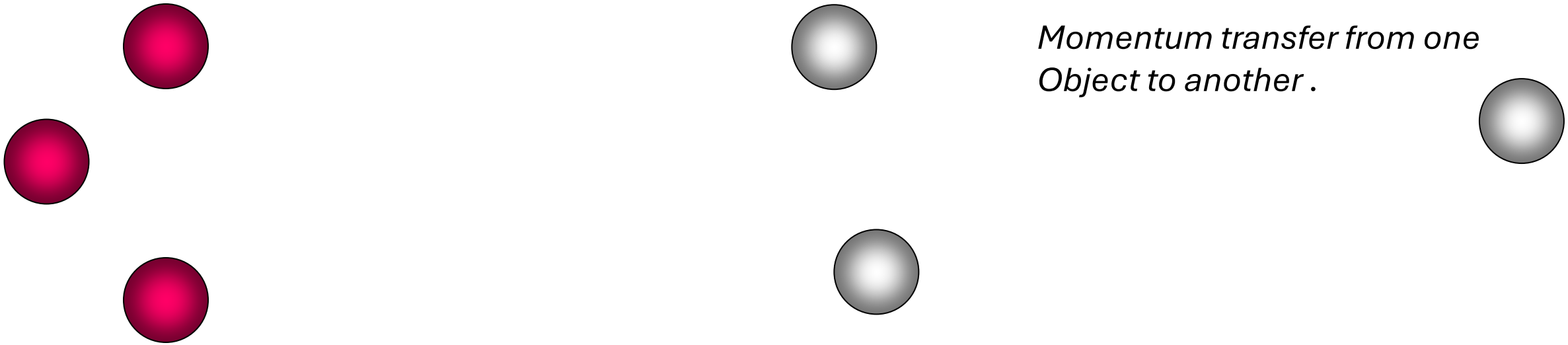
The collision is defined as two objects strike one another, and in which the net external force is either zero or negligibly small. For example, if two train cars roll along on a level track and hit one another, this is a collision.

Time of collision is short enough that external forces may be ignored. As result, the momentum of the two-car system is conserved.

9.6 Momentum and Kinetic Energy in Collisions

Types of Collisions

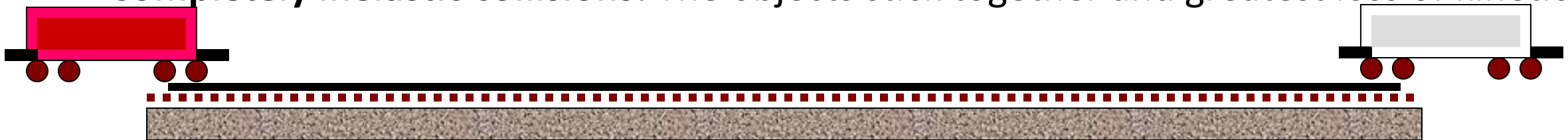
- **Elastic collisions:** Total kinetic energy is unchanged (conserved) it is the same before and after the collision)



Momentum transfer from one Object to another .

- **Inelastic collisions:** some energy is transferred into a different form

Completely inelastic collisions: The objects stick together and greatest loss of kinetic



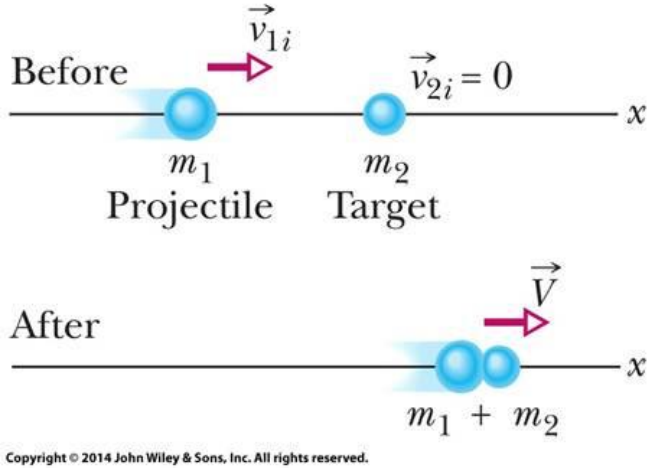
9.6 Momentum and Kinetic Energy in Collisions

In an **inelastic collision**, the total kinetic energy after the collision is less than before the collision. Inelastic collision: momentum is conserved but kinetic energy is not
A collision in which the bodies stick together is called a **completely inelastic collision**.

