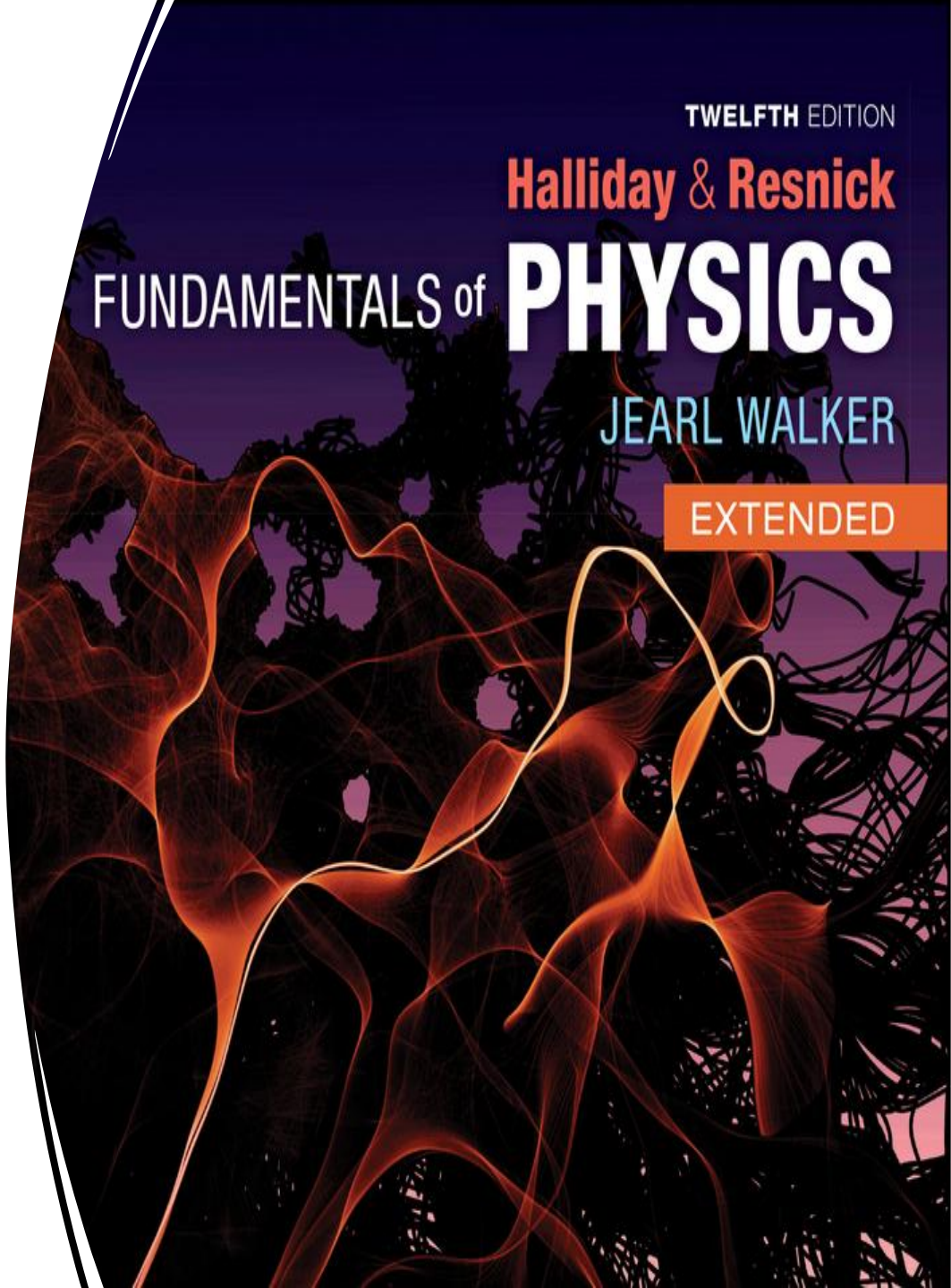
Chapter 8

Potential Energy and Conservation of Energy



Units of Chapter 8

8.1. Potential Energy

8.2. Conservation of Mechanical Energy

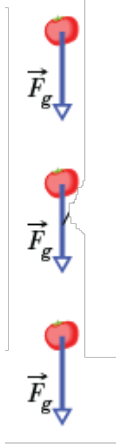
8.4. Work on a System by an External Force

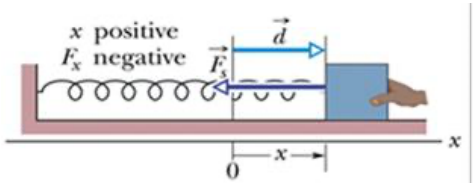
8.5. Conservation of Energy

8.1 Potential Energy

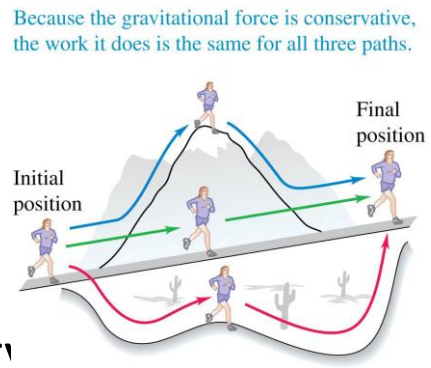


- **Potential energy** *U* is energy that can be associated with the configuration of a system of objects that exert forces on one another
- A system of objects may be:
  - Earth and a bungee jumper
  - **Gravitational potential energy** transforms into kinetic energy increase during the fall
  - **Elastic potential energy** transforms into deceleration by the bungee cord
- Physics determines how potential energy is calculated, to account for stored energy



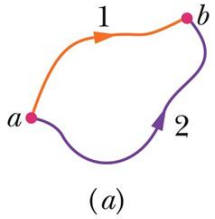


Conservative and Non-Conservative Forces



Conservative forces are forces for which the work done on an object is **independent of the path taken and depends only on the initial and final positions.** These forces allow energy to be converted between kinetic energy and potential energy.

- Examples: gravitational force, spring force
- Permits relationship between work and potential energies



The force is conservative. Any choice of path between the points gives the same amount of work.

