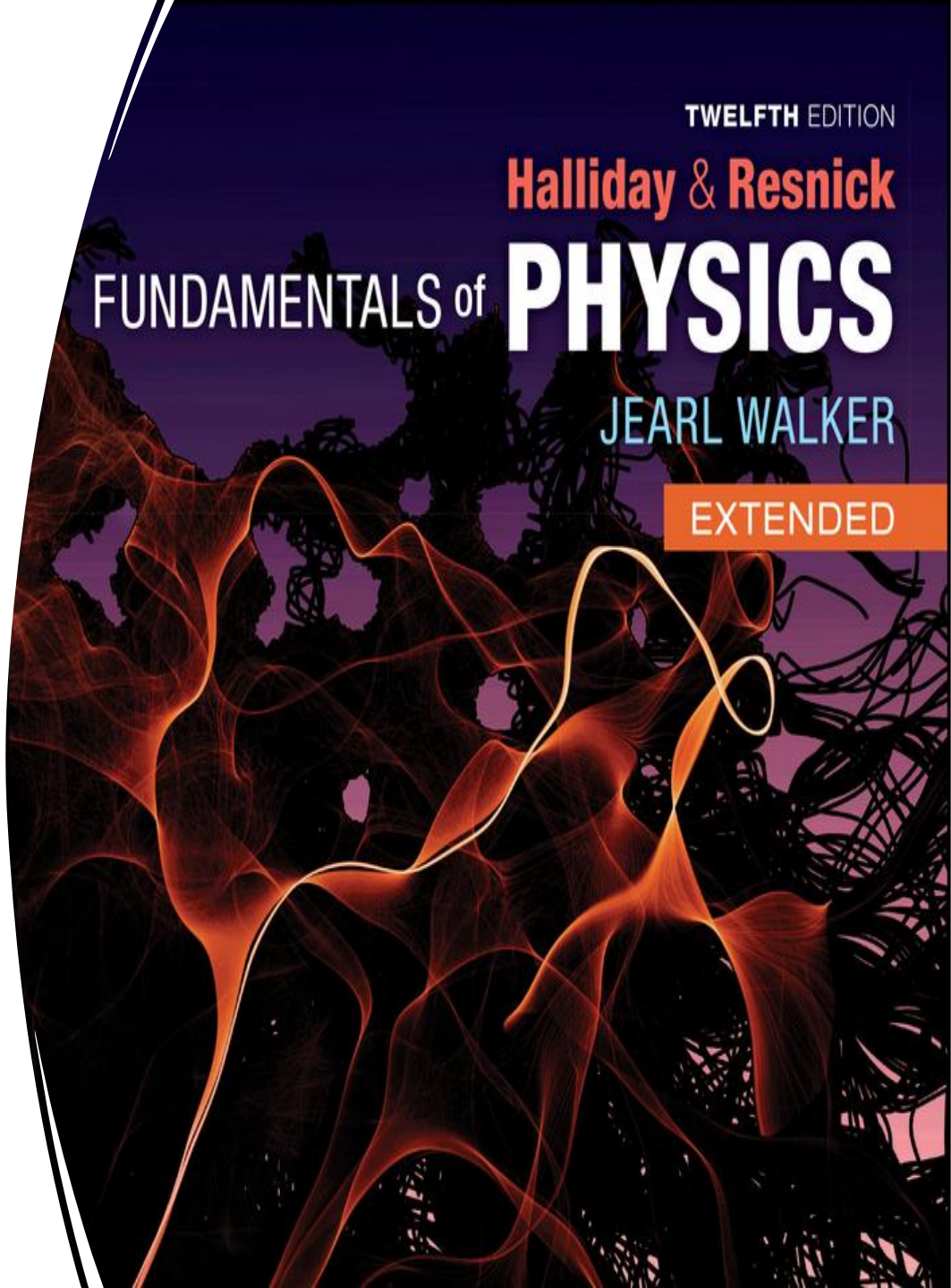
Chapter 6

Force and Motion—II



Units of Chapter 6

6.1. Friction

6.2. Uniform Circular Motion

6.1 friction

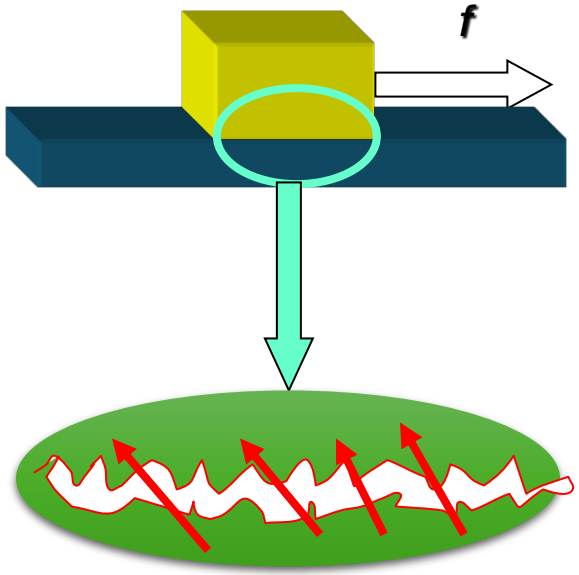
Friction has its basis in surfaces that are not completely smooth

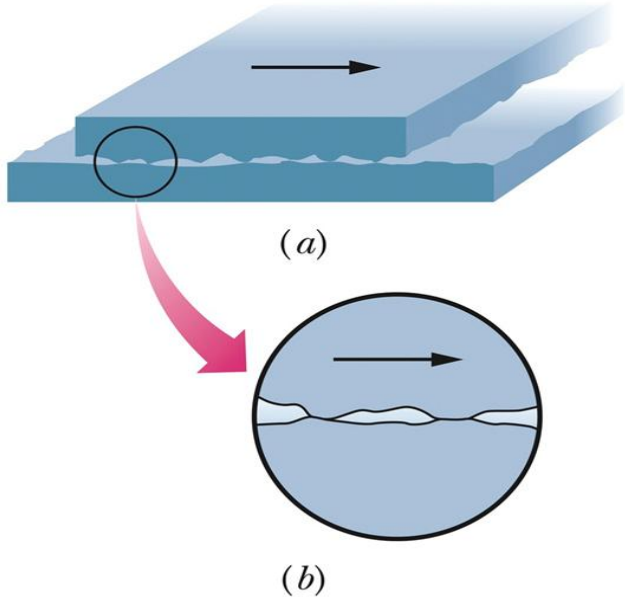
Friction-producing Microscopic Surface Asperities

Even smooth surfaces have irregularities when viewed at the microscopic level.
This type of roughness contributes to friction.