One-Dimensional Completely Inelastic Collision

Figure shows two bodies before and after they have a completely inelastic collision. The body with mass $m2$ happens to be initially at rest (v_{2i} = 0). After the collision, the stuck-together bodies move with velocity V.

In a completely inelastic collision, the bodies stick together.



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Before $p_{\mathrm{i}} = m_1 v_{1,\mathrm{i}} + m_2 v_{2,\mathrm{i}}$

After $p_{\mathrm{f}} = (m_1 + m_2)\, V$

$$V = \frac{m_1 v_{1,\mathrm{i}} + m_2 v_{2,\mathrm{i}}}{m_1 + m_2}$$

A special case *at rest*: v_{2i} = 0

$$V = \frac{m_1 v_{1,i}}{m_1 + m_2}$$

Example 11: A 1200 kg car moving at 2.5 m/s is struck in the rear by a 2600 kg truck moving at 6.2 m/s.
If the vehicles stick together after the collision, what is their speed immediately after colliding ? (Assume that external forces may be ignored)



Answer: 5.0 m/s

The speed of the vehicles immediately after the collision can be found from the momentum conservation:

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$$

$$v = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$v = \frac{(1200kg)\left(2.5\frac{m}{s}\right) + (2600kg)(6.2\frac{m}{s})}{1200 + 2600} = 5.0\, m/s$$

Example 12: On a touchdown attempt, a 95.0 kg running back runs toward the end zone at 3.75 m/s. A 111 kg line backer moving at 4.10 m/s meets the runner in a head-on collision and locks his arms around the runner.

(a) Find their velocity immediately after the collisions.

(b) Find the initial and final kinetic energies and the energy lost in the collision.

Answers: $v_f = -0.48$ m/s, $K_i = 1600$ J, $K_f = 23.7$ J ,$\Delta K = -1576$ J

SOLUTION

Part (a)

1. Set the initial momentum equal to the final momentum:

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}_f$$

2. Solve for the final velocity and substitute numerical values, being careful to use the appropriate signs:

$$\vec{v}_f = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

$$= \frac{(95.0\text{ kg})(3.75\text{ m/s})\hat{x} + (111\text{ kg})(-4.10\text{ m/s})\hat{x}}{95.0\text{ kg} + 111\text{ kg}}$$

$$= (-0.480\text{ m/s})\hat{x}$$

Part (b)

3. Calculate the initial kinetic energy of the two players:

$$K_i = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2$$

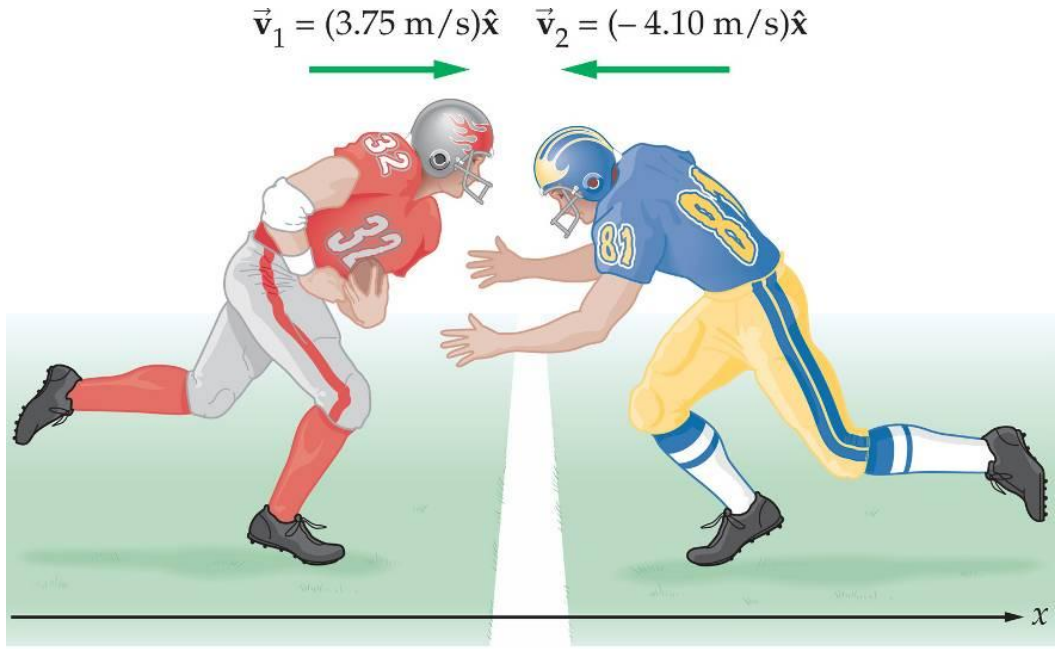
$$= \tfrac{1}{2}(95.0\text{ kg})(3.75\text{ m/s})^2 + \tfrac{1}{2}(111\text{ kg})(-4.10\text{ m/s})^2$$