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Nonconservative forces

- Examples: kinetic friction force and drag force, where kinetic energy of a moving particle is transferred to heat by friction
- A force (such as friction) that is not conservative is called a **nonconservative force**, or a **dissipative force**. It is a force whose work depends on the path taken

Gravitational Potential Energy

$$W = -\Delta U$$

When a conservative force does an amount of work, the corresponding potential energy *U* is changed according to the following definition

$$\Delta U = mg(y_f - y_i)$$

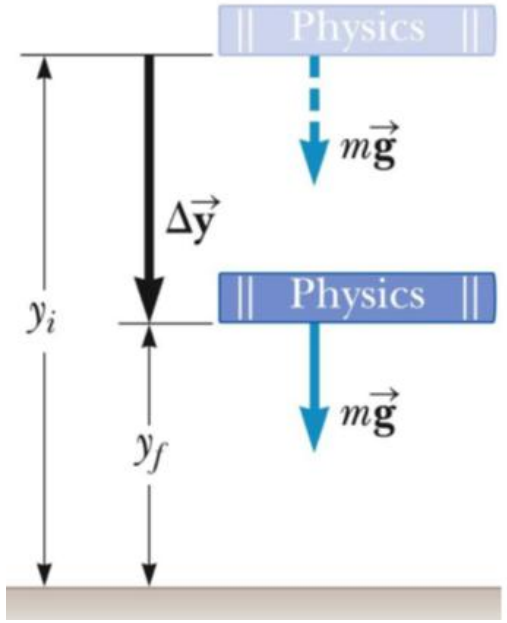
$$U_f - U_i = mg(y_f - y_i)$$

Using this to calculate gravitational potential energy, relative to a **reference configuration** with **reference point** $y_i = 0$:

$$U(y) = mgy$$

SI unit: joule (J)

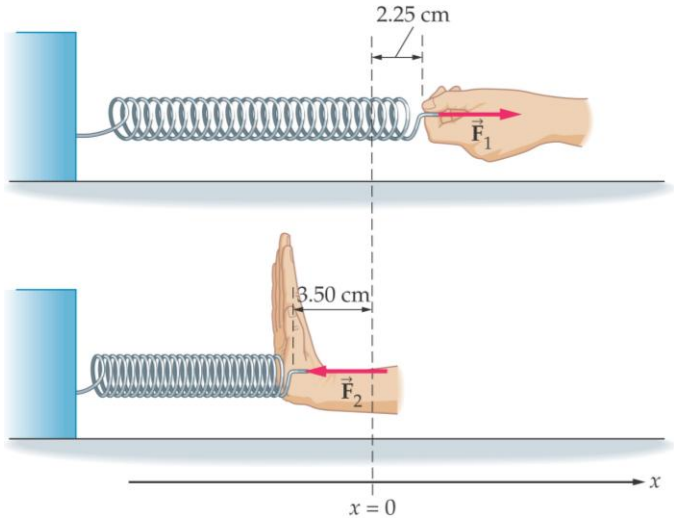
The work done by the gravitational force as the book falls equals $mgy_i - mgy_f$.



Elastic Potential Energy

Consider a spring that is stretched from its equilibrium position a distance *x*. The work required to cause this stretch is

$$W = \frac{1}{2}kx^2 = U_f - U_i$$



With reference point $x_i = 0$ for a relaxed spring:

$$U(x) = \frac{1}{2}kx^2$$

Example 1. An 82.0 kg mountain climber is in the final stage of ascent of 4301 m high Pikes peak.

A)What is the change in gravitational potential energy as the climber gains the last 100m of altitude? Let U=0 be at sea level?

B) What is the change in gravitational potential energy as the climber gains the last 100m of altitude? Let U=0 be at the top of peak?

Answer: $\Delta U = 80{,}400\ J$

SOLUTION

Part (a)

1. Calculate ΔU with $y_f = 4301$ m and $y_i = 4201$ m: $\Delta U = mgy_f - mgy_i$

$= (82.0\ \text{kg})(9.81\ \text{m/s}^2)(4301\ \text{m})$

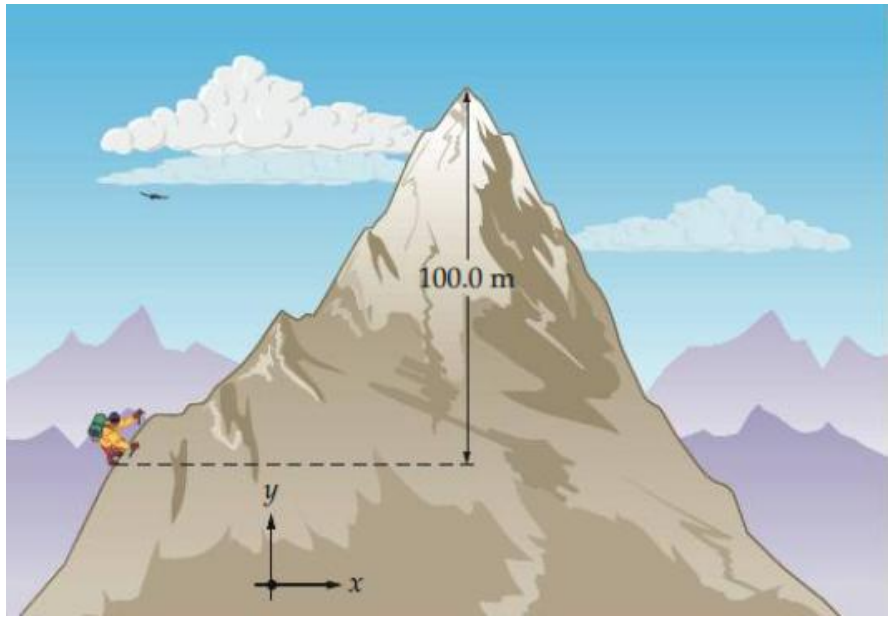
$- (82.0\ \text{kg})(9.81\ \text{m/s}^2)(4201\ \text{m}) = 80{,}400\ \text{J}$

Part (b)

