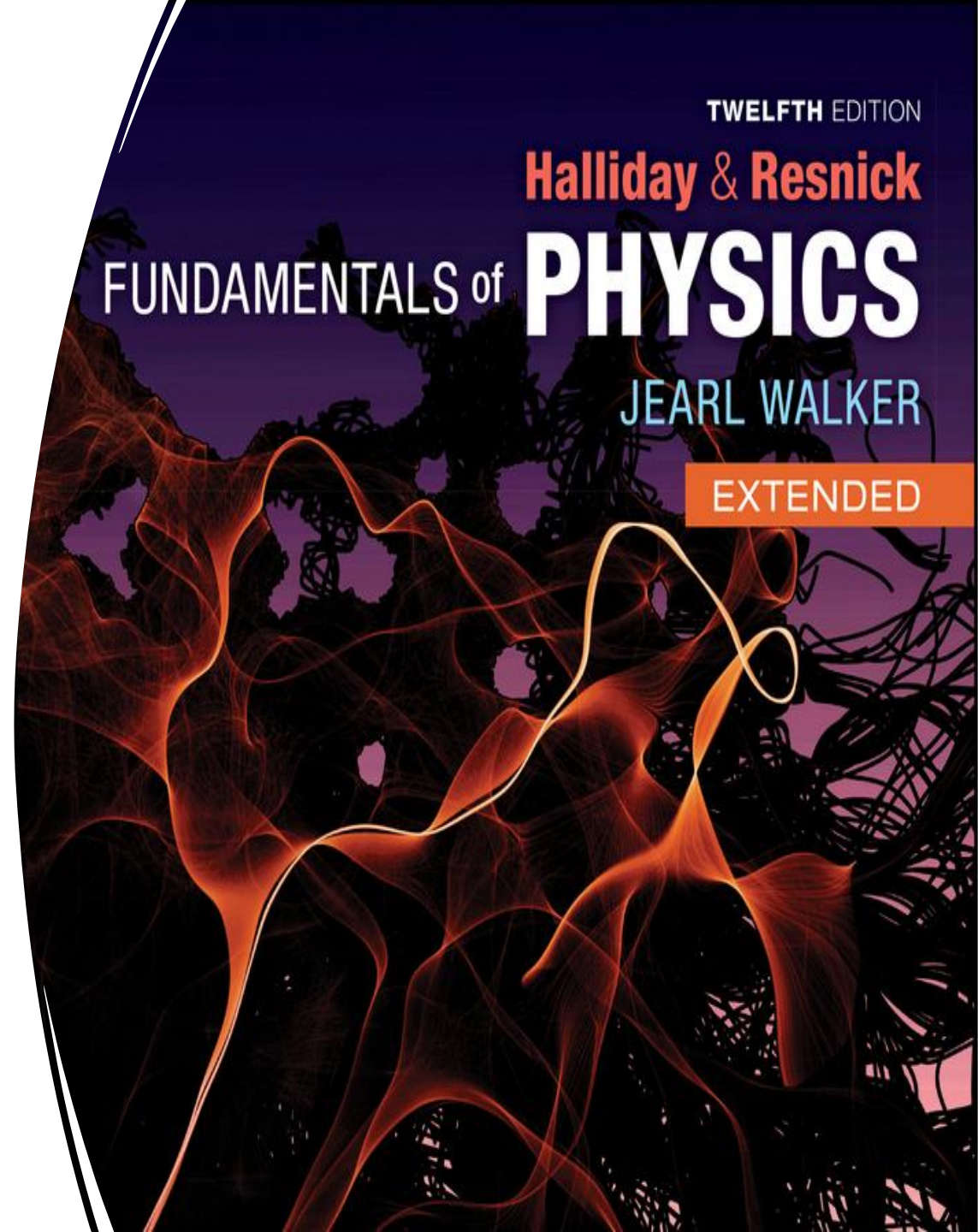


Chapter 5

Force and Motion—I



Units of Chapter 5

5.1. Newton's First and Second Laws

5.2. Some particular Forces

5.3. Application of Newton's Laws

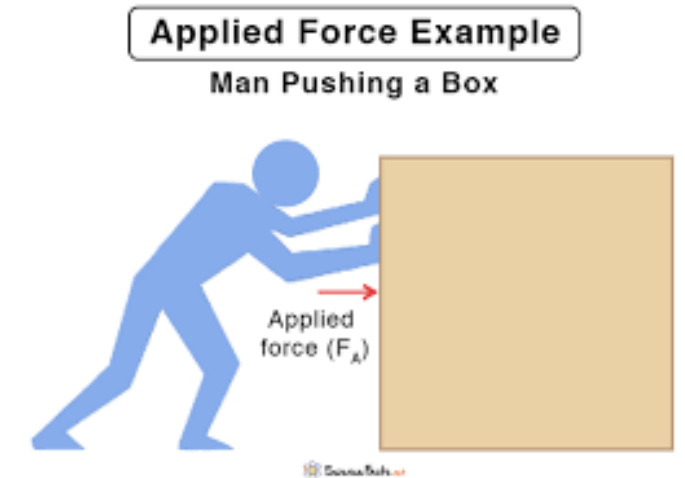
5.1 Newton's First and Second Laws

Force

Force is a **push or pull** upon an object resulting from the object's **interaction** with another object.

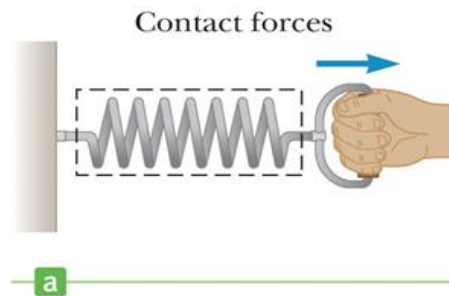
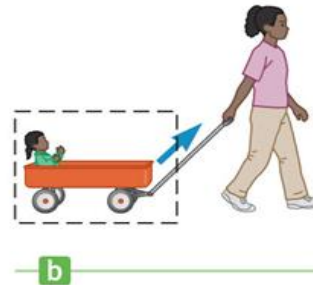
Characteristics of Forces

- Unit: N, the newton; $1 \text{ N} = 1 \text{ kg m/s}^2$
- Acceleration of a mass is proportional to the exerted force
- Forces are vectors - it has magnitude (strength of force) and direction.
- **Net force** is the vector sum of all forces on an object
- **Principle of superposition for forces:**
 - A net force has the same impact as a single force with identical magnitude and direction

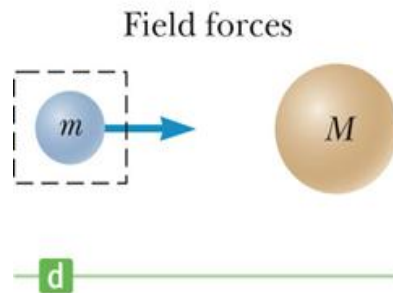
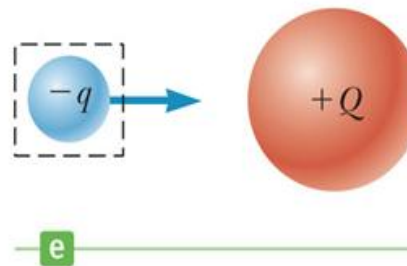
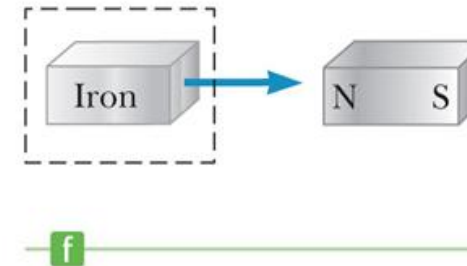


- Two categories of forces:

- **Contact forces** result from physical contact between two objects. Examples: pushing or pulling objects, friction, normal force



- **Field forces** (action at a distance). Examples: gravity, electrical forces, magnetic forces



Newtonian mechanics

The relation between a force and the acceleration it causes was first understood by Isaac Newton (1642–1727) and is the subject of this chapter. The study of that relation, as Newton presented it, is called Newtonian mechanics. We shall focus on its three primary laws of motion.

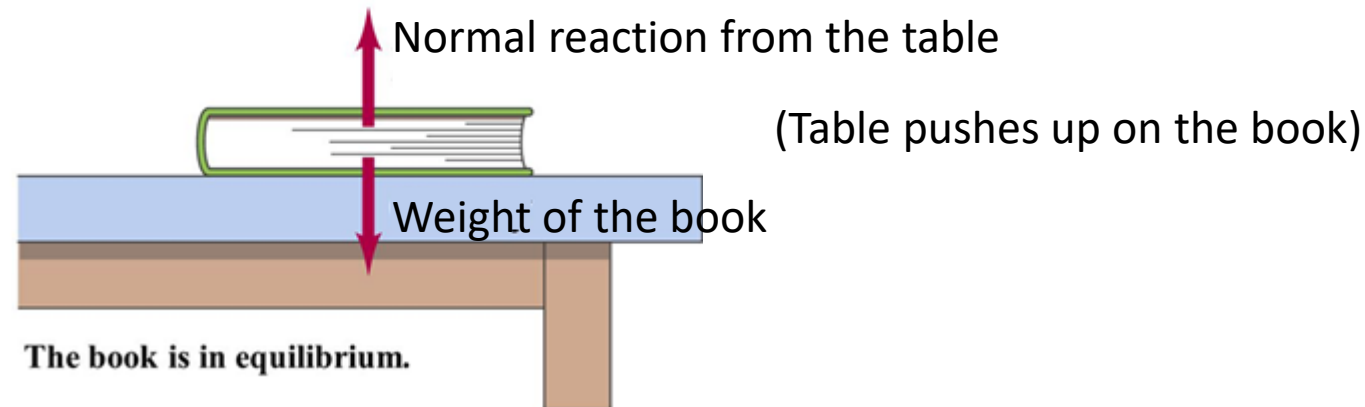
Newton's First Law

- **Definition:** If no net force acts on a body, the body's velocity cannot change; that is, the body cannot accelerate.
- **In other words:** In the absence of a net external force, an object at rest will remain at rest, and an object in motion will continue to move at a constant velocity (constant speed in the same direction)
- In this case, the object is said to be in equilibrium.