No surface is perfectly smooth

- On the other hand, Friction can be helpful/indispensable in another situation.
- Ex: Walk, Run, Starting or Stopping a car,…etc.

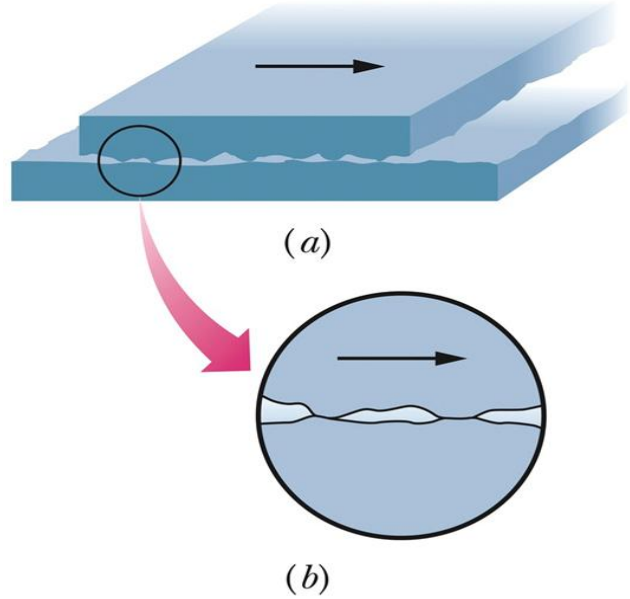




Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

Chemical and Microstructural Origins of Frictional Force

- Microscopic picture: surfaces are bumpy
- Friction occurs as contact points slide over each other
- Two specially prepared metal surfaces can cold-weld together and become impossible to slide, because there is so much contact between the surfaces
- Greater force normal to the contact plane increases the friction because the surfaces are pressed together and make more contact
- Sliding that is jerky, due to the ridges on the surface, produces squeaking/squealing/sound



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

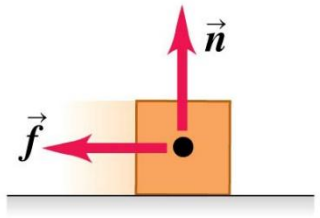
Definition: friction is the force that bodies can impart on each other when they are in contact.

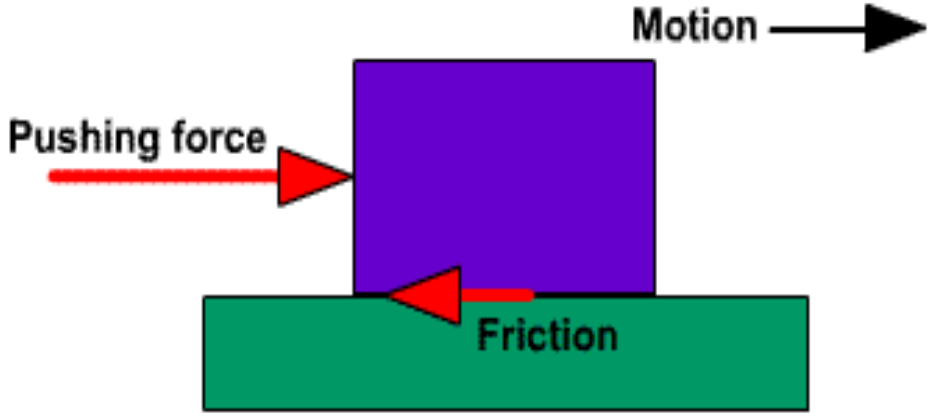
The friction force on an object is **opposite** to its motion or impending motion relative to the surface.

The direction of Friction Force is **parallel** to the surface in contact and occur when:

- One body slides over the other.
- They cling together despite external force.

(b) **Friction force $\vec{f}$:** In addition to the normal force, a surface may exert a friction force on an object, directed parallel to the surface.





Static vs. Kinetic Friction

Two types of friction:

The static frictional force:

- The opposing force that prevents an object from moving
- Can have any magnitude from 0 N up to a maximum
- Once the maximum is reached or exceeded, forces are no longer in equilibrium and the object slides or begins to move.

The kinetic frictional force:

- The opposing force that acts on an object in motion
- Has only one value
- Generally smaller than the maximum static frictional force.

There is no attempt at sliding. Thus, no friction and no motion.

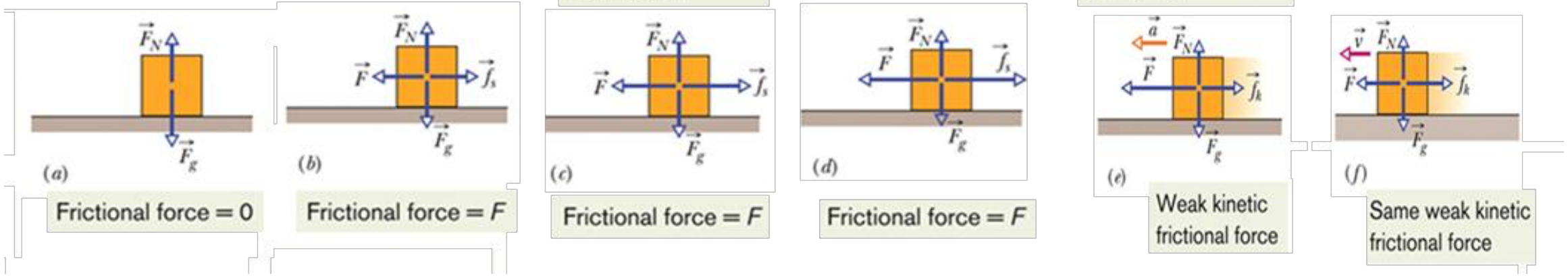
Force $\vec{F}$ attempts sliding but is balanced by the frictional force. No motion.