$$= 1600\text{ J}$$

4. Calculate the final kinetic energy of the players, noting that they both move with the same velocity after the collision:

$$K_f = \tfrac{1}{2}(m_1 + m_2)v_f^2$$

$$= \tfrac{1}{2}(95.0\text{ kg} + 111\text{ kg})(-0.480\text{ m/s})^2 = 23.7\text{ J}$$

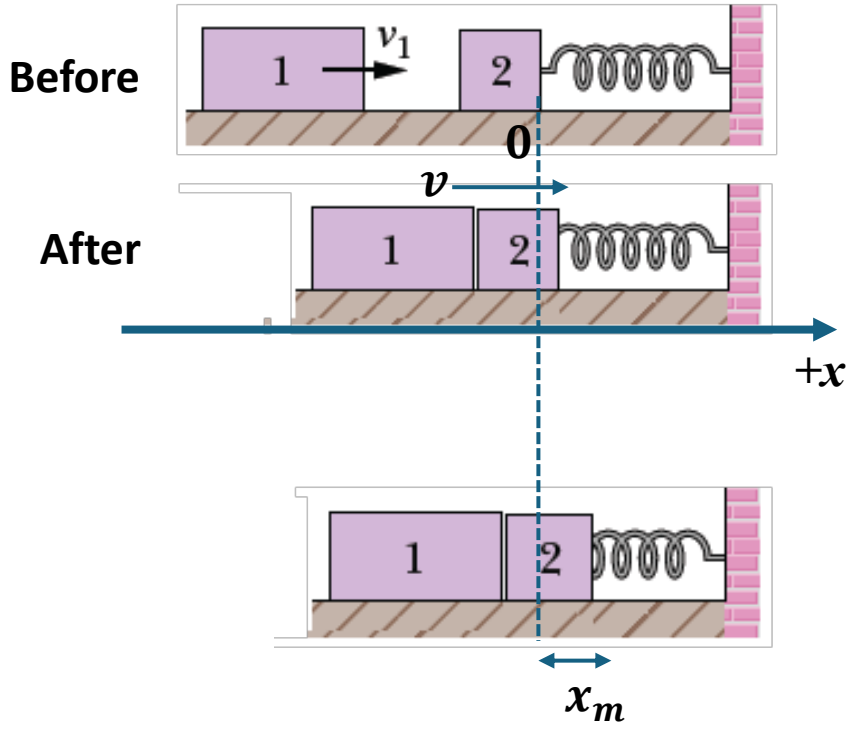


Example 13: In figure, block 2 (mass 1.0 kg) is at rest on a frictionless surface and touching the end of an unstretched spring of spring constant 200 N/m. The other end of the spring is fixed to a wall. Block 1 (mass 2.0 kg), traveling at speed *v*1 = 4.0 m/s, collides with block 2, and the two blocks stick together.

(a) Find their velocity immediately after the collision

(b) When the blocks momentarily stop, by what distance is the spring compressed?

Answer: $V = 2.7$ m/s, $x_m = 0.33\ m$



$m_1 v_1 = (m_1+m_2)v \Rightarrow (2.0\ \text{kg})(4.0\ \text{m/s}) = (3.0\ \text{kg})\vec{v}$ which yields $\vec{v} = 2.7\ \text{m/s}$.