Newton's first law: $\sum \vec{F} = 0$... must be zero if body is in equilibrium.
Net force on a body ...

- In order to change the velocity of an object—magnitude or direction—a net force is required.

- Several forces can act on an object. The **net force $\Sigma \vec{F}$ is their vector sum.**
- A zero net force does not necessarily mean that no forces are acting on the object.
 - In the figure, even though forces act on the book, they are **balanced**. Therefore, there is no **net force** on the book ($\Sigma \vec{F} = 0$).



- A non-zero net force changes an object's state of motion, resulting in acceleration.

Definition of Mass

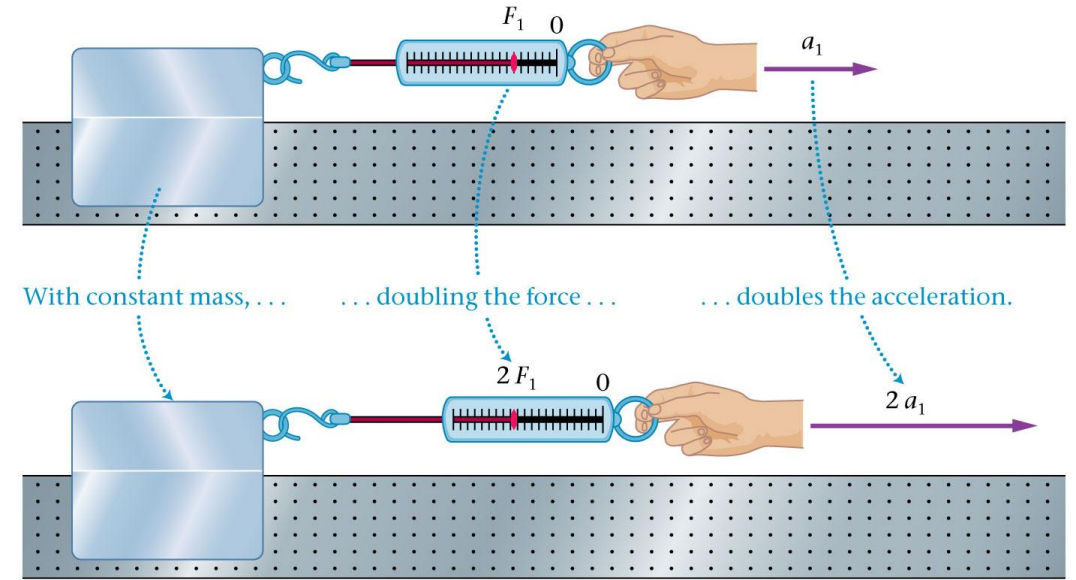
- Mass is a measure of a body’s resistance to a change in motion (change in velocity). Mass specifies how much inertia an object has.
- Mass can also be thought of as a measure of the quantity of matter in an object.
- Mass is an intrinsic property of a body and is independent of any external factor and **constant** at any place and time
- **Mass** is a **scalar quantity**, measured in kilograms (**kg**) in SI units **using balance**.
- Note that mass is different from weight (the force of gravity). **Weight** is a **vector quantity**, measured in newtons (**N**) in SI units.

TABLE **Typical Masses in Kilograms (kg)**

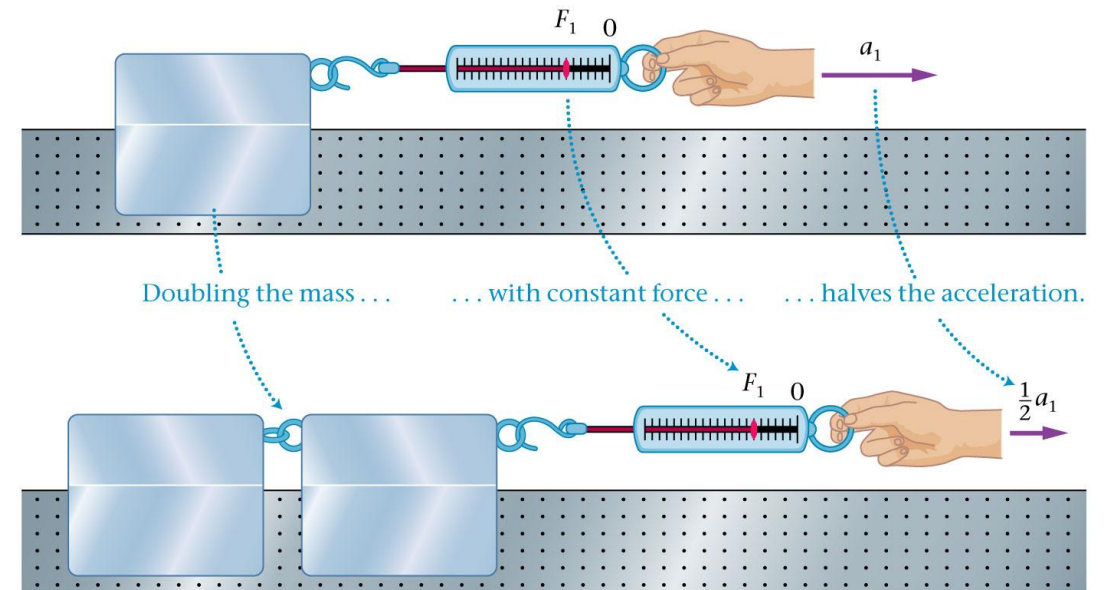
Earth	5.97×10^{24}
Space shuttle	2,000,000
Blue whale (largest animal on Earth)	105,000
Whale shark (largest fish)	18,000
Elephant (largest land animal)	5400
Automobile	1200
Human (adult)	70
Gallon of milk	3.9
Quart of milk	1.0
Baseball	0.145
Honeybee	0.00015
Bacterium	10^{-15}

Effect of net force on acceleration

- Acceleration is proportional to the net force:



- Acceleration is inversely proportional to mass. The same force on a larger mass produces a smaller acceleration.



Newton's Second Law Statement

Newton's Second Law: The net force on a body is equal to the product of the body's mass and its acceleration.

- As an equation, we write:

Newton's second law:
If there is a net force on a body ... $\sum \vec{F} = m\vec{a}$... the body accelerates in same direction as the net force.
Mass of body

- Remember: forces are vector quantities. Dealing with the vector components:
- Identify the body in question, and only include forces that act on that body!
- Separate the problem axes (they are independent):

$$F_{\text{net},x} = ma_x, \quad F_{\text{net},y} = ma_y, \quad \text{and} \quad F_{\text{net},z} = ma_z.$$

Vector Nature of Force and Acceleration

The acceleration component along a given axis is caused only by the sum of the force components along that same axis, and not by force components along any other axis.

If the net force on a body is zero:

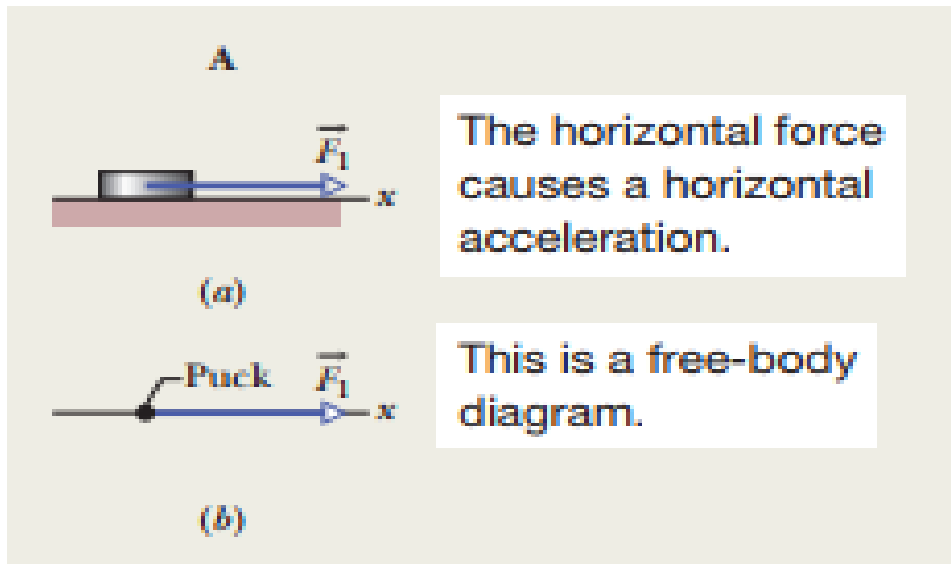
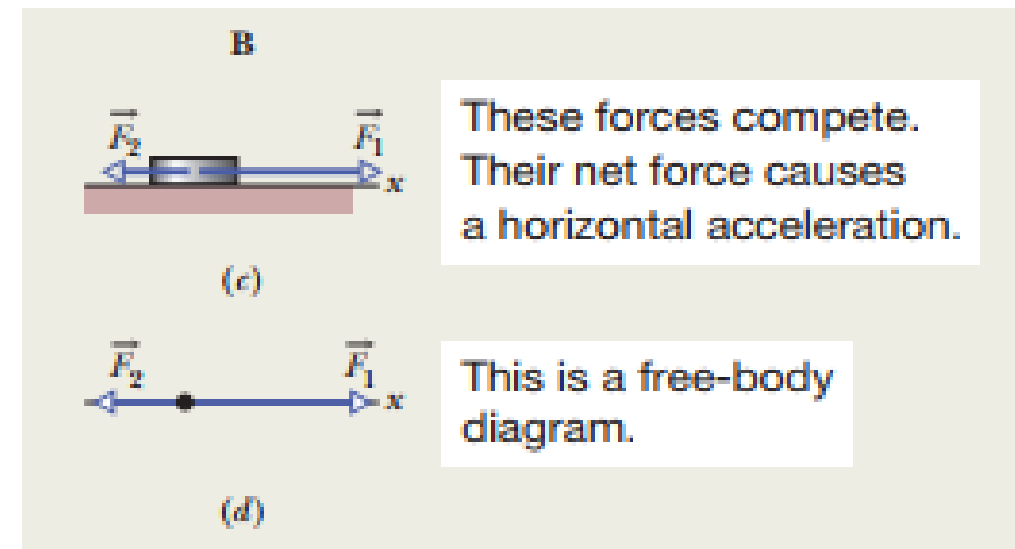
- Its acceleration is zero

- The forces and the body are in equilibrium

- There may still be forces that are present, but they cancel out!