2. Calculate ΔU with $y_f = 0$ and $y_i = -100.0$ m: $\Delta U = mgy_f - mgy_i$

$= (82.0\ \text{kg})(9.81\ \text{m/s}^2)(0)$

$- (82.0\ \text{kg})(9.81\ \text{m/s}^2)(-100.0\ \text{m}) = 80{,}400\ \text{J}$

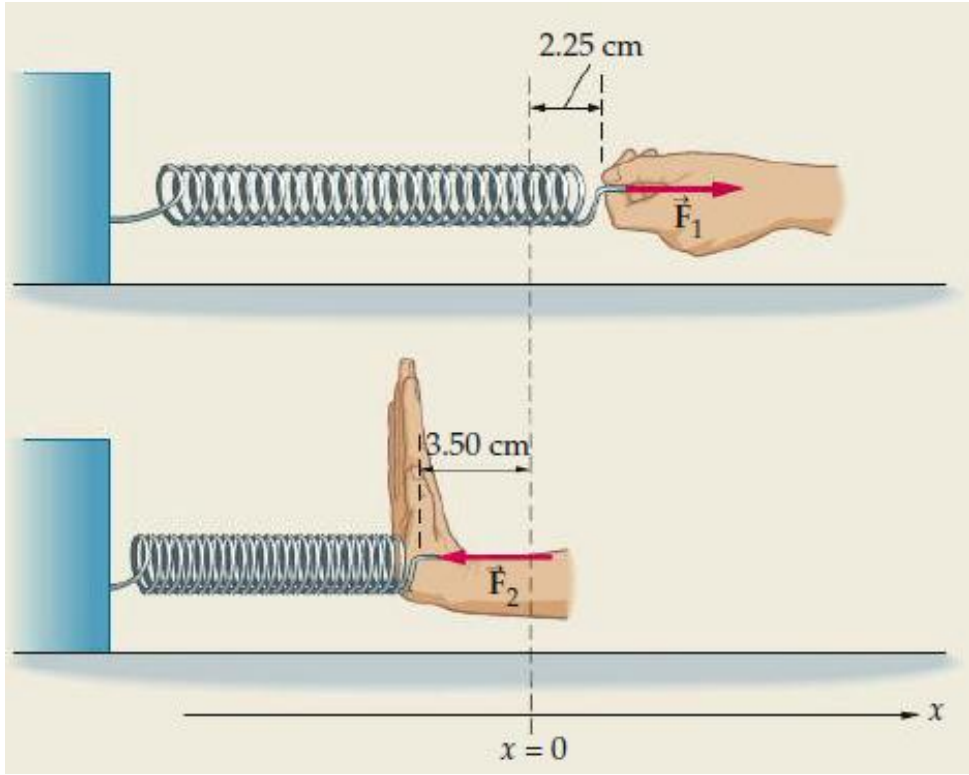


Example 2. If you lift a 2.0 kg box by 0.50 m (constant speed), how much work have you done and what is the change in the box's potential energy?

Answer: $W = 9.8\,J\ and\ \Delta U = 9.8J$

Example 3. When a force of 120.0 N is applied to a certain spring, it causes a stretch of 2.25 cm. What is the potential energy of this spring when it is **(a)** compressed by 3.50 cm or **(b)** expanded by 7.00 cm?

Answer: (a) $U = 3.26\,J$, $(b)\ U = 13.1J$



SOLUTION

1. Solve $F = kx$ for the spring constant, k:

$$F = kx$$
$$k = \frac{F}{x} = \frac{120.0\text{ N}}{0.0225\text{ m}} = 5330\text{ N/m}$$

Part (a)

2. Substitute $k = 5330$ N/m and $x = -0.0350$ m into the potential energy expression, $U = \frac{1}{2}kx^2$:

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(5330\text{ N/m})(-0.0350\text{ m})^2 = 3.26\text{ J}$$

Part (b)

3. Substitute $k = 5330$ N/m and $x = 0.0700$ m into the potential energy expression, $U = \frac{1}{2}kx^2$:

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(5330\text{ N/m})(0.0700\text{ m})^2 = 13.1\text{ J}$$

8.2 Conservation of Mechanical Energy

The **mechanical energy** E_{mec} of a system is the sum of its potential energy *U* and the kinetic energy *K* of the objects within it:

$$E_{mec} = K + U$$

Work done by conservative forces increases *K* and decreases *U* by that amount, so:

$$\Delta K = \Delta U$$

Using subscripts to refer to different instants of time:

$$K_2 + U_2 = K_1 + U_1$$

8.2 Conservation of Mechanical Energy

Statement of Mechanical Energy Conservation: In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.

This is the principle of the **conservation of mechanical energy**:

$$\Delta E_{mec} = \Delta K + \Delta U = 0$$

- This is very powerful tool:

When the mechanical energy of a system is conserved, we can relate the sum of kinetic energy and potential energy at one instant to that at another instant without considering the intermediate motion and without finding the work one by the forces involved.

- One application:
  - Choose the lowest point in the system as $U = 0$
  - Then at the highest point U = max, and K = min

Example 4: At the end of a graduation ceremony, the graduates fling their caps into the air. Suppose a 0.120 kg cap is thrown straight upward with a speed of 7.85 m/s and that frictional forces can be ignored.

(a) Use kinematics to find the speed of the cap when it has risen 1.18 m above the fling point.

(b) Show that the total mechanical energy of the cap is unchanged.

(Ans. (a) v = 6.2 m/s), (b) E = 3.7 J)

SOLUTION

Part (a)

1. Use kinematics to solve for v_y:

$$v_y^2 = v_{0y}^2 + 2a_y\,\Delta y$$
$$v_y = \pm\sqrt{v_{0y}^2 + 2a_y\,\Delta y}$$

2. Substitute $v_{0y} = 7.85$ m/s, $\Delta y = 1.18$ m, and $a_y = -g$ to find v_y. Choose the plus sign, since we are interested only in the speed:

$$v_y = \sqrt{v_{0y}^2 + 2a_y\,\Delta y}$$
$$= \sqrt{(7.85\text{ m/s})^2 + 2(-9.81\text{ m/s}^2)(1.18\text{ m})} = 6.20\text{ m/s}$$