The second part involves mechanical energy conservation:

$$\frac{1}{2}(3.0\ \text{kg})(2.7\ \text{m/s})^2 = \frac{1}{2}(200\ \text{N/m})x_m^2$$ which gives the result $x_m = 0.33$ m.

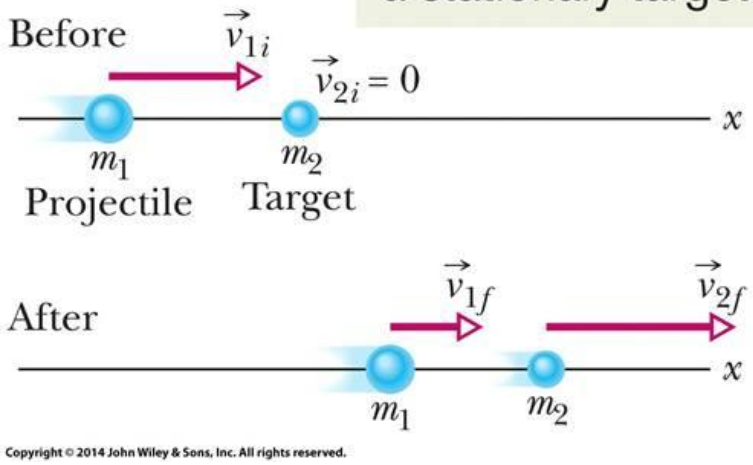
9.7 Elastic Collisions in One Dimension

- Total kinetic energy is conserved in elastic collisions. In an elastic collision, the kinetic energy of each colliding body may change, but the total kinetic energy of the system does not change.
- For a stationary target (**special case:** If $\boldsymbol{v}_{1,i} = \boldsymbol{v}_0$ and $\boldsymbol{v}_{2,i} = 0$: (that is, initially one mass is moving and the second mass is at rest), conservation laws give:

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \quad \text{(linear momentum)}.$$

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad \text{(kinetic energy)}.$$

Here is the generic setup for an elastic collision with a stationary target.



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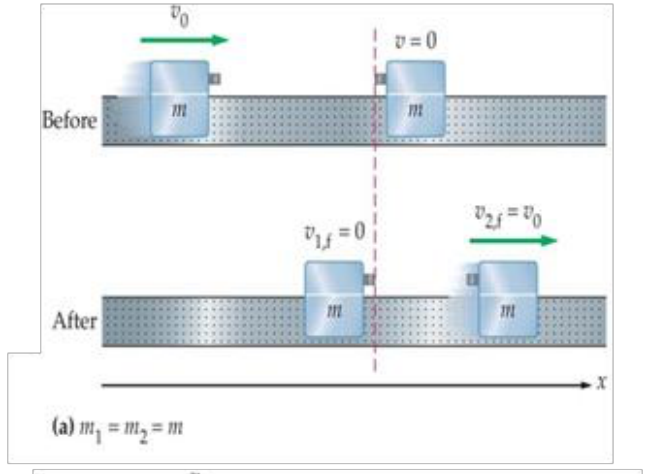
- With some algebra we get:

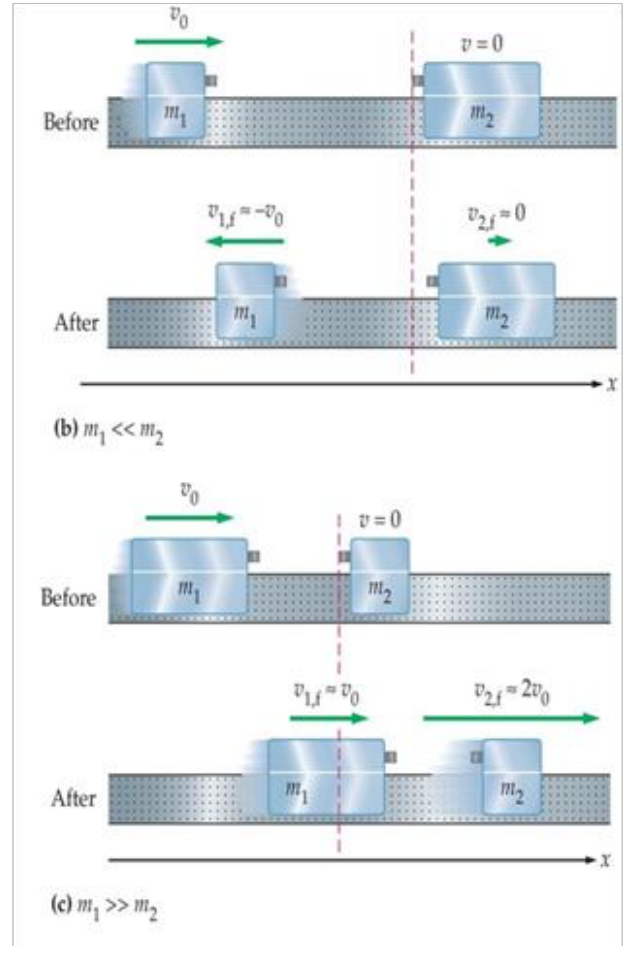
$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}.$$