Force $\vec{F}$ is now stronger but is still balanced by the frictional force. No motion.

Force $\vec{F}$ is now even stronger but is still balanced by the frictional force. No motion.

Finally, the applied force has overwhelmed the static frictional force. Block slides and accelerates.

To maintain the speed, weaken force $\vec{F}$ to match the weak frictional force.



Frictional force = 0

Frictional force = F

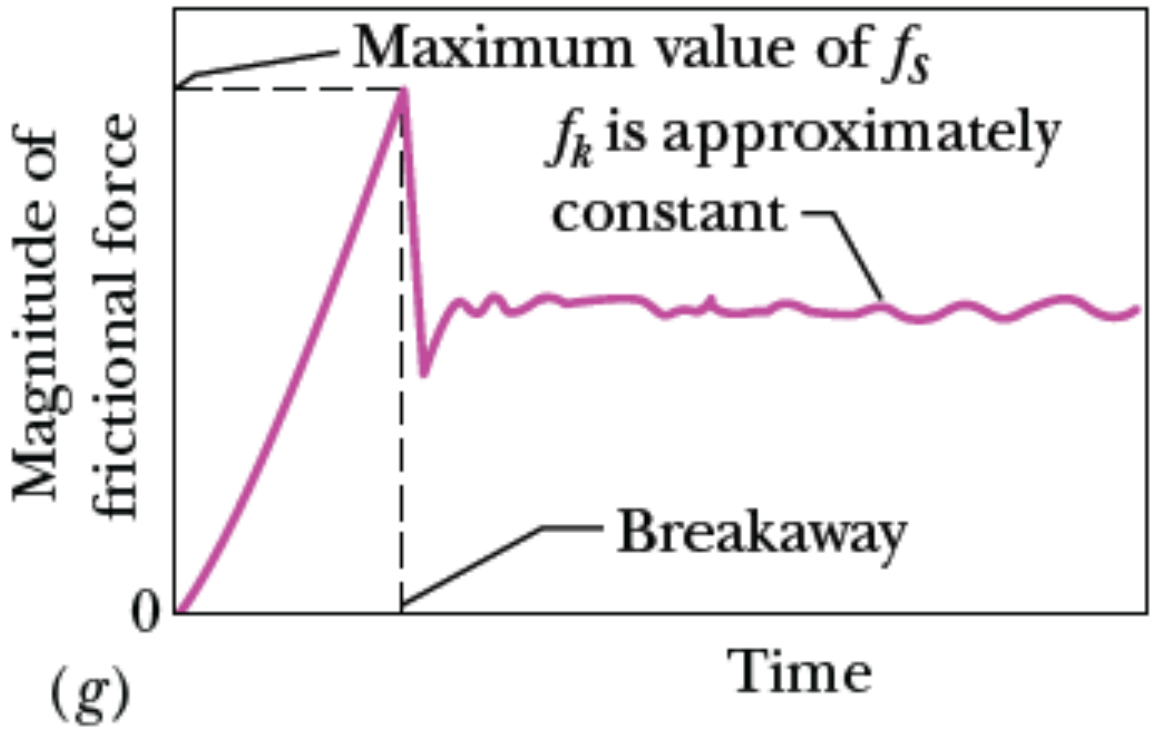
Frictional force = F

Frictional force = F

Weak kinetic frictional force

Same weak kinetic frictional force

Static frictional force can only match growing applied force.



Kinetic frictional force has only one value (no matching).

✓ **The static frictional force** keeps an object from **starting to move** when a force is applied.

Friction Force: Stationary Conditions

The properties of friction:

1. If the body does not move, then the applied force and frictional force balance along the direction parallel to the surface: equal in magnitude, opposite in direction
2. If the applied force is equal in magnitude or less than the frictional force, then the object will stay in a static position.
3. The magnitude of f_s has a maximum $f_{s,\,\max}$ given by:

$$f_{s,\max} = \mu_s F_N ,$$

where μ_s is the **coefficient of static friction**. If the applied force increases past $f_{s,\,\max}$, sliding begins.

Friction Force: Moving Conditions

Once sliding begins, the frictional force decreases to f_k given by:

$$f_k = \mu_k F_N,$$

where μ_k is the **coefficient of kinetic friction**.

- Magnitude F_N of the normal force measures how strongly the surfaces are pushed together
- The values of the friction coefficients are unitless and must be determined experimentally

TABLE 6–1 Typical Coefficients of Friction

μ_s and μ_k..... Are dimensionless.