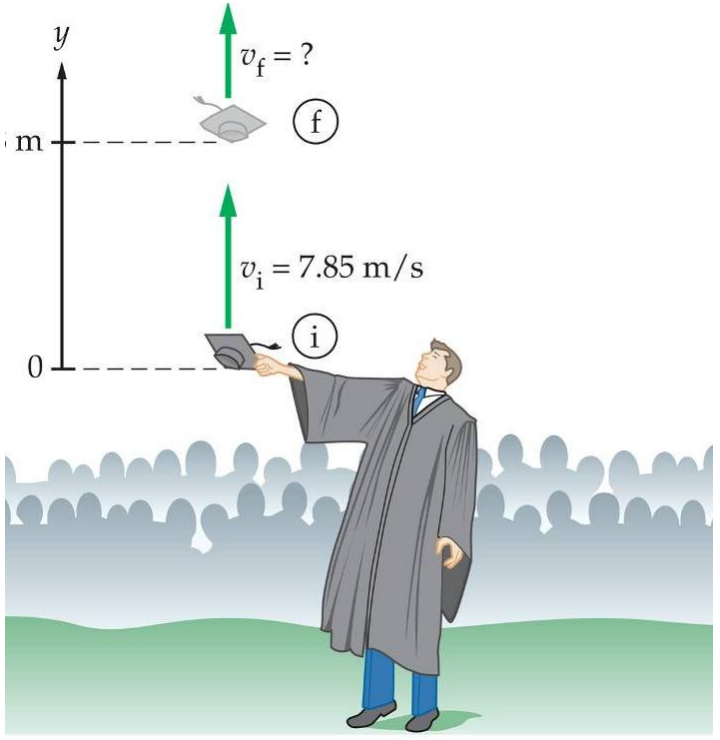
Part (b)

3. Calculate E_i. At this point $y_i = 0$ and $v_i = 7.85$ m/s:

$$E_i = U_i + K_i = mgy_i + \tfrac{1}{2}mv_i^2$$
$$= 0 + \tfrac{1}{2}(0.120\text{ kg})(7.85\text{ m/s})^2 = 3.70\text{ J}$$

4. Calculate E_f. At this point $y_f = 1.18$ m and $v_f = 6.20$ m/s:

$$E_f = U_f + K_f = mgy_f + \tfrac{1}{2}mv_f^2$$
$$= (0.120\text{ kg})(9.81\text{ m/s}^2)(1.18\text{ m}) + \tfrac{1}{2}(0.120\text{ kg})(6.20\text{ m/s})^2$$
$$= 1.39\text{ J} + 2.31\text{ J} = 3.70\text{ J}$$

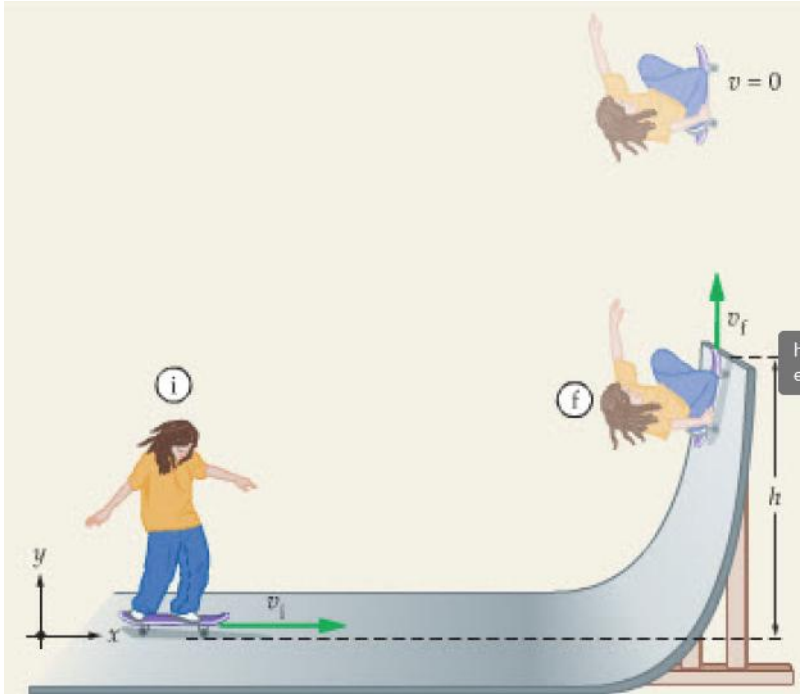


EXAMPLE 5: A 55-kg skateboarder enters a ramp moving horizontally with a speed of 6.5 m/s and leaves the ramp moving vertically with a speed of 4.1 m/s.
Find the height of the ramp, assuming no energy loss to frictional forces.

Ans. h = 1.3 m

SOLUTION

1. Write expressions for U_i and K_i: $U_i = mg \cdot 0 = 0$ $\quad K_i = \frac{1}{2}mv_i^2$
2. Write expressions for U_f and K_f: $U_f = mgh$ $\quad K_f = \frac{1}{2}mv_f^2$
3. Set the total mechanical energy at point i, $E_i = U_i + K_i$, equal to the total mechanical energy at point f, $E_f = U_f + K_f$: $U_i + K_i = U_f + K_f$, $0 + \frac{1}{2}mv_i^2 = mgh + \frac{1}{2}mv_f^2$
4. Solve for h. Note that m cancels: $mgh = \frac{1}{2}mv_i^2 - \frac{1}{2}mv_f^2$, $h = \frac{v_i^2 - v_f^2}{2g}$
5. Substitute numerical values: $h = \frac{(6.5\ \text{m/s})^2 - (4.1\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = 1.3\ \text{m}$



EXAMPLE 6: In figure, a child of mass *m* is released from rest at the top of a water slide, at height *h* = 8.5 m above the bottom of the slide. Assuming that the slide is frictionless because of the water on it, find the child's speed at the bottom of the slide.

Ans. v = 13 m/s

To show both kinds of mechanical energy, we have

$$K_b + U_b = K_t + U_t, \qquad (8.2.9)$$

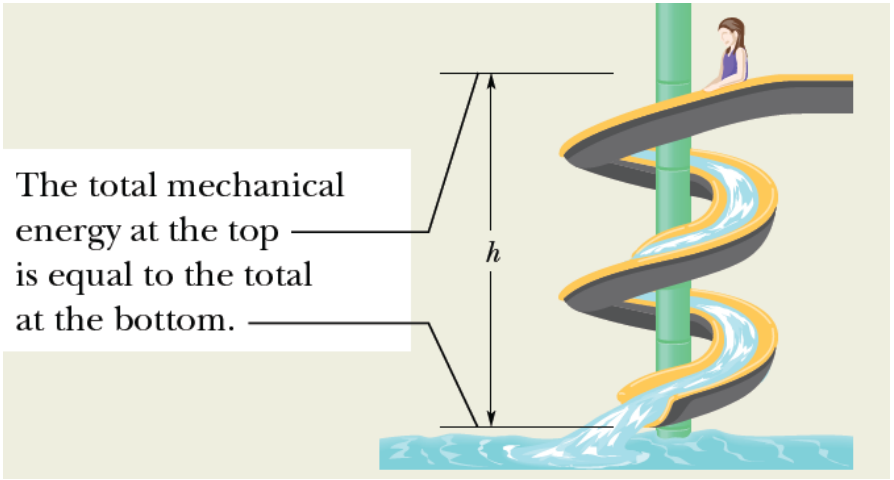
or $$\tfrac{1}{2}mv_b^2 + mgy_b = \tfrac{1}{2}mv_t^2 + mgy_t.$$