Special Conditions of Mass & Velocity in Elastic Collisions

- Equal masses: $v_{1f} = 0$, $v_{2f} = v_{1i}$: the first object stops
- Massive target, $m_2 >> m_1$: the first object just bounces back, speed mostly unchanged
- Massive projectile: $v_{1f} \approx v_{1i}$, $v_{2f} \approx 2v_{1i}$: the first object keeps going, the target flies forward at about twice its speed





Example 14: At an amusement park, a 96 kg bumper car moving with a speed of 1.24 m/s bounces elastically off a 135 kg bumper car at rest.
Find the final velocities of the cars.

Answers : $V_{f,1}$= – 0.209 m/s, $V_{f,2}$= 1.03 m/s



$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}.$$

SOLUTION

Using Equations 9–12, we find the final velocities to be $v_{1,f} = -0.209$ m/s and $v_{2,f} = 1.03$ m/s. Note that the direction of travel of car 1 has been reversed.

Chapter 9 Summary: Linear Momentum Definition and Newton's Second Law

- Linear momentum defined as: $\vec{p} = m\vec{v}$
- Write Newton's 2nd law: $\vec{F}_{\text{net}} = \dfrac{d\vec{P}}{dt}$

Collision and Impulse Defined as: $\vec{J} = \vec{F}_{avg}\Delta t$

- Impulse causes changes in linear momentum

Conservation of Linear Momentum $\vec{P} = \text{constant} \quad (\text{closed, isolated system}).$

Inelastic Collision in 1D : Momentum conserved along that dimension

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}.$$

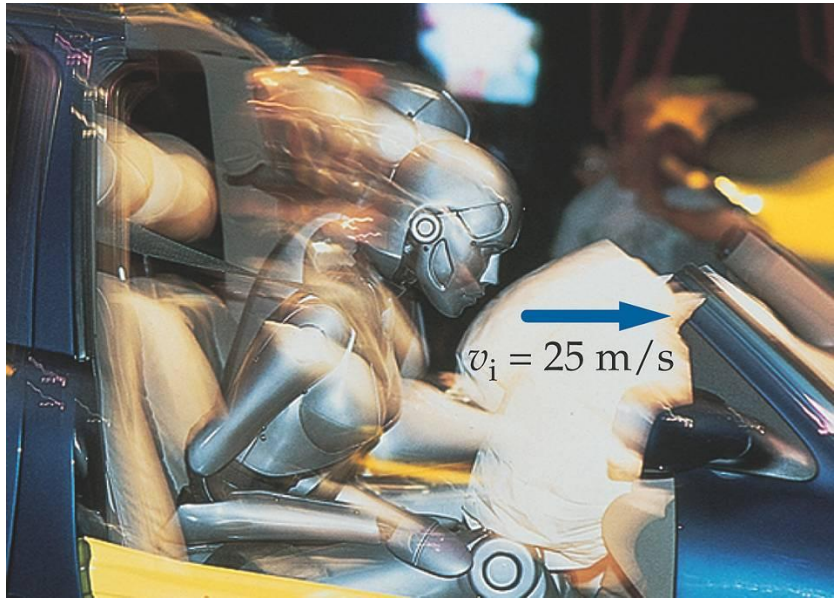
Elastic Collisions in One Dimension

- *K* is also conserved

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \qquad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}.$$

Additional Exercises

A car equipped with an 80 kg crash dummy drives into a massive concrete wall at 25 m/s and the dummy stops in 0.08 s. Estimate the average force exerted on the dummy.