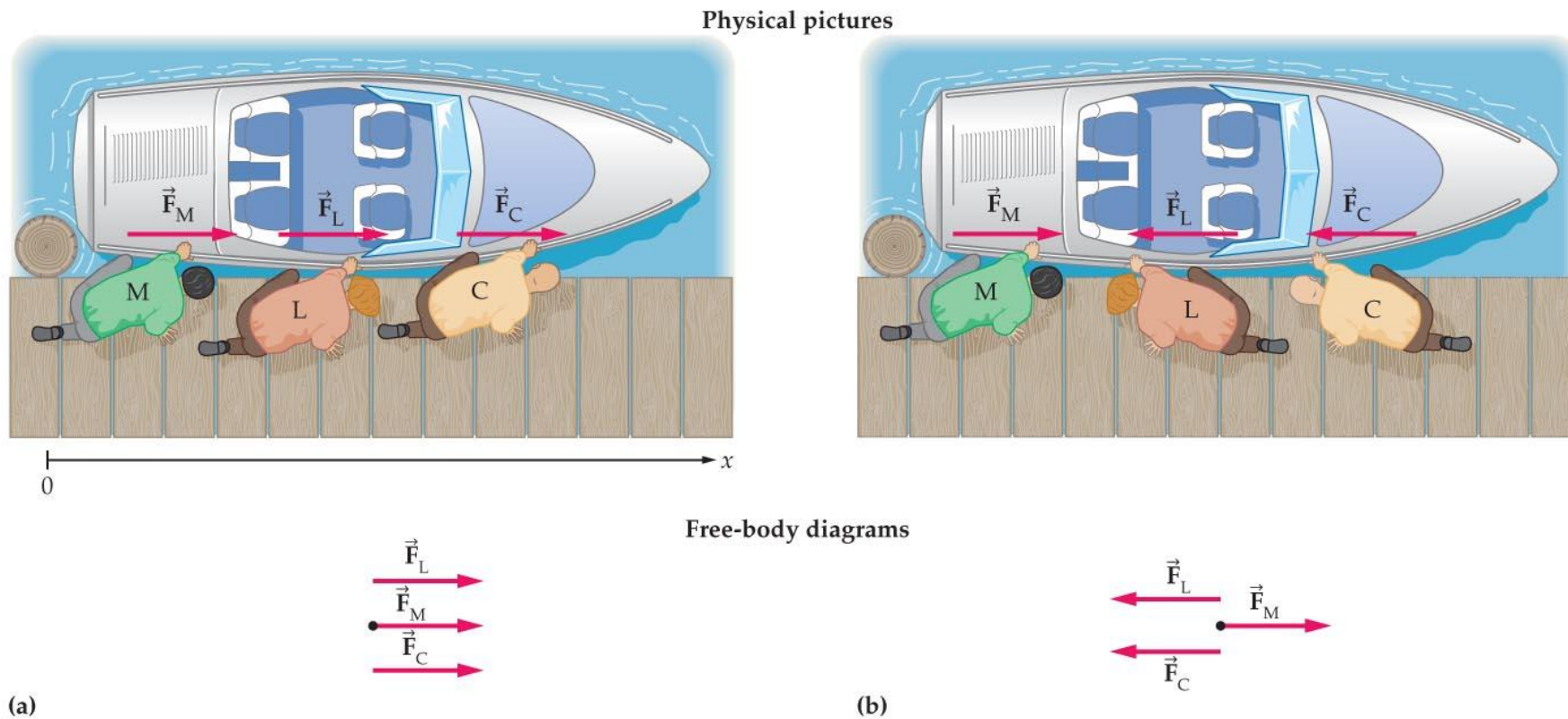
Free Body Diagram

- To solve problems with forces, we often draw a **free body diagram**
- The only body shown is the one we are solving for
- Forces are drawn as vector arrows with their tails on the body
- Coordinate system shown
- Acceleration is Never part of a free body diagram – only forces on a body are present.



Example 1: Three people push on a 752 kg boat, each exerting an 80.5 N force parallel to the dock.

- (a) What is the acceleration of the boat if they all push in the same direction? [Ans.: 0.321 m/s^2]
(b) What is the acceleration if one pushes in the opposite direction from the other two as shown? [Ans.: -0.107 m/s^2]



SOLUTION

Part (a)

1. Write out the x component for each of the three forces:
2. Sum the x components of force and set equal to ma_x :
3. Divide by the mass to find a_x . Since a_x is positive, the acceleration is to the right, as expected:

$$F_{M,x} = F_{L,x} = F_{C,x} = 80.5 \text{ N}$$

$$\sum F_x = F_{M,x} + F_{L,x} + F_{C,x} = 241.5 \text{ N} = ma_x$$

$$a_x = \frac{\sum F_x}{m} = \frac{241.5 \text{ N}}{752 \text{ kg}} = 0.321 \text{ m/s}^2$$

Part (b)

4. Again, start by writing the x component for each force:
5. Sum the x components of force and set equal to ma_x :
6. Solve for a_x . In this case a_x is negative, indicating an acceleration to the left:

$$F_{M,x} = 80.5 \text{ N}$$

$$F_{L,x} = F_{C,x} = -80.5 \text{ N}$$

$$\begin{aligned}\sum F_x &= F_{M,x} + F_{L,x} + F_{C,x} \\ &= 80.5 \text{ N} - 80.5 \text{ N} - 80.5 \text{ N} = -80.5 \text{ N} = ma_x\end{aligned}$$

$$a_x = \frac{\sum F_x}{m} = \frac{-80.5 \text{ N}}{752 \text{ kg}} = -0.107 \text{ m/s}^2$$

Example 2. If the 1 kg standard body has an acceleration of 2.00 m/s^2 at 20.0° to the positive direction of an axis, what are (a) the x component and (b) the y component of the net force acting on the body, and (c) what is the net force in unit-vector notation?

[Ans.: $F_x = 1.88 \text{ N}$; $F_y = 0.684 \text{ N}$; $\vec{F} = (1.88)\hat{i} + (0.684)\hat{j}$]

(a) We find the x component of the force is

$$F_x = ma_x = ma \cos 20.0^\circ = (1.00 \text{ kg}) (2.00 \text{ m/s}^2) \cos 20.0^\circ = 1.88 \text{ N}.$$

(b) The y component of the force is

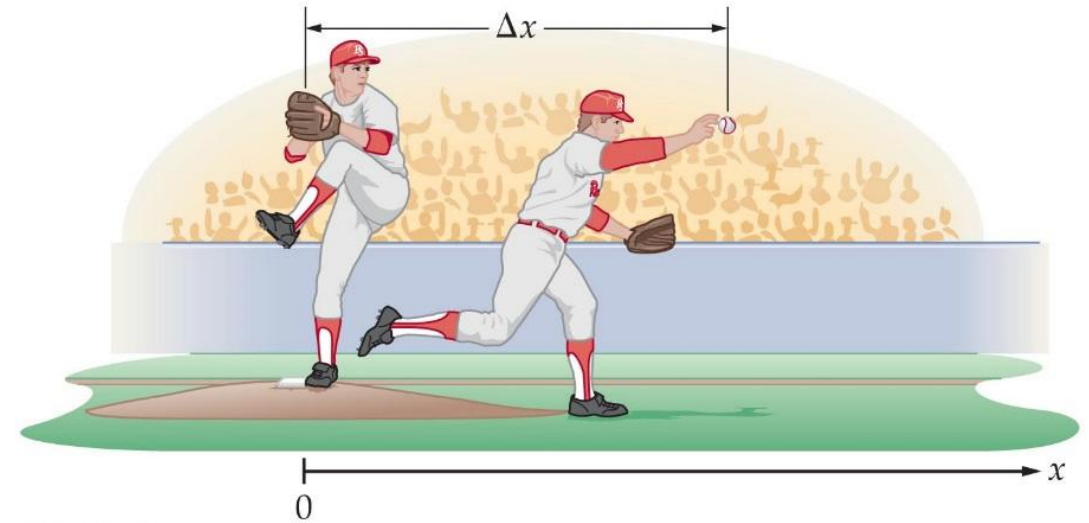
$$F_y = ma_y = ma \sin 20.0^\circ = (1.0 \text{ kg}) (2.00 \text{ m/s}^2) \sin 20.0^\circ = 0.684 \text{ N}.$$

(c) In unit-vector notation, the force vector is

$$\vec{F} = F_x \hat{i} + F_y \hat{j} = (1.88 \text{ N})\hat{i} + (0.684 \text{ N})\hat{j}.$$

Example 3: A pitcher throws a 0.15-kg baseball, accelerating it from rest to a speed of about 90 mi/h. Estimate the force exerted by the pitcher on the ball. The distance over which the pitcher accelerates the ball is 2 m.