Materials	Kinetic, μ_k	Static, μ_s
Rubber on concrete (dry)	0.80	1–4
Steel on steel	0.57	0.74
Glass on glass	0.40	0.94
Wood on leather	0.40	0.50
Copper on steel	0.36	0.53
Rubber on concrete (wet)	0.25	0.30
Steel on ice	0.06	0.10
Waxed ski on snow	0.05	0.10
Teflon on Teflon	0.04	0.04
Synovial joints in humans	0.003	0.01

$\mu_k < \mu_s$

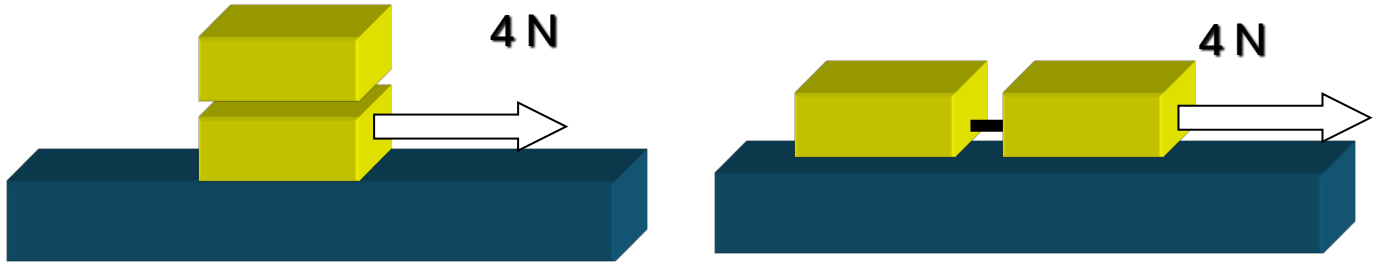
f_k ----is the **kinetic friction** force that exists once the object begins to move. Its value remains fairly constant, as the **velocity of the object increases.**

The coefficient of kinetic friction μ_k is always **positive** and typical values range between **zero and 1.**

The coefficient of static friction μ_s is always **positive** and can have any value between **0 and greater than > 1**

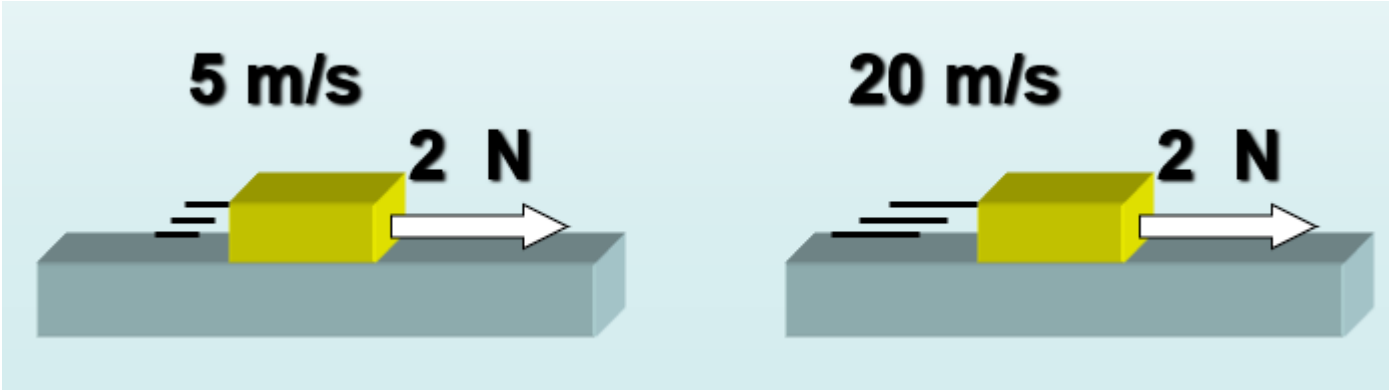
Friction forces are independent of area

If the total mass pulled is constant, the same force (4 N) is required to overcome friction even with twice the area of contact.



Friction forces are independent of speed

The force of kinetic friction is the same at 5 m/s as it is for 20 m/s. Again, we must assume that there are no chemical or mechanical changes due to speed.



Friction Strength

The magnitude of the friction force is proportional to:

1. How hard the two bodies are pressed together (the normal force*, N).*

2. The materials from which the bodies are made (the coefficient of friction).

Attributes that have little or no effect:

- Sliding speed
- Contact area



Photo 8.1 The coefficient of static friction between a package and the inclined conveyer belt must be sufficiently large to enable the package to be transported without slipping.

Example 1. A flatbed truck slowly tilts its bed upward to dispose of a 95 kg crate. For small angles of tilt, the crate stays put, but when the tilt angle exceeds 23.2º, the crate begins to slide. What is the coefficient of static friction between the bed of the truck and the crate.

Answer: $\mu_s = 0.429$



SOLUTION

1. Resolve the three force vectors acting on the crate into x and y components:

$$N_x = 0 \qquad N_y = N$$
$$f_{s,x} = -f_{s,\max} = -\mu_s N \qquad f_{s,y} = 0$$
$$W_x = mg \sin\theta \qquad W_y = -mg \cos\theta$$

2. Set $\Sigma F_y = ma_y = 0$, since $a_y = 0$.

$$\sum F_y = N_y + f_{s,y} + W_y = N + 0 - mg\cos\theta = ma_y = 0$$

Solve for the normal force, N:

$$N = mg\cos\theta$$

3. Set $\Sigma F_x = ma_x = 0$, since the crate is at rest, and use the result for N obtained in Step 2:

$$\sum F_x = N_x + f_{s,x} + W_x = ma_x = 0$$
$$= 0 - \mu_s N + mg\sin\theta$$
$$= 0 - \mu_s mg\cos\theta + mg\sin\theta$$