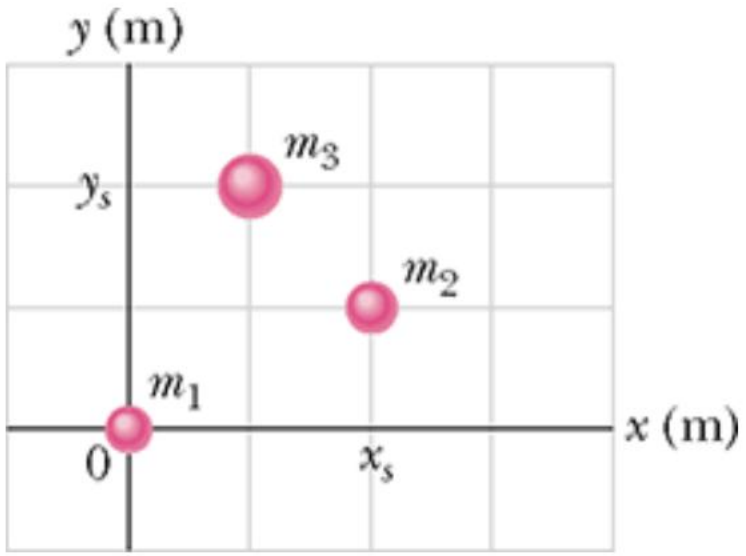
Answers

$$\vec{I} = \vec{F}_{av}\Delta t = \Delta\vec{p} = m\Delta\vec{v}$$
$$= (80\text{ kg})(-25\text{ m/s}\,\hat{x}) = -2000\text{ N s}\,\hat{x}$$

$$\vec{F}_{av} = I/\Delta t = (-2000\text{ N s}\,\hat{x})/(0.080\text{ s}) = -25{,}000\text{ N}\,\hat{x}$$



2 E Figure 9.13 shows a three-particle system, with masses m_1 = 3.0 kg, m_2 = 4.0 kg, and m_3 = 8.0 kg. The scales on the axes are set by x_s = 2.0 m and y_s = 2.0 m. What are (a) the x coordinate and (b) the y coordinate of the system's center of mass?



Solution

(a) The x coordinate of the center of mass is

$$x_{com} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{0 + (4.0\text{ kg})(2.0\text{ m}) + (8.0\text{ kg})(1.0\text{ m})}{3.0\text{ kg} + 4.0\text{ kg} + 8.0\text{ kg}} = 1.1\text{ m}.$$

(b) The y coordinate of the center of mass is

$$y_{com} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{0 + (4.0\text{kg})(1.0\text{m}) + (8.0\text{kg})(2.0\text{m})}{3.0\text{kg} + 4.0\text{kg} + 8.0\text{kg}} = 1.3\text{m}.$$

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Example : A 2.00 kg particle has the *xy* coordinates (−1.20 m, 0.500 m), and a 4.00 kg particle has the *xy* coordinates (0.600 m, −0.750 m). Both lie on a horizontal plane. At what (a) *x* and (b) *y* coordinates must you place a 3.00 kg particle such that the center of mass of the three-particle system has the coordinates (−0.500 m, −0.700 m)?

(a) The x coordinate of the system's center of mass is:

$$x_{com} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{(2.00\text{ kg})(-1.20\text{ m}) + (4.00\text{ kg})(0.600\text{ m}) + (3.00\text{ kg})x_3}{2.00\text{ kg} + 4.00\text{ kg} + 3.00\text{ kg}}$$
$$= -0.500\text{ m}.$$

Solving the equation yields $x_3 = -1.50$ m.

(b) The y coordinate of the system's center of mass is:

$$y_{com} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{(2.00\text{ kg})(0.500\text{ m}) + (4.00\text{ kg})(-0.750\text{ m}) + (3.00\text{ kg})y_3}{2.00\text{ kg} + 4.00\text{ kg} + 3.00\text{ kg}}$$
$$= -0.700\text{ m}.$$

Solving the equation yields $y_3 = -1.43$ m.