[Ans.: $v = 40 \text{ m/s}$; $a_x = 400 \text{ m/s}^2$; $F_x = 60 \text{ N}$]



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SOLUTION

1. Starting with the fact that $60 \text{ mi/h} = 1 \text{ mi/min}$, perform a rough back-of-the-envelope conversion of 90 mi/h to meters per second:
2. Solve $v^2 = v_0^2 + 2a_x \Delta x$ for the acceleration, a_x . Use the estimates $\Delta x \approx 2.0 \text{ m}$ and $v \approx 40 \text{ m/s}$:
3. Find the corresponding force with $F_x = ma_x$:

$$v \approx 90 \text{ mi/h} = \frac{1.5 \text{ mi}}{\text{min}} \approx \frac{2400 \text{ m}}{60 \text{ s}} = 40 \text{ m/s}$$

$$a_x = \frac{v^2 - v_0^2}{2 \Delta x} \approx \frac{(40 \text{ m/s})^2 - 0}{2(2.0 \text{ m})} = 400 \text{ m/s}^2$$

$$F_x = ma_x \approx (0.15 \text{ kg})(400 \text{ m/s}^2) = 60 \text{ N}$$

5.2 Some Particular Forces

Weight

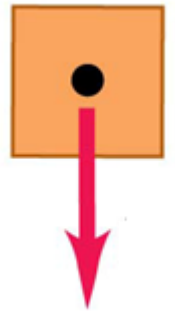
- The weight W of a body is the magnitude of the net force required to prevent the body from falling freely, as measured by someone on the ground.

$$W = F_g \text{ (weight, with ground as inertial frame).}$$

$$W = mg \text{ (mass – weight relationship)}$$

- The weight W of a body is equal to the magnitude F_g of the gravitational force on the body.
- The weight is a vector quantity, pointing downward.

$$\vec{F}_g = \vec{W} = m\vec{g}$$



Weight Depends on gravity at the place and is measured in Newton not in kilograms using scales or spring scales

Apparent weight:

Do you feel lighter/heavier when you are riding an elevator?

Your perception of your weight is based on the contact forces between you and your surroundings.

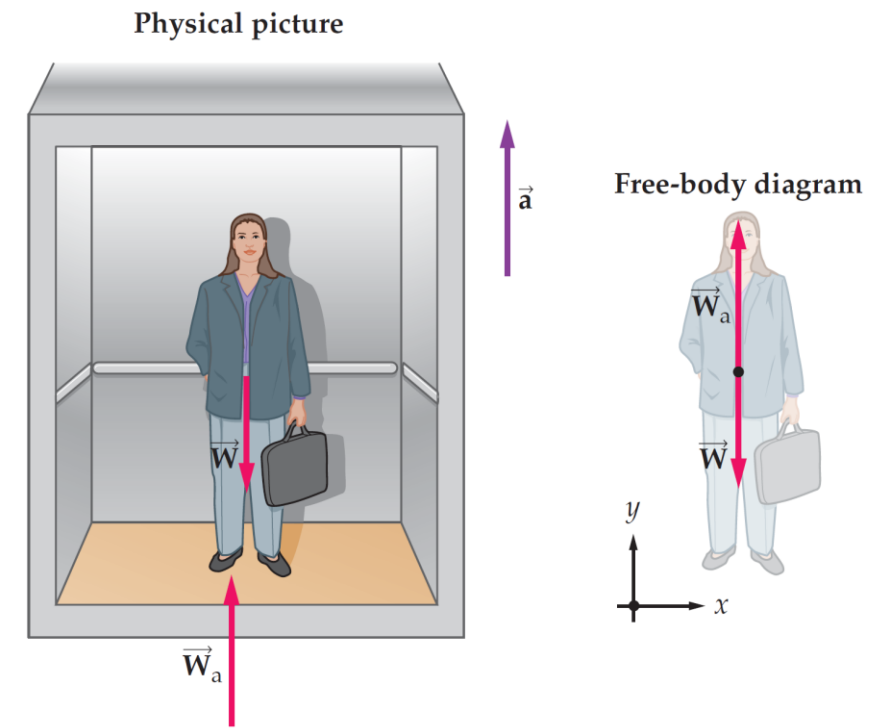
If the floor is accelerating, your apparent weight may be more or less than your actual weight.

Sensation of your weight = **Apparent weight (W_a)**. (Also called Normal force)

Applying Newton's Second law $\Sigma F_y = ma_y$: $W_a - mg = ma_y$
 $W_a = m(g + a_y)$

a_y is positive when the elevator accelerates upward and negative when it accelerates downward.

- Elevator **upward acceleration**:
 - Apparent weight is greater than actual weight ($W_a > W$)
 - Note: velocity can be upward (increasing speed) or downward (slowing down).
- Elevator **downward acceleration**:
 - Apparent weight is less than actual weight ($W_a < W$)
 - Note: velocity can be upward (slowing down) or downward (increasing speed).



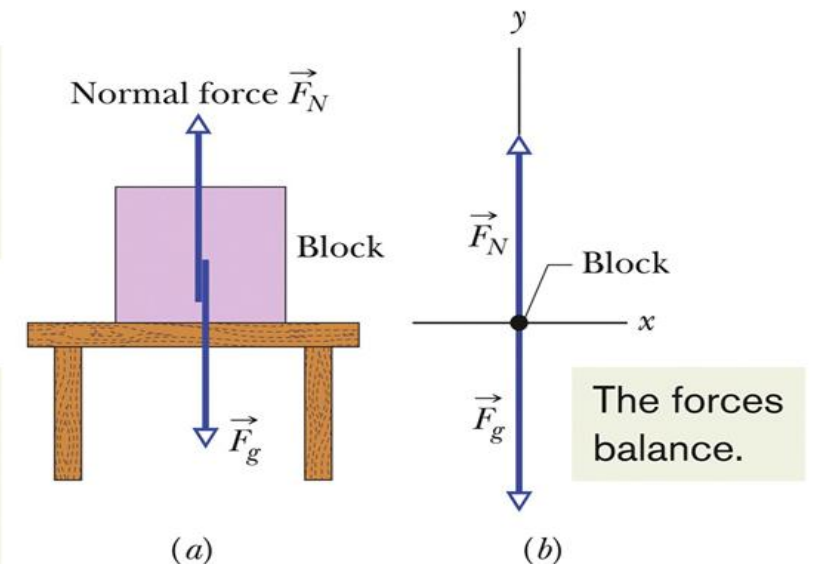
Normal Force

- If you are standing on a surface, the push back on you from the surface (due to deformation) is the normal force
- Normal means perpendicular
- The **normal force** is the force **exerted by a surface on an object**.
- The normal force is always **perpendicular** to the surface.
- The normal force may be **equal** to, **greater** than, or **less** than the **weight**.

When a body presses against a surface, the surface pushes on the body with a normal force that is perpendicular to the surface.

The normal force is the force on the block from the supporting table.

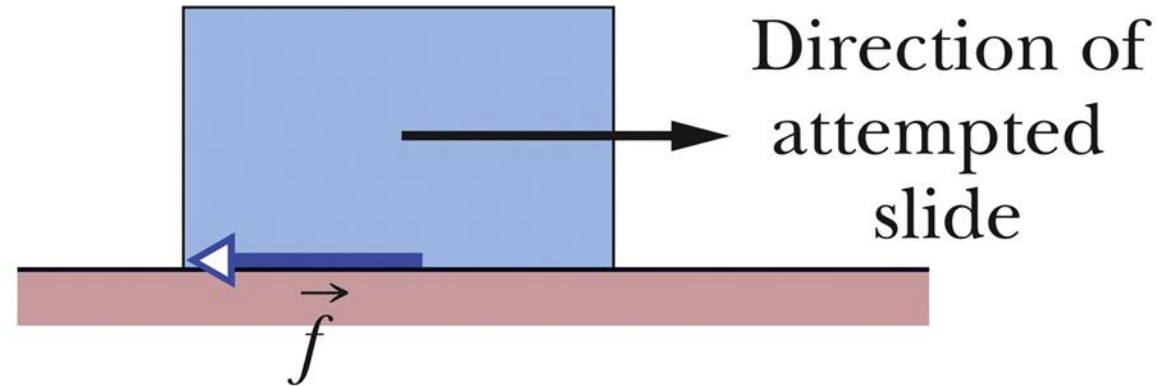
The gravitational force on the block is due to Earth's downward pull.



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Friction

- **Frictional force or friction:**
 - Occurs when one object slides or attempts to slide over another
 - Directed along the surface, opposite to the direction of intended motion



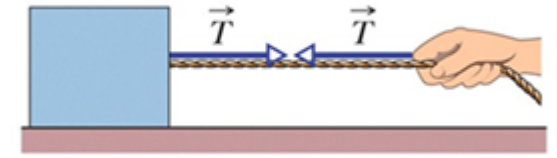
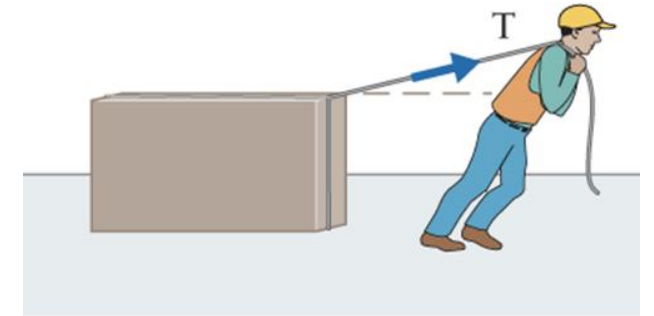
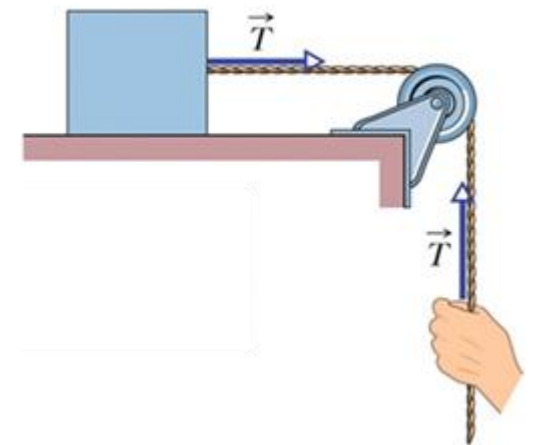
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Tension

- A cord (or rope, etc.) is attached to a body and pulled taut
- Cord pulls on the body with force $\vec{T}$ directed along the cord
- The cord is said to be under tension
- The tension in the cord is T

A massless and unstretchable cord exists only as a connection between two bodies

- It pulls on both with the same force, T
- True even if the bodies and cord are accelerating, and even if the cord runs around a massless, frictionless pulley
- These are useful simplifying assumptions
- All ropes, wires, strings are **massless** and hence, that the tension is the **same** throughout their length
- In the ideal case, a pulley has no mass and no friction in its bearings.