Dividing by m and rearranging yield

$$v_b^2 = v_t^2 + 2g(y_t - y_b).$$

Putting $v_t = 0$ and $y_t - y_b = h$ leads to

$$v_b = \sqrt{2gh} = \sqrt{(2)(9.8\ \text{m/s}^2)(8.5\ \text{m})}$$
$$= 13\ \text{m/s}. \qquad \text{(Answer)}$$



8.4 Work Done on a System By an External Force

No Friction

We can extend work on an object to work on a system:

- Work is energy transferred to or from a system by means of an external force acting on that system.
- For a system of more than 1 particle, work can change both *K* and *U*, or other forms of energy of the system
- For a frictionless system:

$$W = \Delta K + \Delta U,$$

$$W = \Delta E_{\text{mec}}$$

8.4 Work Done on a System By an External Force

Friction

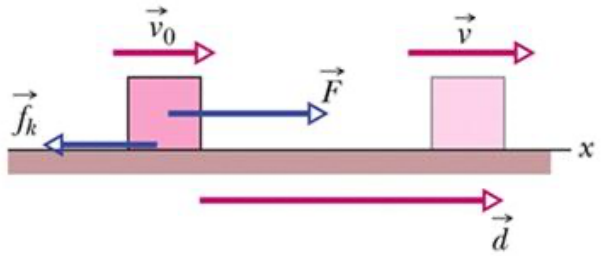
For a system with friction:

$$W_{nc} = f_k d$$ (here W_{nc} is the work by the non-conservative forces (friction))

The work of nonconservative force is equal the change in the mechanical energy.

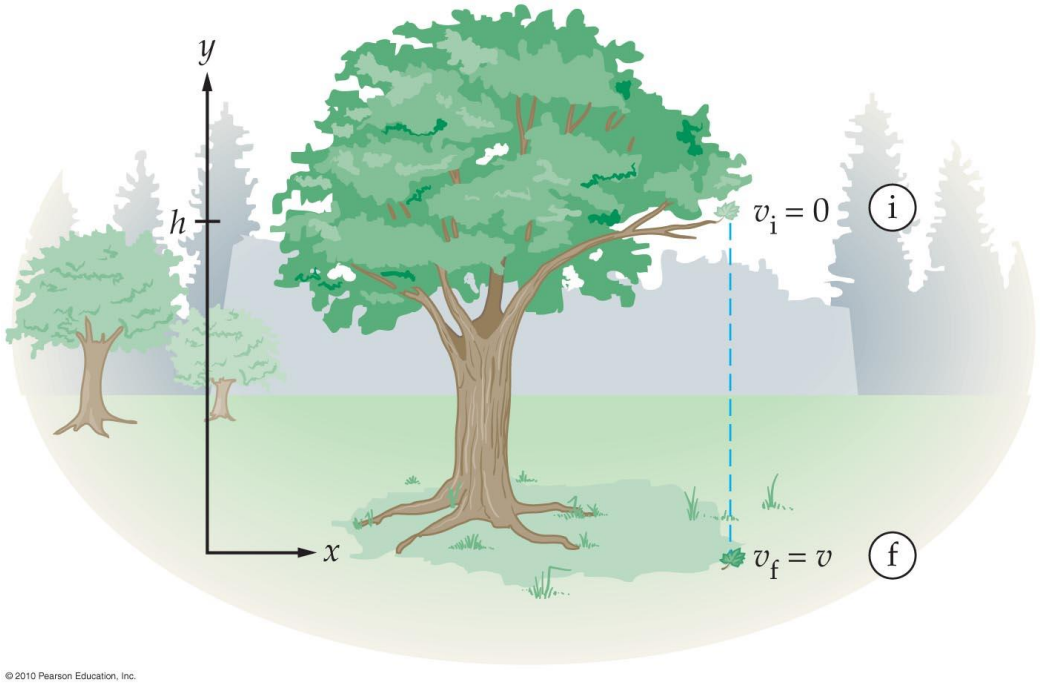
$$W_{nc} = \Delta E_{mec} = \Delta K + \Delta U = 0$$

The applied force supplies energy. The frictional force transfers some of it to thermal energy.



Example 7: Deep in the forest, a 17.0 g leaf falls from a tree and drops straight to the ground. If its initial height was 5.30 m and its speed on landing was 1.3 m/s, how much non conservative work was done on the leaf?

Answers: E_i= 0.884 J , E_f= 0.014 J , W_{nc} = - 0.870 J



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SOLUTION

1. Evaluate U_i, K_i, and E_i:

$$U_i = mgh = (0.0170\ \text{kg})(9.81\ \text{m/s}^2)(5.30\ \text{m}) = 0.884\ \text{J}$$
$$K_i = \tfrac{1}{2}m \cdot 0^2 = 0$$
$$E_i = U_i + K_i = 0.884\ \text{J}$$

2. Next, evaluate U_f, K_f, and E_f.

$$U_f = mg \cdot 0 = 0$$
$$K_f = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.0170\ \text{kg})(1.3\ \text{m/s})^2 = 0.014\ \text{J}$$
$$E_f = U_f + K_f = 0.014\ \text{J}$$

3. Use $W_{nc} = \Delta E$ to find the nonconservative work:

$$W_{nc} = \Delta E = E_f - E_i = 0.014\ \text{J} - 0.884\ \text{J} = -0.870\ \text{J}$$