4. Solve the expression for the coefficient of static friction, μ_s:

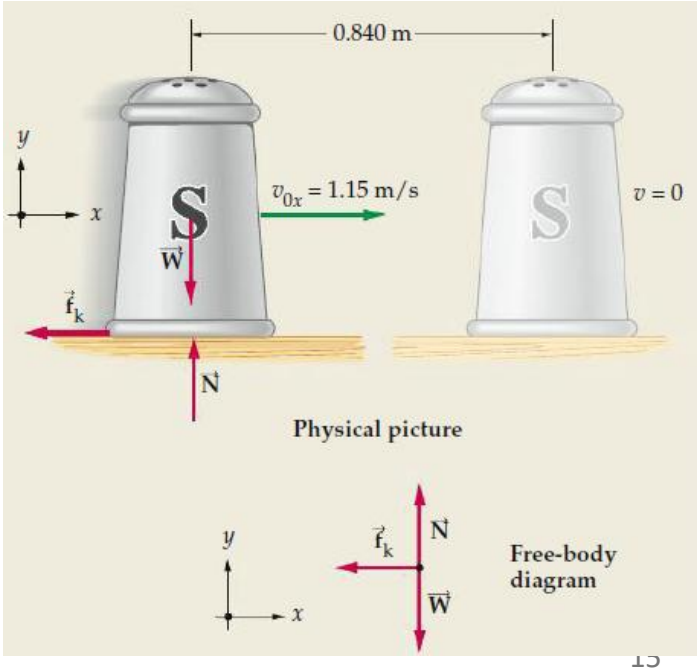
$$\mu_s mg\cos\theta = mg\sin\theta$$
$$\mu_s = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta = \tan 23.2^\circ = 0.429$$

Example 2. Someone at the other end of table asks you to pass the salt. Feeling quiet dashing, you slide the 50g saltshaker in their direction, giving it an initial speed of 1.15m/s.

(a) If the shaker comes to rest with constant acceleration in 0.840 m , what is the coefficient of kinetic friction between the shaker and the table?

(b) How much time is required for the shaker to come to rest if you slide it with an initial speed of 1.32 m/s?

Answer : a_x = -0.787 m/s^2 , μ_k = 0.0802 , t=1.68s



SOLUTION

Part (a)

1. Set $\Sigma F_x = ma_x$ to find f_k in terms of a_x:

$$\sum F_x = -f_k = ma_x \quad \text{or} \quad f_k = -ma_x$$

2. Determine a_x by using the kinematic equation relating velocity to position, $v_x^2 = v_{0x}^2 + 2a_x\Delta x$:

$$v_x^2 = v_{0x}^2 + 2a_x\Delta x$$

$$a_x = \frac{v_x^2 - v_{0x}^2}{2\Delta x} = \frac{0 - (1.15\ \text{m/s})^2}{2(0.840\ \text{m})} = -0.787\ \text{m/s}^2$$

3. Set $\Sigma F_y = ma_y = 0$ to find the normal force, N:

$$\sum F_y = N + (-W) = ma_y = 0 \quad \text{or} \quad N = W = mg$$

4. Substitute $N = mg$ and $f_k = -ma_x$ (with $a_x = -0.787\ \text{m/s}^2$) into $\mu_k = f_k/N$ to find μ_k:

$$\mu_k = \frac{f_k}{N} = \frac{-ma_x}{mg} = \frac{-a_x}{g} = \frac{-(-0.787\ \text{m/s}^2)}{9.81\ \text{m/s}^2} = 0.0802$$

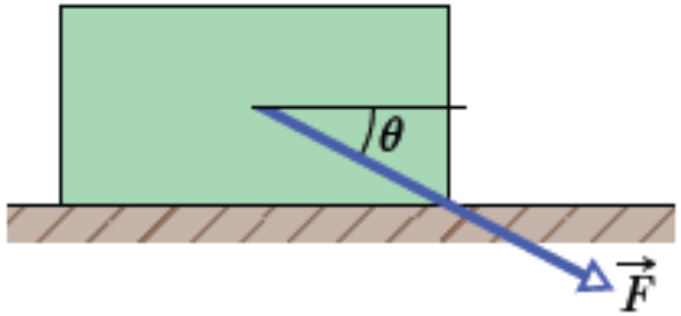
Part (b)

5. Use $a_x = -0.787\ \text{m/s}^2$, $v_{0x} = 1.32\ \text{m/s}$, and $v_x = 0$ in $v_x = v_{0x} + a_x t$ to solve for the time, t:

$$v_x = v_{0x} + a_x t \quad \text{or}$$

$$t = \frac{v_x - v_{0x}}{a_x} = \frac{0 - (1.32\ \text{m/s})}{-0.787\ \text{m/s}^2} = 1.68\ \text{s}$$

Example 3. A 3.5 kg block is pushed along a horizontal floor by a force $\vec{F}$ of magnitude 15 N at an angle $\theta = 40°$ with the horizontal. The coefficient of kinetic friction between the block and the floor is 0.25.
(a) Calculate the magnitude of the frictional force on the block from the floor.
(b) Calculate the magnitude of the block's acceleration



Solution

9. We choose the $+x$ direction horizontally rightward and $+y$ upward and observe that the 15 N force has components $F_x = F\cos\theta$ and $F_y = -F\sin\theta$.

(a) We apply Newton's second law to the y axis:

$$F_N - F\sin\theta - mg = 0 \;\Rightarrow\; F_N = (15\text{ N})\sin 40° + (3.5\text{ kg})(9.8\text{ m/s}^2) = 44\text{ N}.$$

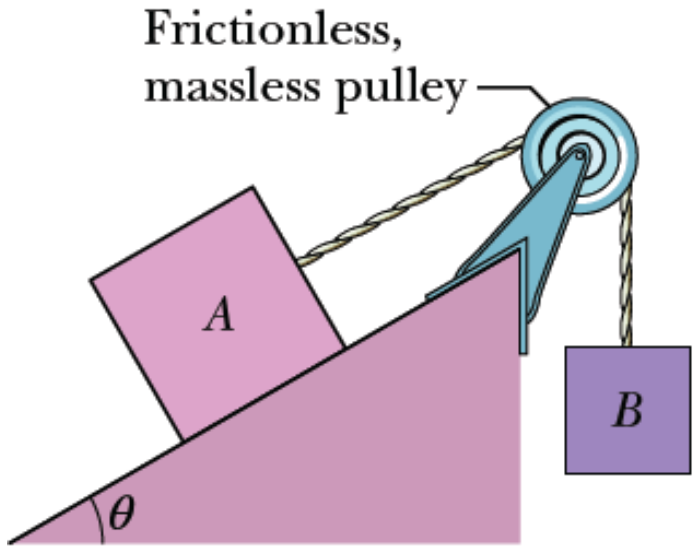
With $\mu_k = 0.25$, Eq. 6.1.2 leads to $f_k = 11$ N.