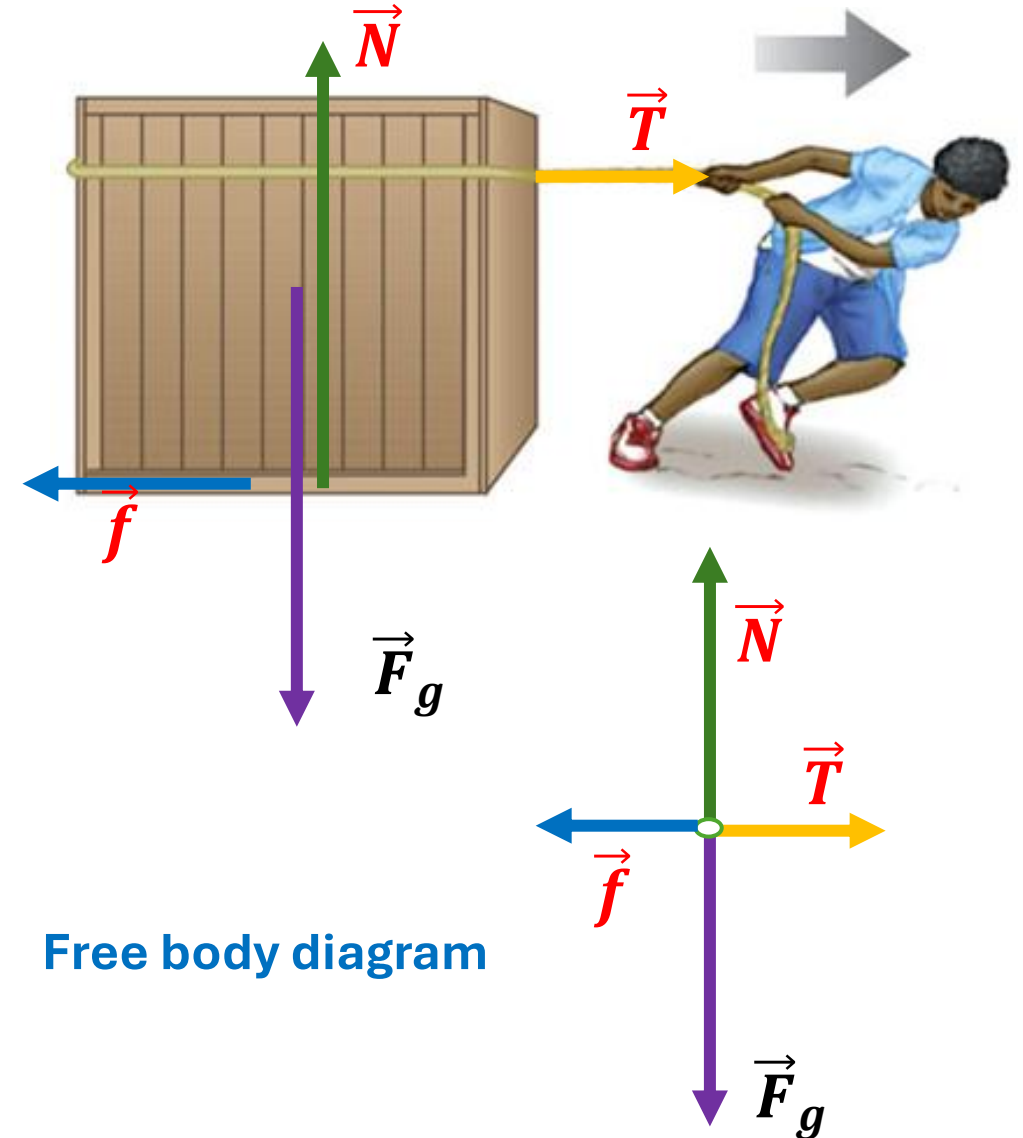


Free Body Diagram

Consider a crate being pulled to the right on a frictionless, horizontal surface.

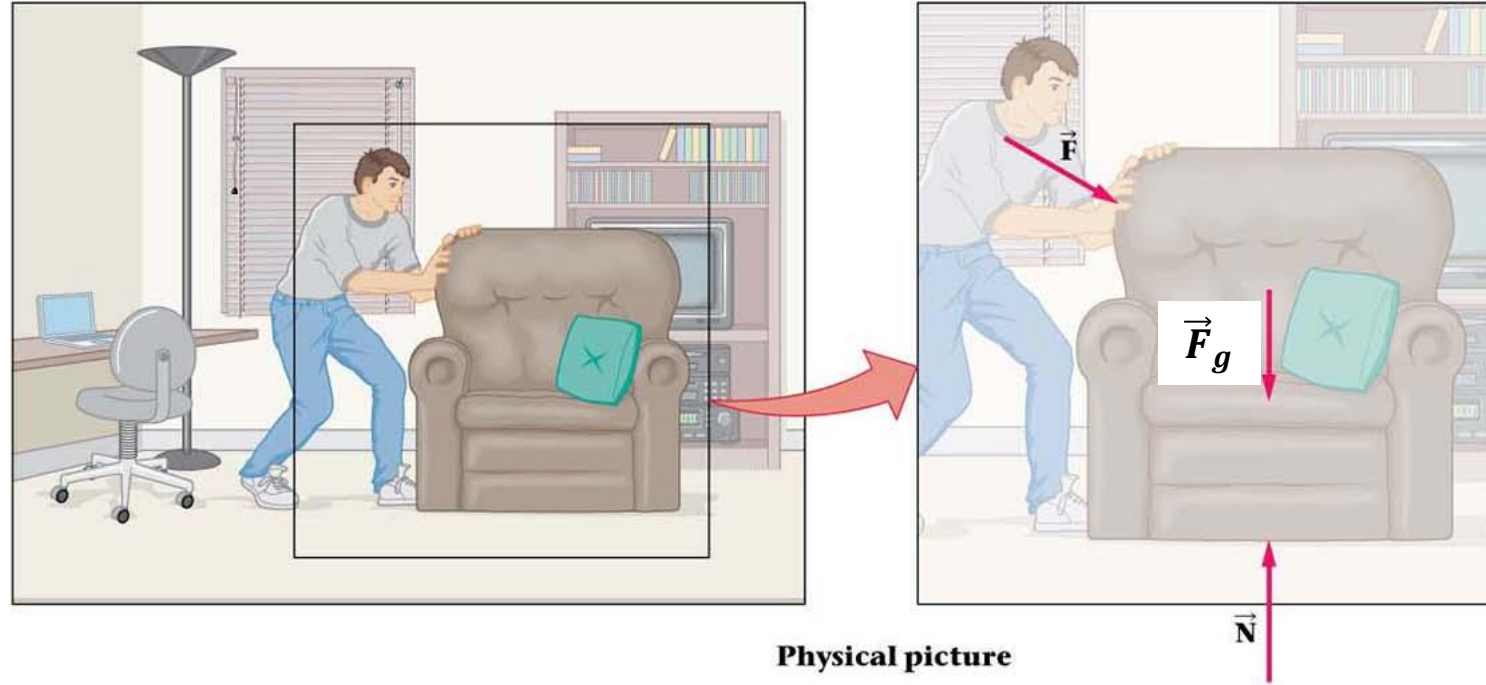
The force $\vec{T}$ is the tension acting on the box. The tension is the same at all points along the rope the force diagram for the crate includes the force of gravity $\vec{F}_g$ exerted by Earth the normal force $\vec{N}$ exerted by the floor.

Such a force diagram is called a **free body diagram** because the environment is replaced by a series of forces on an otherwise free body.

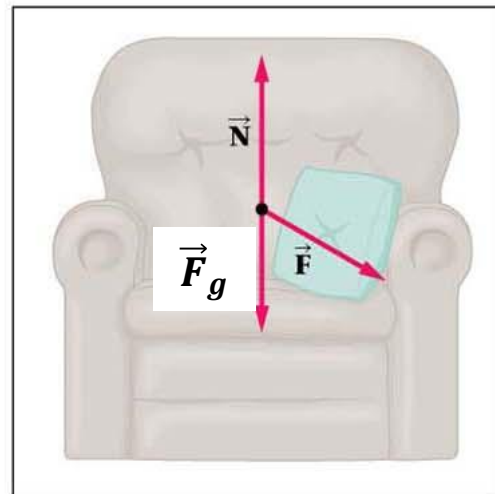


- (a) Sketch the forces
- (b) Isolate the object of interest
- (c) Choose a convenient coordinate system
- (d) Resolve the forces into components
- (e) Apply Newton's second law to each coordinate direction