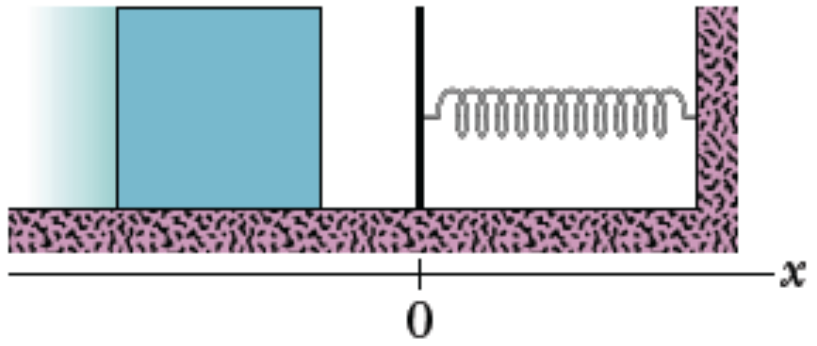
Example 8: In figure, a block of mass m = 2.5 kg slides head on into a spring of spring constant k = 320 N/m. When the block stops, it has compressed the spring by 7.5 cm. The coefficient of kinetic friction between block and floor is 0.25. While the block is in contact with the spring and being brought to rest.

(a) what is the work done by the spring force

(b) What is the work done by friction on the block, given the friction between the block and the floor?

(c) What is the block's speed just as it reaches the spring

Answers: (a) W_s = -0.90 J, (b) W_{nc} = -0.46 J, (c) v_i = 1.04m/s



55. (a) With $x = 0.075$ m and $k = 320$ N/m, Eq. 7.4.7 yields $W_s = -\frac{1}{2}kx^2 = -0.90$ J.

For later reference, this is equal to the negative of ΔU.

(b) Analyzing forces, we find $F_N = mg$, which means $f_k = \mu_k F_N = \mu_k mg$. With $d = x$, Eq. 8.4.7 yields

$$W_{nc} = -f_k d = -\mu_k mgx = -(0.25)(2.5)(9.8)(0.075) = -0.46\ \text{J}.$$

(c) the initial kinetic energy is

$$K_i = \Delta U + W_{nc} = 0.90 + 0.46 = 1.36\ \text{J}$$

which leads to $v_i = \sqrt{2K_i/m} = 1.0$ m/s.

8.5 Conservation of Energy

- Energy transferred between systems can always be accounted for
- The **law of conservation of energy** concerns
  - The **total energy** *E* of a system

The total energy *E* of a system can change only by amounts of energy that are transferred to or from the system.

An isolated system is one for which there can be no external energy transfer

The total energy *E* of an isolated system cannot change.

Energy transfers may happen internal to the system

We can write:

$$\Delta E_{mec} = \Delta K + \Delta U = 0$$

$$E_{mec,2} = E_{mec,1}$$

Power and Energy Definitions

We can expand the definition of power
In general, power is the rate at which energy is transferred by a force
from one type to another
If energy ΔE is transferred in time Δt, the **average power** is:

$$P_{\text{avg}} = \frac{\Delta E}{\Delta t}.$$

And the **instantaneous power** is:

$$P = \frac{dE}{dt}.$$

Example 9: The luxury liner Queen Elizabeth 2 has a diesel-electric power plant with a maximum power of $92 \times 10^6\ W$ at a cruising speed of 60.19 km/h.
What forward force is exerted on the ship at this speed?

We use $P = Fv$ to compute the force:

$$F = \frac{P}{v} = \frac{92\times10^6\ \text{W}}{\left(60.19\ \frac{\text{km}}{\text{h}}\right)\left(\frac{1000\ \text{m/km}}{3600\ \text{s/h}}\right)} = 5.5\times10^6\ \text{N}.$$

Chapter 8 Summary: Gravitational and Elastic Potential Energies

Gravitational Potential Energy

$$U(y) = mgy$$

- Energy associated with Earth + a nearby particle

Elastic Potential Energy

- Energy associated with compression or extension of a spring $U(x) = \frac{1}{2}kx^2$

Mechanical Energy

$$E_{\text{mec}} = K + U$$

Work Done on a System by an External Force

- Without/with friction: $W = \Delta E_{\text{mec}}$

Conservation of Energy

- The **total energy** can change only by amounts transferred in or out of the system

$$\Delta E_{mec} = \Delta K + \Delta U = 0$$

Additional Exercises

How much potential energy is lost by a 5Kg object to kinetic energy due a decrease in height of 4.5 m?

Ans. $PE = 225\ J$

As an Acapulco cliff driver drops to the water from a height of 46 m, his gravitational potential energy decreases by 25,000 J. Find the divers weight in Newton's?

Ans. : 540 N

Q24) At an amusement park, a swimmer uses a water slide to enter the main pool, If the swimmer starts at rest, slides without friction, and descends through a vertical height of 2.31 m, what is her speed at the bottom of slide?

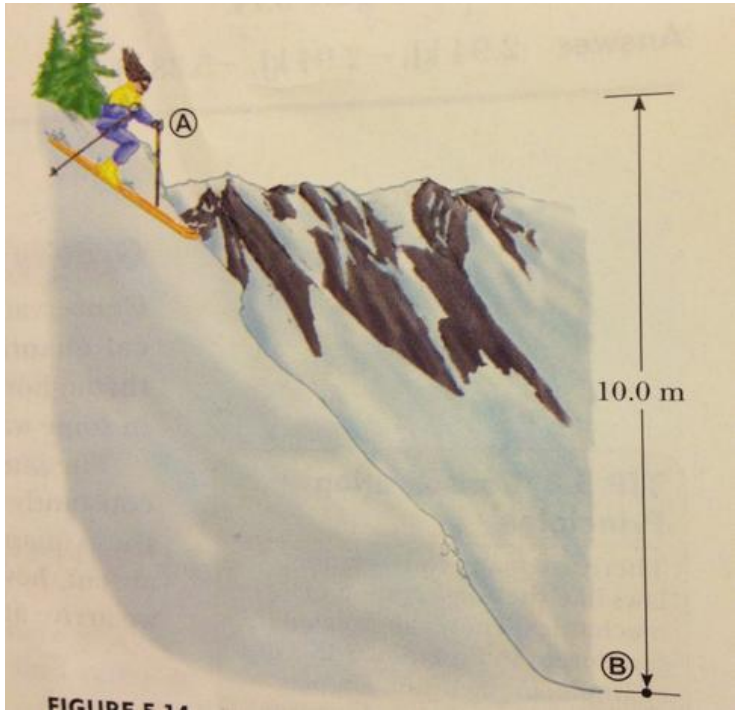
Ans. : 6.73 m/s

Q33) A 5.76-kg rock is dropped and allowed to fall freely. Find the initial kinetic energy, the final kinetic energy, and the change in kinetic energy for the first 2.00 m of fall