Impulse

Example: A 10 kg cart collides with a wall and changes its direction, as shown. What is its change in x-momentum Δp_x?

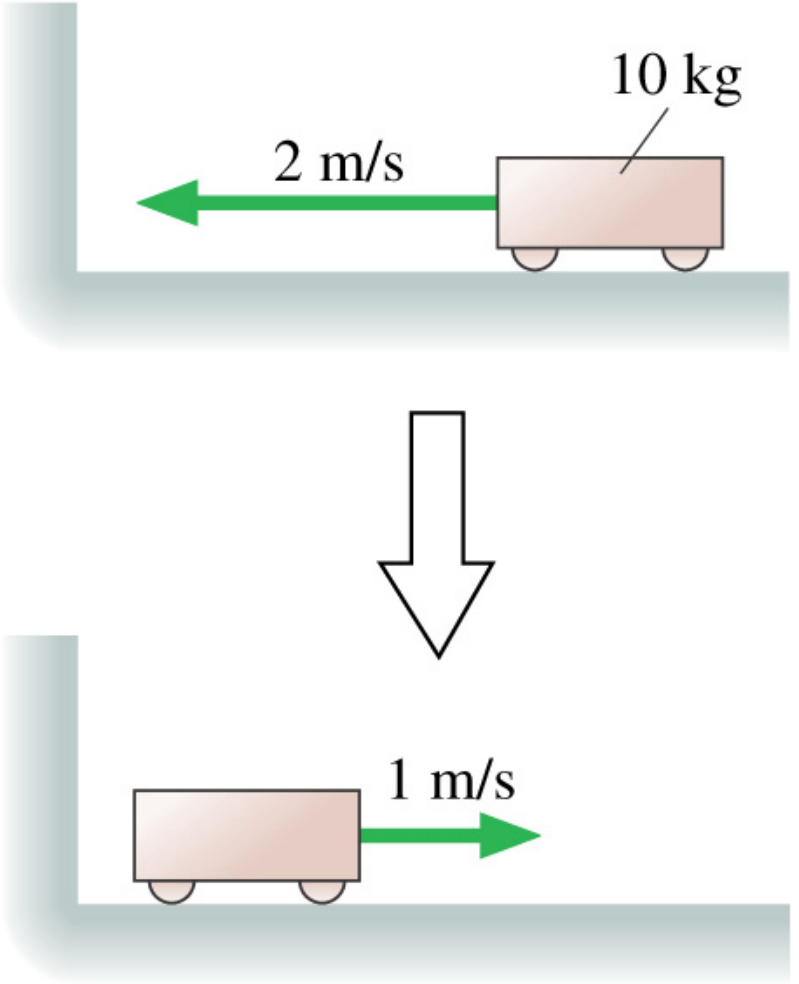
a) 30 kg.m/s left

b) 10 kg.m/s left

c) 10 kg.m/s right

d) 20 kg.m/s right

e) 30 kg.m/s right



Problems

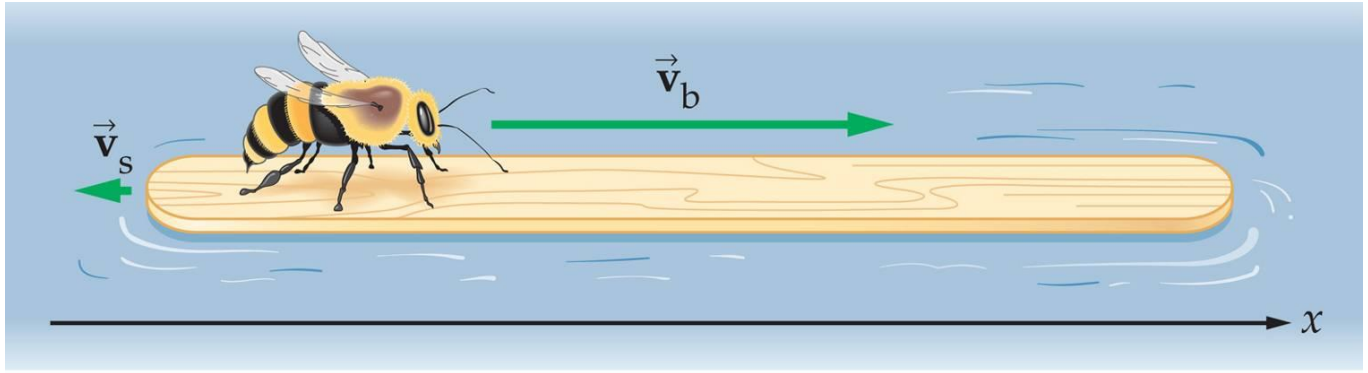
Problem 13: Find the magnitude of the impulse delivered to a soccer ball when a player kicks it with a force of 1250 N. Assume that the player's foot is in contact with the ball for 5.95×10^{-3} s.