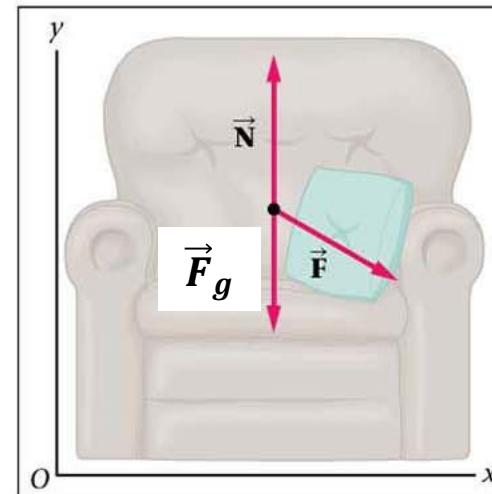
(a) Sketch the forces



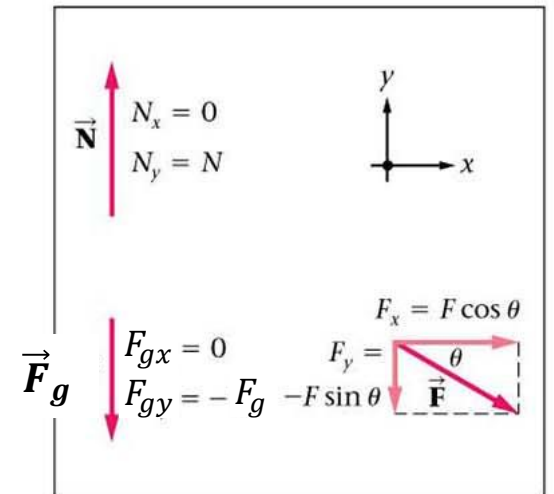
(b) Isolate the object of interest



(c) Choose a convenient coordinate system

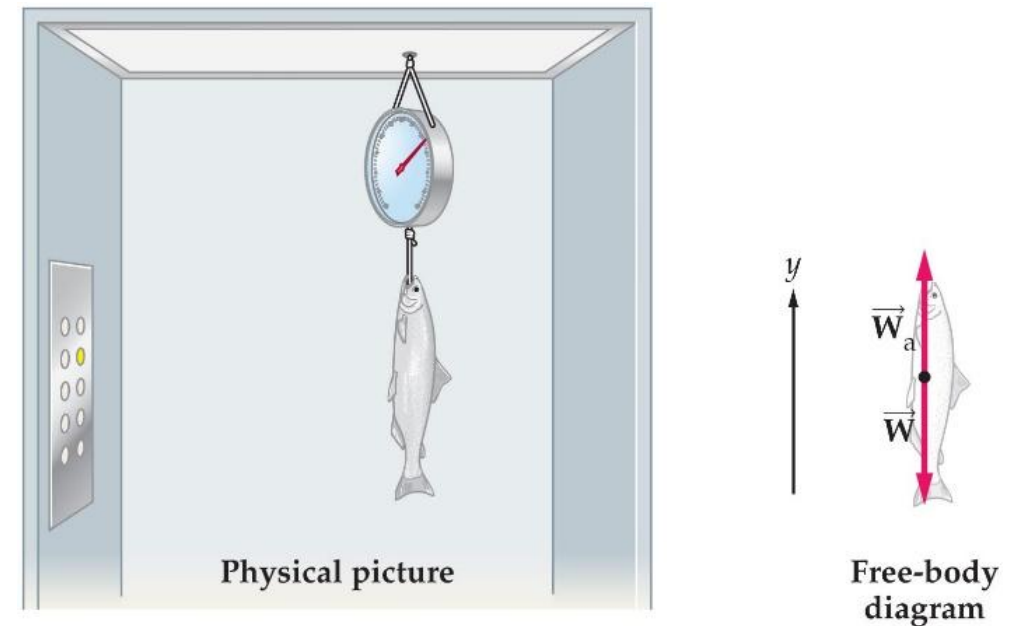


(d) Resolve forces into their components



Example 4: As part of an attempt to combine physics and biology in the same class, an instructor asks students to weigh a 5.0 kg salmon by hanging it from a fish scale attached to the ceiling of an elevator. What is the apparent weight of the salmon if the elevator **(a)** is at rest **(b)** moves with an upward acceleration of 2.5 m/s^2 , or **(c)** moves with downward acceleration of 3.2 m/s^2 ?

[Ans.: (a) 49 N, (b) 62 N, (c) 33 N]



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SOLUTION**Part (a)**

1. Sum the y component of the forces and set equal to mass times the y component of acceleration, with $a_y = 0$:
2. Solve for W_a , then write the vector $\vec{W}$:

$$\sum F_y = W_a - W = ma_y = 0$$

$$W_a = W = mg = (5.0 \text{ kg})(9.81 \text{ m/s}^2) = 49 \text{ N}$$

$$\vec{W}_a = (49 \text{ N})\hat{y}$$

Part (b)

3. Again, sum the forces and set equal to mass times acceleration, this time with $a_y = a = 2.5 \text{ m/s}^2$:
4. Solve for W_a , then write the vector $\vec{W}_a$:

$$\sum F_y = W_a - W = ma_y = ma$$

$$W_a = W + ma$$

$$= mg + ma = 49 \text{ N} + (5.0 \text{ kg})(2.5 \text{ m/s}^2) = 62 \text{ N}$$

$$\vec{W}_a = (62 \text{ N})\hat{y}$$

Part (c)

5. Finally, sum the forces and set equal to mass times acceleration, with $a_y = -a = -3.2 \text{ m/s}^2$:
6. Solve for W_a , then write the vector $\vec{W}_a$:

$$\sum F_y = W_a - W = ma_y = -ma$$

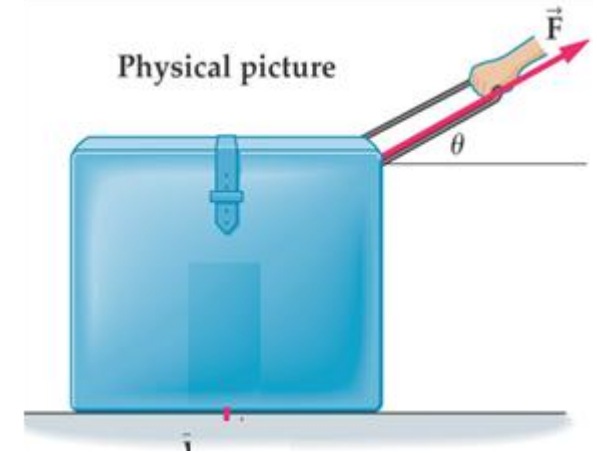
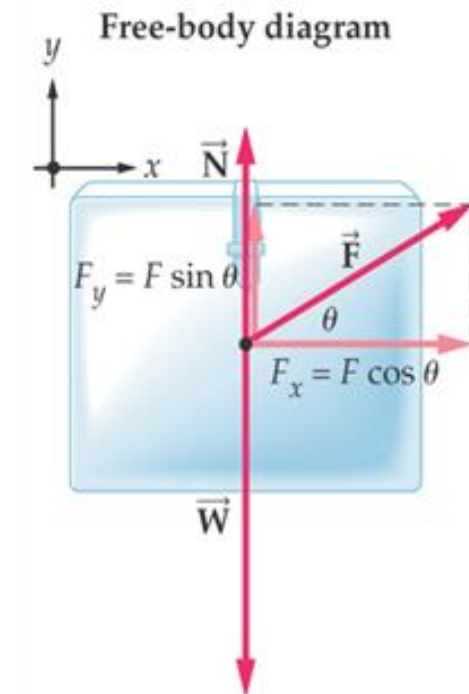
$$W_a = W - ma$$

$$= mg - ma = 49 \text{ N} - (5.0 \text{ kg})(3.2 \text{ m/s}^2) = 33 \text{ N}$$

$$\vec{W}_a = (33 \text{ N})\hat{y}$$

Example 5: Consider pulling a 12.0-kg suitcase across a smooth floor by exerting a force $\vec{F}$ of magnitude $F = 45.0$ N, at an angle 20° above the horizontal. Calculate the normal force exerted on the suitcase.

[Ans.: $N = 103$ N]



$$\sum F_y = W_y + F_y + N_y = -mg + F \sin 20.0^\circ + N = 0$$

Solving for N gives

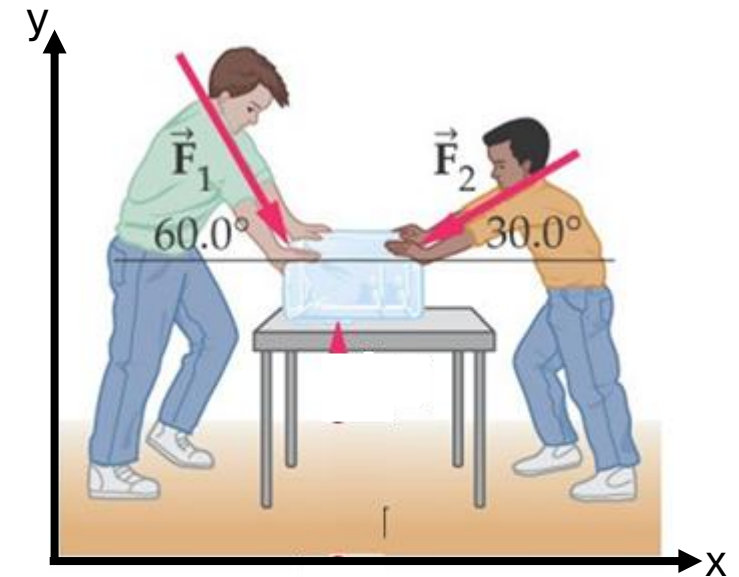
$$N = mg - F \sin 20.0^\circ = 118 \text{ N} - 15.4 \text{ N} = 103 \text{ N}$$

In vector form,

$$\vec{N} = N_y \hat{y} = (103 \text{ N}) \hat{y}$$

Example 6: A 6.0-kg block of ice is acted on by two forces $\vec{F}_1$ and $\vec{F}_2$, as shown in the diagram. If the magnitudes of the forces are $F_1 = 13 \text{ N}$, $F_2 = 11 \text{ N}$, calculate (a) the acceleration of the ice and (b) the normal force exerted on it by the table?