Ans. : Ki=0j, Kf=113J, deltaE=113J



Exercise



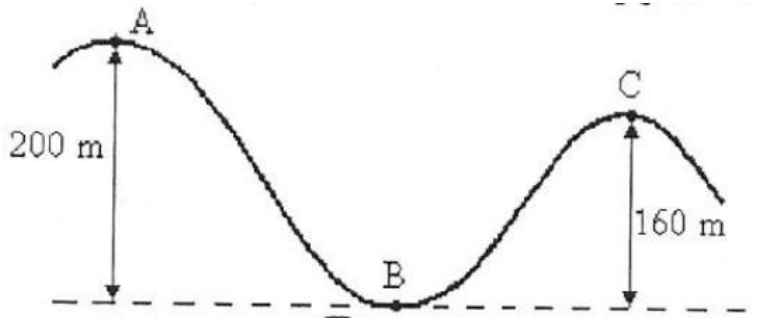
A 60.0 kg skier is at the top of a slope, as shown in Figure. At the initial point (A), she is 10.0m vertically above point (B).

a) Setting the zero level for gravitational potential energy at (B), find the gravitational potential energy of this system when the skier is at (A)and then at (B), finally, find the change in potential energy at the skier-Earth system as the skier goes from point (A) to point (B).

b) Repeat this problem with the zero level at point (A).

c) Repeat again, with the zero level 2.00m higher than point (B). Answer : $\Delta U = -5886$ J

Example: A bead is moving with a speed of 20 m/s at position A on the track shown in the Figure below. This track is friction-free, and there is no appreciable air resistance. What is the speed of the bead at point C?



$$E_{Ti} = E_{Tf}$$

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_A^2 + mgy_A = \frac{1}{2}mv_C^2 + mgy_c$$

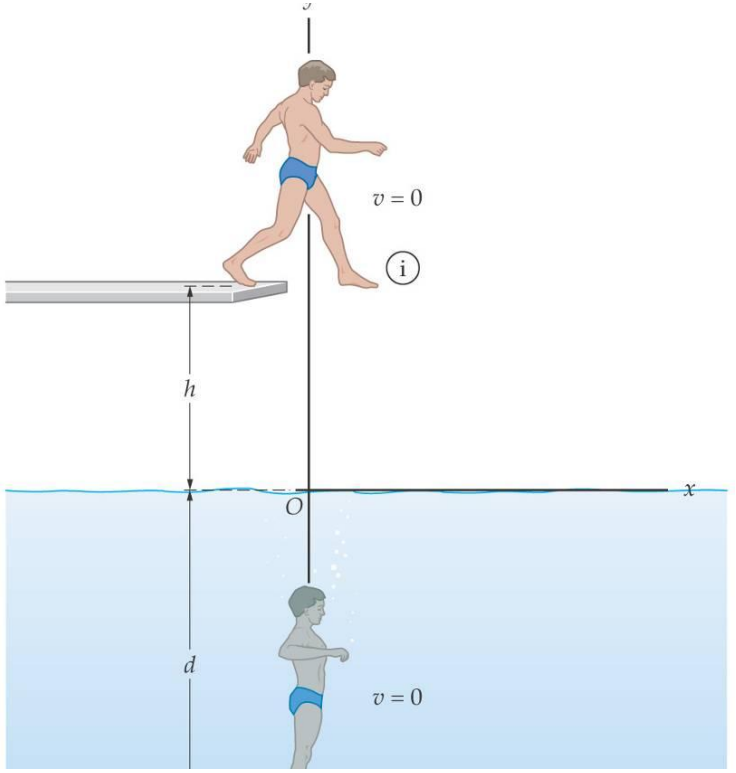
$$\frac{1}{2}v_A^2 + gy_A = \frac{1}{2}v_C^2 + gy_c$$

$$v_c = \sqrt{v_A^2 + 2gy_A - 2gy_C} = \sqrt{20^2 + 2(9.81)(200) - 2(9.81)(160)} = 34.4\ m/s$$

Exercise

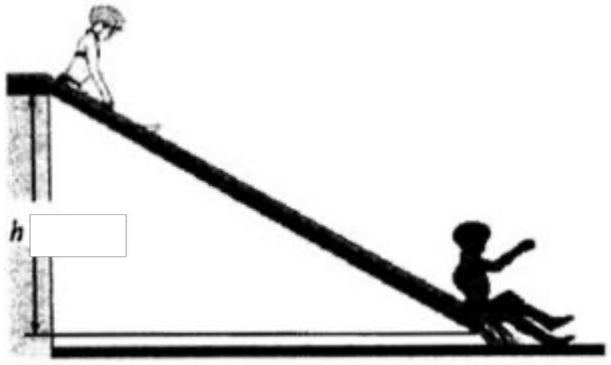
Example: A 95.0 kg diver steps off a diving board and drops into the water, 3.00 m below. At some depth d below the water's surface, the diver comes to rest. If the nonconservative work done on the diver is $W_{\text{nc}} = -5120$ J, what is the depth d?

(Ans. 2.49 m)



Example: Q41) At a playground, a 19-kg child plays on a slide that drops through a height of *h* = 2.3 m. The child starts at rest at the top of the slide. On the way down, the slide does a nonconservative work of on the child. What is the child's speed at the bottom of the slide?

(Ans. 2.7 m/s)



Example. An ideal spring is used to fire a 15.0-g block horizontally. The spring has a spring constant of 20 N/m and is initially compressed by 7.0 cm. calculate the kinetic energy of the block as it leaves the spring.

Answer: $K_f = 0.049\ J$

The kinetic energy (KE) of the block as it leaves the spring can be found using the conservation of energy. The potential energy stored in the compressed spring is converted into kinetic energy.

The potential energy (PE_{spring}) stored in the spring is given by:

$$PE_{\text{spring}} = \frac{1}{2}kx^2$$

where:
k is the spring constant (20 N/m),
x is the compression of the spring (7.0 cm converted to meters, so $0.07\ \text{m}$).