Answer: 7.44 kg.m/s

Problem 14: In a typical golf swing, the club is in contact with the ball for about 0.0010 s. If the 45 g ball acquires a speed of 67 m/s, estimate the magnitude of the force exerted by the club on the ball.

Problem 15: A 0.50 kg croquet ball is initially at rest on the grass. When the ball is struck by a mallet, the average force on it is 230 N. If the ball's speed after being struck is 3.2 m/s, how long was the mallet in contact with the ball?

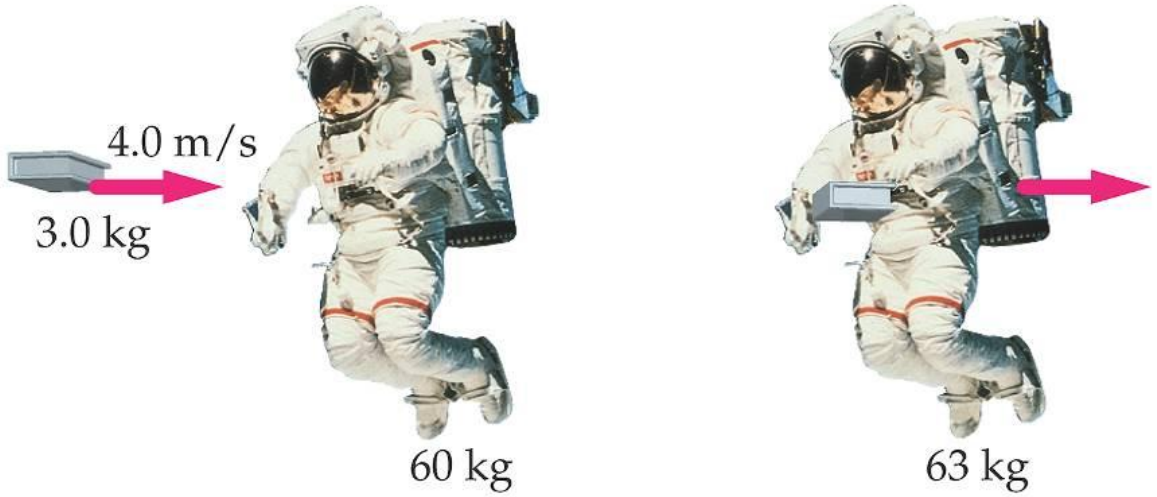
Conservation of Linear Momentum



Example: A honeybee with a mass of 0.150 g lands on one end of a floating 4.75 g popsicle stick. After sitting at rest for a moment, it runs to the other end of the stick with a velocity $\vec{v}_b$ relative to still water. The stick moves in the opposite direction with a velocity of 0.120 cm/s. Find the velocity $\vec{v}_b$ of the bee.

Answer: 3.8 cm/s

Inelastic Collisions in One Dimension



An astronaut of mass 60 kg is on a spacewalk to repair a communications satellite, when he realizes that he needs to consult the repair manual. You throw it to him at a speed of 4.0 m/s relative to the spacecraft. He is at rest when he catches the 3.0 kg book

(a) Find his velocity just after he catches the book.

(b) Find the initial and final kinetic energies of the book-astronaut system.

(c) Find the impulse exerted by the book on the astronaut.

Answers:

$v_f = 0.19$ m/s $\qquad K_i = 24$ J, $K_f = 1.1$ J $\qquad I = 11.4$ N.s