[Ans.: (a) $\vec{a} = (-0.50 \text{ m/s}^2) \hat{x}$, (b) $\vec{N} = (75 \text{ N}) \hat{y}$]



Physical picture

SOLUTION**Part (a)**

1. Write out the x component of each force:

$$F_{1,x} = F_1 \cos 60.0^\circ = (13 \text{ N}) \cos 60.0^\circ = 6.5 \text{ N}$$

$$F_{2,x} = -F_2 \cos 30.0^\circ = -(11 \text{ N}) \cos 30.0^\circ = -9.5 \text{ N}$$

$$N_x = 0 \quad W_x = 0$$

2. Sum the x components of force:

$$\begin{aligned} \sum F_x &= F_{1,x} + F_{2,x} + N_x + W_x \\ &= 6.5 \text{ N} - 9.5 \text{ N} + 0 + 0 = -3.0 \text{ N} \end{aligned}$$

3. Divide by the mass to obtain the acceleration:

$$a_x = \frac{\sum F_x}{m} = \frac{-3.0 \text{ N}}{6.0 \text{ kg}} = -0.50 \text{ m/s}^2$$

$$\vec{a} = (-0.50 \text{ m/s}^2)\hat{x}$$

Part (b)

4. Write out the y component of each force:

The only force we don't know is the normal.
We represent its magnitude by N :

$$F_{1,y} = -F_1 \sin 60^\circ = -(13 \text{ N}) \sin 60.0^\circ = -11 \text{ N}$$

$$F_{2,y} = -F_2 \sin 30^\circ = -(11 \text{ N}) \sin 30.0^\circ = -5.5 \text{ N}$$

$$N_y = N \quad W_y = -W = -mg$$

5. Sum the y components of force:

$$\begin{aligned} \sum F_y &= F_{1,y} + F_{2,y} + N_y + W_y \\ &= -11 \text{ N} - 5.5 \text{ N} + N - mg \end{aligned}$$

6. Set this sum equal to 0 since the acceleration in the y direction is zero, and solve for N :

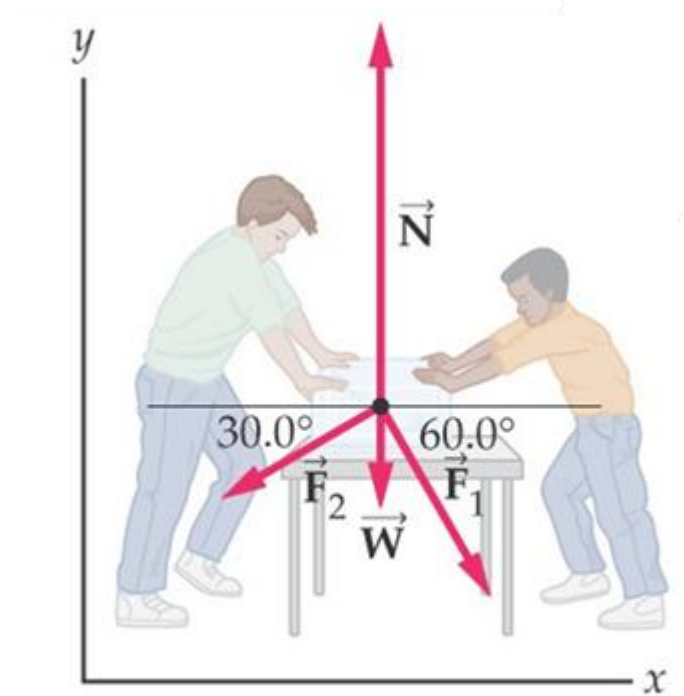
$$-11 \text{ N} - 5.5 \text{ N} + N - mg = 0$$

$$N = 11 \text{ N} + 5.5 \text{ N} + mg$$

$$= 11 \text{ N} + 5.5 \text{ N} + (6.0 \text{ kg})(9.81 \text{ m/s}^2) = 75 \text{ N}$$

7. Finally, we write the normal force in vector form:

$$\vec{N} = (75 \text{ N})\hat{y}$$

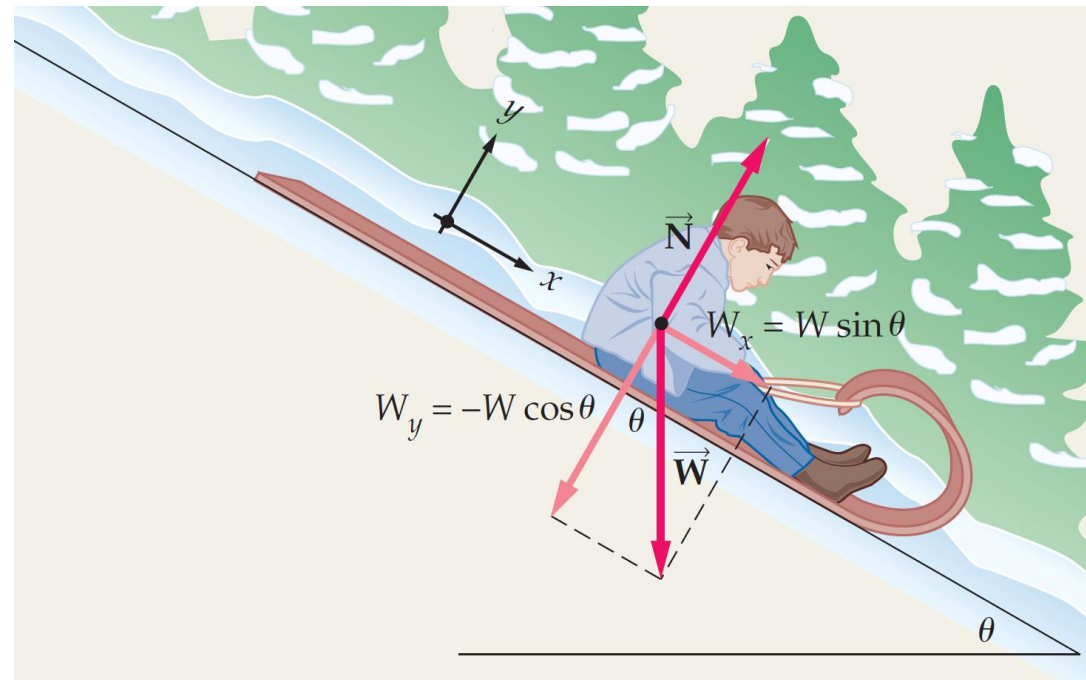


Free-body diagram

Example 7: A child of mass $m = 50\text{ kg}$ rides on a toboggan down a slick, ice-covered hill inclined at an angle $\theta = 30^\circ$ with respect to the horizontal.

- (a) What is the acceleration of the child?
- (b) What is the normal force exerted on the child?

[Ans.: (a) $a_x = g \sin \theta = 4.9 \text{ m/s}^2$, (b) $\vec{N} = (mg \cos \theta)\hat{y} = (425)\hat{y}$]



SOLUTION

Part (a)

1. Write out the x components of the forces acting on the child:
2. Sum the x components of the forces and set equal to ma_x :
3. Divide by the mass m to find the acceleration in the x direction:

$$N_x = 0 \quad W_x = W \sin \theta = mg \sin \theta$$

$$\sum F_x = N_x + W_x = mg \sin \theta = ma_x$$

$$a_x = \frac{\sum F_x}{m} = \frac{mg \sin \theta}{m} = g \sin \theta$$

Part (b)

4. Write out the y components of the forces acting on the child:
5. Sum the y components of the forces and set the sum equal to zero, since $a_y = 0$:
6. Solve for the magnitude of the normal force, N :
7. Write the normal force in vector form:

$$N_y = N \quad W_y = -W \cos \theta = -mg \cos \theta$$

$$\begin{aligned} \sum F_y &= N_y + W_y = N - mg \cos \theta \\ &= ma_y = 0 \end{aligned}$$

$$N - mg \cos \theta = 0 \quad \text{or} \quad N = mg \cos \theta$$

$$\vec{N} = (mg \cos \theta)\hat{y}$$

Example 8. Pulling up on a rope, you lift a 4.35 kg bucket of water from a well with an acceleration of 1.78 m/s^2 . What is the tension in the rope?

[Ans.: $T = 50.4 \text{ N}$]



Solution: 1. Write Newton's Second Law for the bucket:

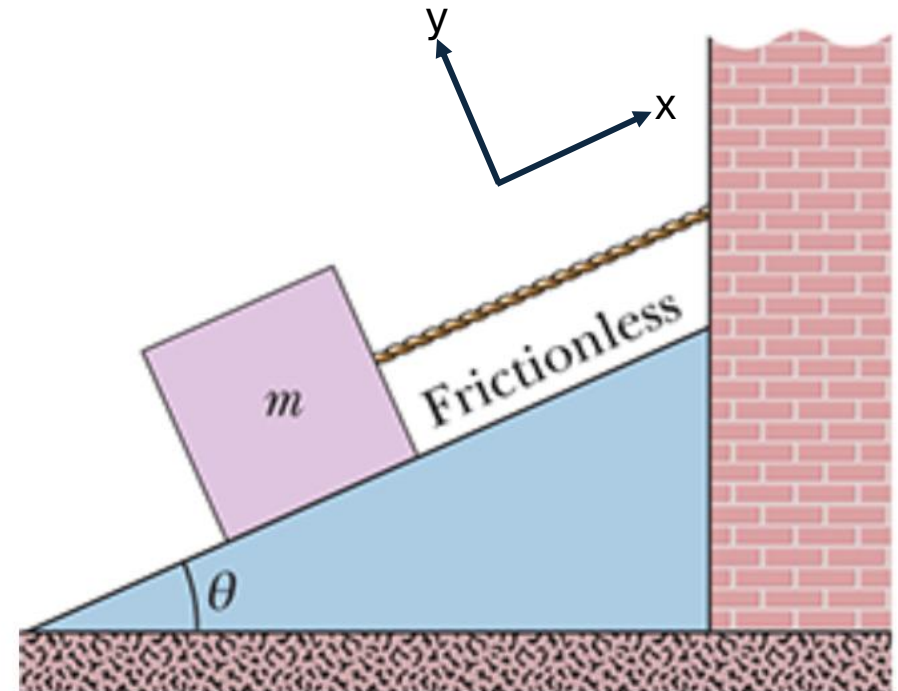
2. Solve the equation for T :

$$\sum F_y = T - W = ma$$

$$\begin{aligned} T &= ma + W = ma + mg = m(a + g) \\ &= (4.35 \text{ kg})(1.78 + 9.81 \text{ m/s}^2) = \boxed{50.4 \text{ N}} \end{aligned}$$

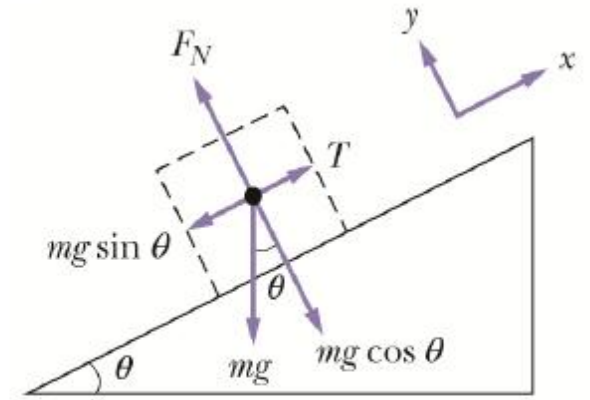
Example 9. In Fig. 5.18, let the mass of the block be 8.5 kg and the angle θ be 30° . Find (a) the tension in the cord and (b) the normal force acting on the block.