(b) We apply Newton's second law to the x axis:

$$F\cos\theta - f_k = ma \;\Rightarrow\; a = \frac{(15\text{ N})\cos 40° - 11\text{ N}}{3.5\text{ kg}} = 0.14\text{ m/s}^2.$$

Since the result has a positive value, the block is accelerating in the $+x$ (rightward) direction.

Example 4. Two blocks are connected over a pulley. The mass of block *A* is 10 kg, and the coefficient of kinetic friction between *A* and the incline is 0.20. Angle θ of the incline is 30°. Block *A* slides down the incline at constant speed. What is the mass of block *B*?
Assume the connecting rope has negligible mass. (The pulley's function is only to redirect the rope.)

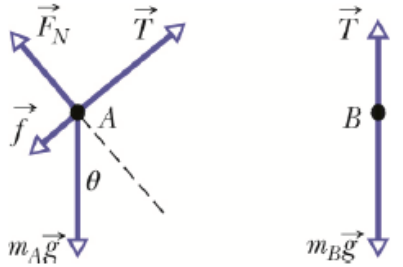


Since *A* is moving down the incline, the force of friction is uphill with magnitude $f_k = \mu_k F_N$ (where $\mu_k = 0.20$). Newton's second law leads to

$$T - f_k + m_A g \sin\theta = m_A a = 0$$
$$F_N - m_A g \cos\theta = 0$$
$$m_B g - T = m_B a = 0$$

for the two bodies (where $a = 0$ is a consequence of the velocity being constant). We solve these for the mass of block *B*:

$$m_B = m_A(\sin\theta - \mu_k \cos\theta) = 3.3 \text{ kg}.$$



5.1 Uniform Circular Motion

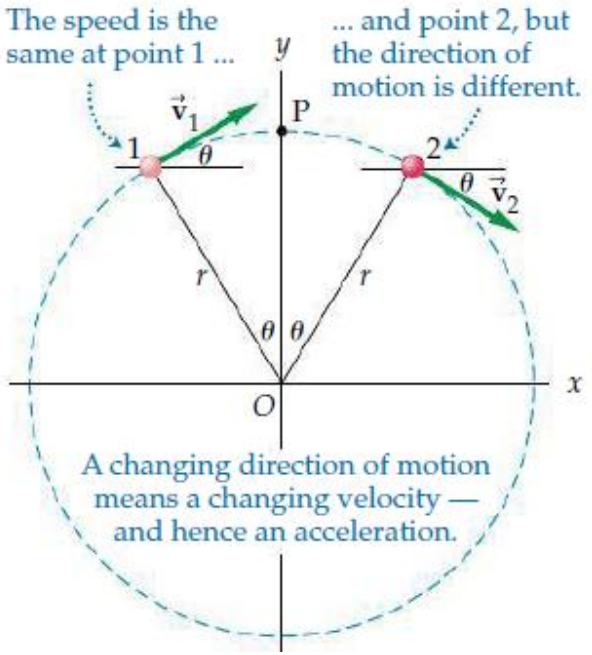
Centripetal Acceleration

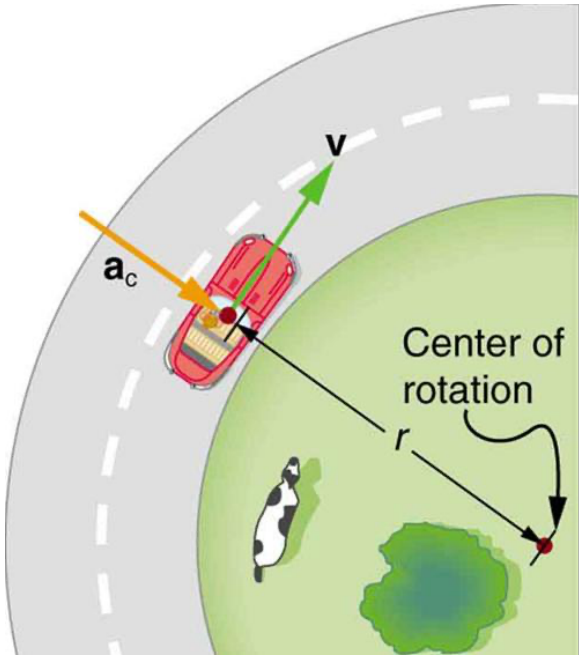
when a body moves in a circle (or a circular arc) at constant speed v, it is said to be in uniform circular motion. Also recall that the body has a centripetal acceleration (directed toward the center of the circle) of constant magnitude given by

- Recall that circular motion requires a centripetal acceleration

$$a_c = \frac{v^2}{R}$$

Where a_{cp} = centripetal acceleration, v = velocity, and R = circular radius



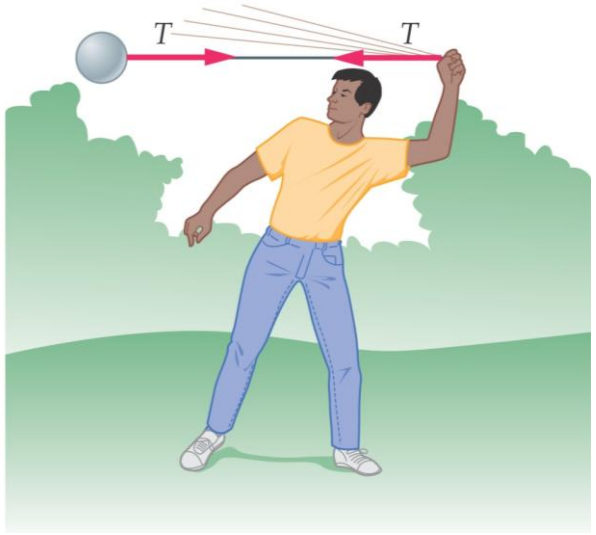


The speed of the particle is constant, but its velocity is constantly changing direction. Because the velocity changes, the particle is accelerating.