The potential energy is converted into kinetic energy, so:

$$KE = PE_{\text{spring}}$$

$$KE = \frac{1}{2}kx^2$$

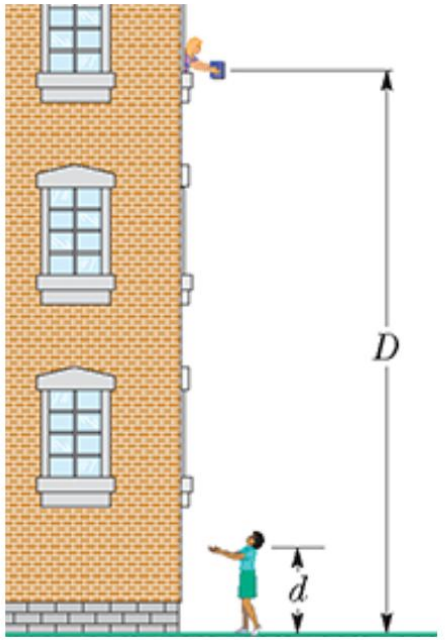
Substitute the known values:

$$KE = \frac{1}{2} \times 20\,\text{N/m} \times (0.07\,\text{m})^2$$

$$KE = 0.049\,\text{J}$$

So, the correct answer from the options provided is $4.9 \times 10^{-2}\,\text{J}$.

3 E You drop a 2.00 kg book to a friend who stands on the ground at distance D = 10.0 m below. If your friend's outstretched hands are at distance d = 1.50 m above the ground (Fig. 8.13), (a) how much work Wg does the gravitational force do on the book as it drops to her hands? (b) What is the change ΔU in the gravitational potential energy of the book–Earth system during the drop? If the gravitational potential energy U of that system is taken to be zero at ground level, what is U (c) when the book is released and (d) when it reaches her hands? Now take U to be 100 J at ground level and again find (e) Wg, (f) ΔU, (g) U at the release point, and (h) U at her hands.



3. (a) Noting that the vertical displacement is 10.0 m – 1.50 m = 8.50 m downward (same direction as $\vec{F}_g$), Eq. 7.3.1 yields

$$W_g = mgd\cos\phi = (2.00\text{ kg})(9.80\text{ m/s}^2)(8.50\text{ m})\cos 0° = 167\text{ J}.$$

(b) One approach (which is fairly trivial) is to use Eq. 8.1.1, but we feel it is instructive to instead calculate this as ΔU where $U = mgy$ (with upward understood to be the $+y$ direction). The result is

$$\Delta U = mg(y_f - y_i) = (2.00\text{ kg})(9.80\text{ m/s}^2)(1.50\text{ m} - 10.0\text{ m}) = -167\text{ J}.$$

(c) In part (b) we used the fact that $U_i = mgy_i$ = 196 J.

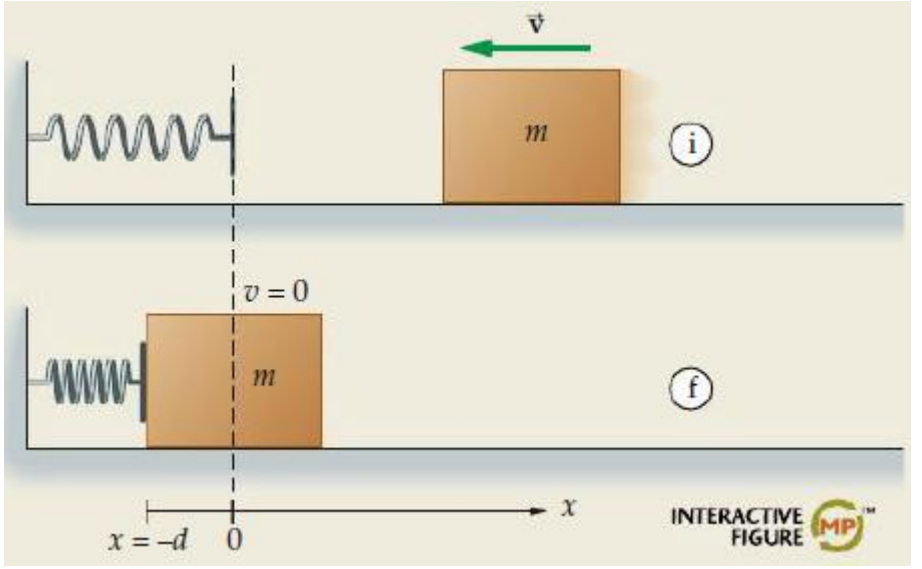
(d) In part (b), we also used the fact $U_f = mgy_f$ = 29 J.

(e) The computation of W_g does not use the new information (that U = 100 J at the ground), so we again obtain W_g = 167 J.

(f) As a result of Eq. 8.1.1, we must again find $\Delta U = -W_g$ = –167 J.

EXAMPLE: A 1.70-kg block slides on a horizontal, frictionless surface until it encounters a spring with a force constant of 955 N/m. The block comes to rest after compressing the spring a distance of 4.60 cm. Find the initial speed of the block. (Ignore air resistance and any energy lost when the block initially contacts the spring.)

Answer: $v_f = 1.09\ m/s$



SOLUTION

1. Write expressions for U_i and K_i. For U, we consider only the potential energy of the spring, $U = \frac{1}{2}kx^2$: $U_i = \frac{1}{2}k \cdot 0^2 = 0 \qquad K_i = \frac{1}{2}mv^2$
2. Do the same for U_f and K_f: $U_f = \frac{1}{2}k(-d)^2 = \frac{1}{2}kd^2 \qquad K_f = \frac{1}{2}m \cdot 0^2 = 0$
3. Set the initial mechanical energy, $E_i = U_i + K_i$, equal to the final mechanical energy, $E_f = U_f + K_f$, and solve for v: $U_i + K_i = U_f + K_f$

$$0 + \frac{1}{2}mv^2 = \frac{1}{2}kd^2 + 0$$

$$v = d\sqrt{\frac{k}{m}}$$

4. Substitute numerical values:

$$v = d\sqrt{\frac{k}{m}} = (0.0460\ \text{m})\sqrt{\frac{955\ \text{N/m}}{1.70\ \text{kg}}} = 1.09\ \text{m/s}$$