Centripetal Force Affects Direction of Motion

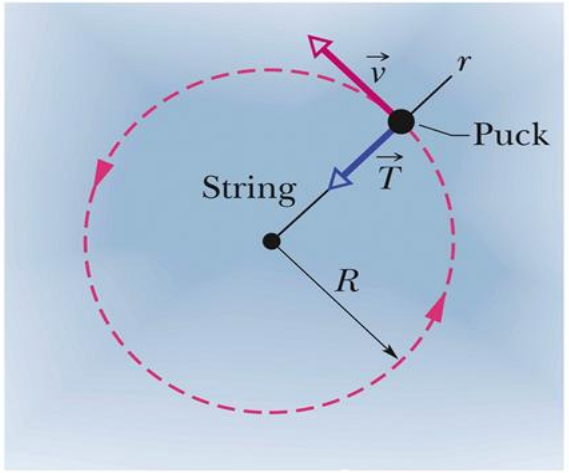
Centripetal force is not a new kind of force, it is simply an application of force

$$F_c = ma_c = m\frac{v^2}{R}$$ (magnitude of centripetal force) .



A centripetal force accelerates a body by changing the direction of the body's velocity without changing the body's speed.

For the puck on a string, the string tension supplies the centripetal force necessary to maintain circular motion



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

The puck moves in uniform circular motion only because of a toward-the-center force.

Example 5. During an Olympic bobsled run, the Jamaican team rounds a curve with a radius of 7.6 m at a speed of 96.6 km/h. What is the magnitude of their acceleration?

solution

44. With $v = 96.6$ km/h $= 26.8$ m/s, Eq. 6.3.1 readily yields

$$a = \frac{v^2}{R} = \frac{(26.8 \text{ m/s})^2}{7.6 \text{ m}} = 94.7 \text{ m/s}^2,$$

Example 6. A 2 kg ball on a string is rotated about a circle of radius 10 m. The maximum tension allowed in the string is 50 N. What is the maximum speed of the ball?

Answer: V = 15.8 m/s

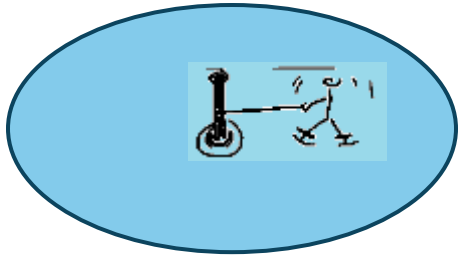
Example 7. A 55.0-kg ice skater is moving at 4.00 m/s when she grabs the loose end of a rope, the opposite end of which is tied to a pole. She then moves in a circle of radius 0.800 m around the pole.

(a) Determine the force exerted by the horizontal rope on her arms.

(b) Compare this force with her weight.

Answers: F_{cp}= 1100 N

W = mg = 540N



Summary of Chapter 6

- Friction is due to microscopic roughness.
- Kinetic friction: $f_{\mathrm{k}} = \mu_{\mathrm{k}} N$
- Static friction: $f_{\mathrm{s,max}} = \mu_{\mathrm{s}} N$
- Tension: the force transmitted through a string.

Summary of Chapter 6

- Centripetal acceleration required to maintain the motion

$$a_c = \frac{v^2}{R}$$

- Corresponds to a centripetal force

$$F_c = m\frac{v^2}{R}$$

- Force points toward the center of curvature

Additional Exercises

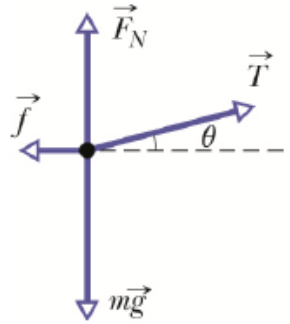
3. A baseball player slides into third base with an initial speed of 4.0 m/s. If the coefficient of kinetic friction between the player and the ground is 0.46,how far does the player slide before coming to rest. (Answer: 1.8 m)

4. A child goes down a playground slide with an acceleration of 1.26 m/s^2. Find the coefficient of kinetic friction between the child and the slide if the slide is inclined at an angle of 33^0 below the horizontal. (Answer: 0.496)

5. Hoping into your Porsche, you floor it and accelerate at 12 m/s^2 without spinning the tires. Determine the minimum coefficient of static friction between the tires and the road needed to make this possible. (Answer: 1.2)

6. A 4 kg ball on a string is rotated about a circle of radius10 m. The maximum tension allowed in the string is 52 N. What is the maximum speed of the ball?(Answer : 11.4 m/s.)

11 E SSM A 68 kg crate is dragged across a floor by pulling on a rope attached to the crate and inclined 15° above the horizontal. (a) If the coefficient of static friction is 0.50, what minimum force magnitude is required from the rope to start the crate moving? (b) If $\mu_k = 0.35$, what is the magnitude of the initial acceleration of the crate?



solution

The equations for the x and the y components of the force according to Newton's second law are

$$T\cos\theta - f = 0, \quad T\sin\theta + F_N - mg = 0,$$

where $\theta = 15°$ is the angle between the rope and the horizontal. The first equation gives $f = T\cos\theta$ and the second gives $F_N = mg - T\sin\theta$. If the crate is to remain at rest, f must be less than $\mu_s F_N$, or $T\cos\theta < \mu_s(mg - T\sin\theta)$. When the tension force is sufficient to just start the crate moving, we must have $T\cos\theta = \mu_s(mg - T\sin\theta)$.

where $\theta = 15°$ is the angle between the rope and the horizontal. The first equation gives $f = T\cos\theta$ and the second gives $F_N = mg - T\sin\theta$. If the crate is to remain at rest, f must be less than $\mu_s F_N$, or $T\cos\theta < \mu_s(mg - T\sin\theta)$. When the tension force is sufficient to just start the crate moving, we must have $T\cos\theta = \mu_s(mg - T\sin\theta)$.

$$T = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta} = \frac{(0.50)(68\text{ kg})(9.8\text{ m/s}^2)}{\cos 15° + 0.50\sin 15°} = 304\text{ N} \approx 3.0\times10^2\text{ N}.$$

(b) The second-law equations for the moving crate are

$$T\cos\theta - f = ma, \quad T\sin\theta + F_N - mg = 0.$$

Now $f = \mu_k F_N$, and the second equation above gives $F_N = mg - T\sin\theta$, which then yields $f = \mu_k(mg - T\sin\theta)$. This expression is substituted for f in the first equation to obtain

$$T\cos\theta - \mu_k(mg - T\sin\theta) = ma,$$

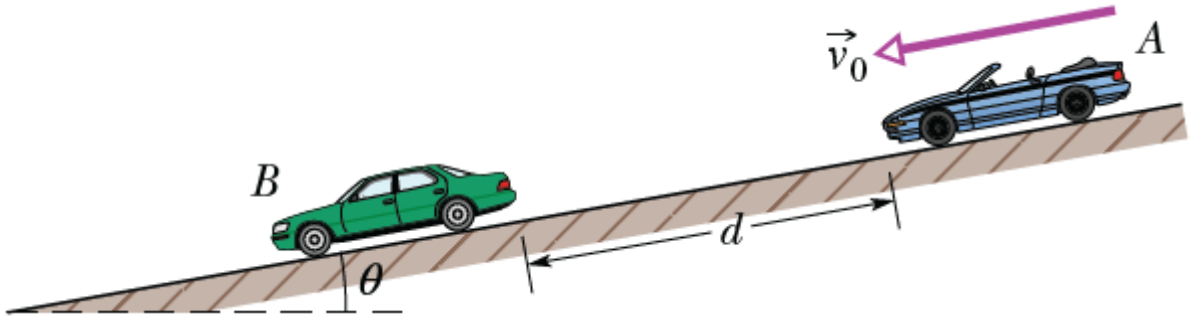
so the acceleration is

$$a = \frac{T(\cos\theta + \mu_k\sin\theta)}{m} - \mu_k g$$
$$= \frac{(304\text{ N})(\cos 15° + 0.35\sin 15°)}{68\text{ kg}} - (0.35)(9.8\text{ m/s}^2) = 1.3\text{ m/s}^2.$$

Copyright © 2022 John Wiley & Sons, Inc.

You testify as an *expert witness* in a case involving an accident in which car *A* slid into the rear of car *B*, which was stopped at a red light along a road headed down a hill. You find that the slope of the hill is θ = 12.0°, that the cars were separated by distance *d* = 24.0 m when the driver of car *A* put the car into a slide (it lacked any automatic anti-brake-lock system), and that the speed of car *A* at the onset of braking was $v_0 = 18.0\ m/s$.
With what speed did car *A* hit car *B* if the coefficient of kinetic friction was (a) 0.60 (dry road surface) and (b) 0.10 (road surface covered with wet leaves)?

solution



18. (a) We apply Newton's second law to the "downhill" direction:

$$mg\sin\theta - f = ma,$$

where

$$f = f_k = \mu_k F_N = \mu_k mg\cos\theta.$$

Thus, with $\mu_k = 0.600$, we have

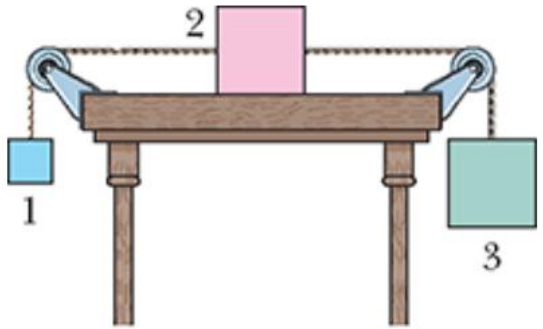
$$a = g\sin\theta - \mu_k\cos\theta = -3.72\ \text{m/s}^2,$$

which means, since we have chosen the positive direction in the direction of motion (down the slope), then the acceleration vector points "uphill"; it is decelerating. With $v_0 = 18.0$ m/s and $\Delta x = d = 24.0$ m, Eq. 2.4.6 leads to

$$v = \sqrt{v_0^2 + 2ad} = 12.1\ \text{m/s}.$$

(b) In this case, we find $a = +1.1\ \text{m/s}^2$, and the speed (when impact occurs) is 19.4 m/s.

23 M When the three blocks in Fig. 6.18 are released from rest, they accelerate with a magnitude of 0.500 m/s^2. Block 1 has mass M, block 2 has $2M$, and block 3 has $2M$. What is the coefficient of kinetic friction between block 2 and the table?



23. Let the tensions on the strings connecting m_2 and m_3 be T_{23} and that connecting m_2 and m_1 be T_{12}, respectively. Applying Newton's second law (and Eq. 6.1.2, with $F_N = m_2 g$ in this case) to the *system*, we have

$$\begin{aligned} m_3 g - T_{23} &= m_3 a \\ T_{23} - \mu_k m_2 g - T_{12} &= m_2 a \\ T_{12} - m_1 g &= m_1 a. \end{aligned}$$

Adding up the three equations and using $m_1 = M, m_2 = m_3 = 2M$, we obtain

$$2Mg - 2\mu_k Mg - Mg = 5Ma.$$

With $a = 0.500$ m/s^2 this yields $\mu_k = 0.372$. Thus, the coefficient of kinetic friction is roughly $\mu_k = 0.37$.