[Ans.: (a) $\vec{T} = 42 \text{ N}\hat{i}$, (b) $\vec{N} = (72)\hat{y}$]



EXPRESS The free-body diagram of the problem is shown in the figure. Since the acceleration of the block is zero, the components of Newton's second law equation yield

$$\begin{aligned}T - mg \sin \theta &= 0 \\F_N - mg \cos \theta &= 0,\end{aligned}$$



ANALYZE (a) Solving the first equation for the tension in the string, we find

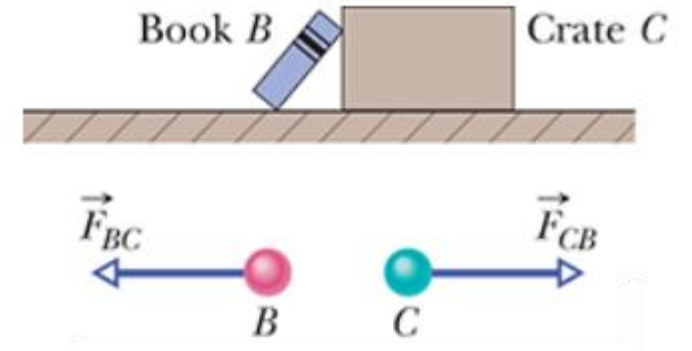
$$T = mg \sin \theta = (8.5 \text{ kg})(9.8 \text{ m/s}^2) \sin 30^\circ = 42 \text{ N}.$$

(b) We solve the second equation above for the normal force F_N :

$$F_N = mg \cos \theta = (8.5 \text{ kg})(9.8 \text{ m/s}^2) \cos 30^\circ = 72 \text{ N}.$$

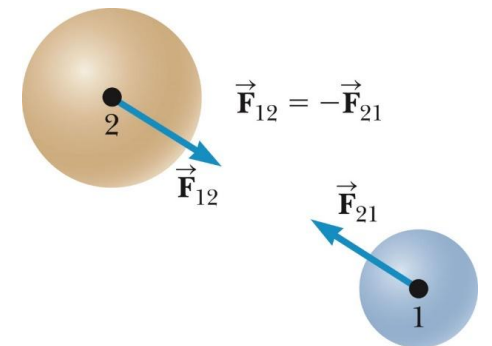
5.3 Applying Newton's Laws

Two bodies are said to interact when they push or pull on each other—that is, when a force acts on each body due to the other body.



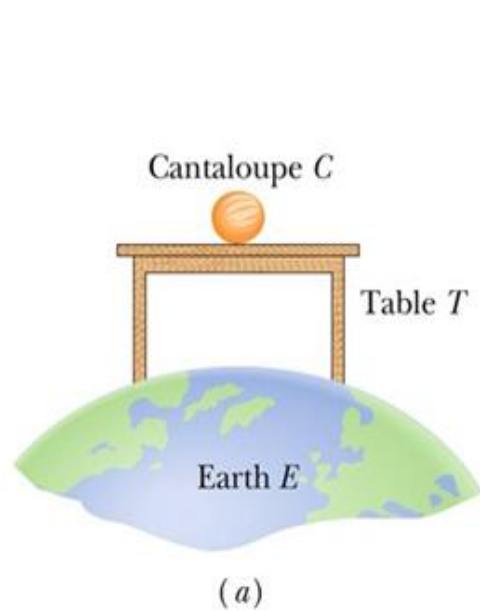
Newton's Third Law: When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.

- We can write this law as a scalar or vector relation: $F_{BC} = F_{CB}$ $\vec{F}_{BC} = -\vec{F}_{CB}$
- **Forces** always come **in pairs**, acting on different objects (the interacting objects): If object 1 exerts a force $\vec{F}$ on object 2, then object 2 exerts a force $-\vec{F}$ on object 1.
- These forces are called **action-reaction pairs**.
- **For every force that acts on an object, there is a reaction force acting on a different object that is equal in magnitude and opposite in direction.**

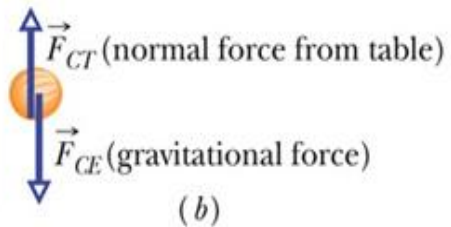


$$\vec{F}_{12} = -\vec{F}_{21}$$

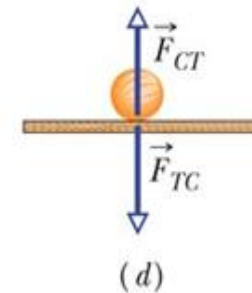
- Some action-reaction pairs:



These forces
just happen
to be balanced.



These are
third-law force
pairs.

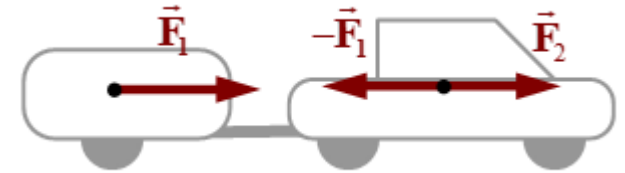


So are these.

Example 10: On vacation, your 1400 kg car pulls a 560 kg trailer away from a stoplight with an acceleration of 1.85 m/s^2 .

- (a) What is the net force exerted on the trailer?
- (b) What force does the trailer exert on the car?
- (c) What is the net force acting on the car?

[Ans.: (a) 1036 N, (b) -1036 N (opposite direction), (c) 2590 N]



Solution: 1. (a) Write Newton's Second Law for the trailer:

$$\sum \vec{F} = \vec{F}_1 = m \vec{a} = (560 \text{ kg})(1.85 \text{ m/s}^2) \hat{x} = \mathbf{1036 \text{ N } \hat{x}}$$

2. (b) Newton's Third Law states that the force the trailer exerts on the car is equal and opposite to the force the car exerts on the trailer:

$$-\vec{F}_1 = -\mathbf{1036 \text{ N } \hat{x}}$$

3. (c) Write Newton's Second Law for the car:

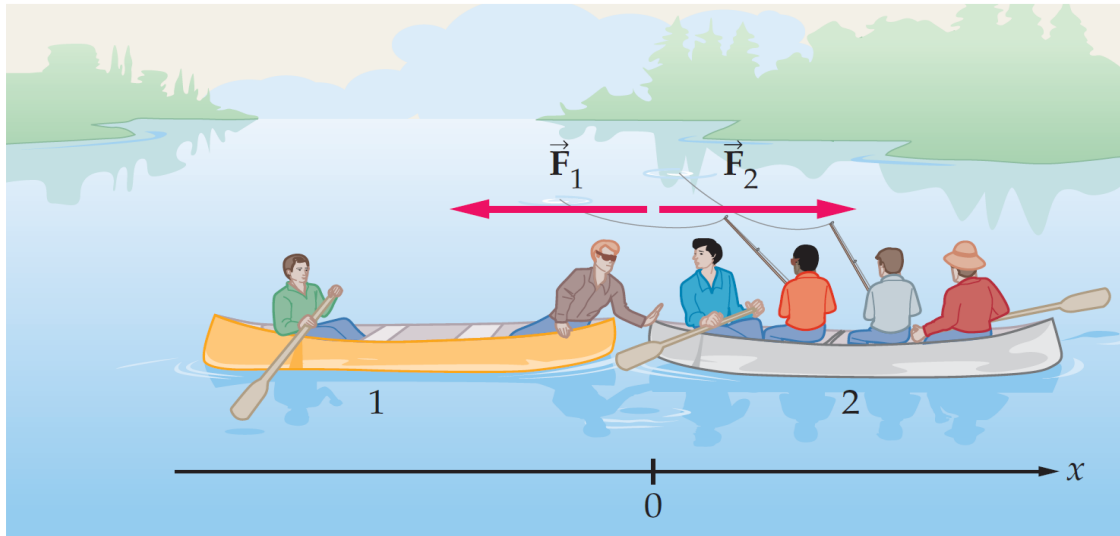
$$\sum \vec{F} = M \vec{a} = (1400 \text{ kg})(1.85 \text{ m/s}^2) \hat{x} = \mathbf{2590 \text{ N } \hat{x}}$$

Example 11: Two groups of canoeists meet in the middle of the lake. After a brief visit, a person in canoe 1 pushes on canoe 2 with a force of 46 N to separate the canoes. If the mass of the canoe 1 and its occupants is $m_1 = 150$ kg and the mass of the canoe 2 and its occupants is $m_2 = 250$ kg:

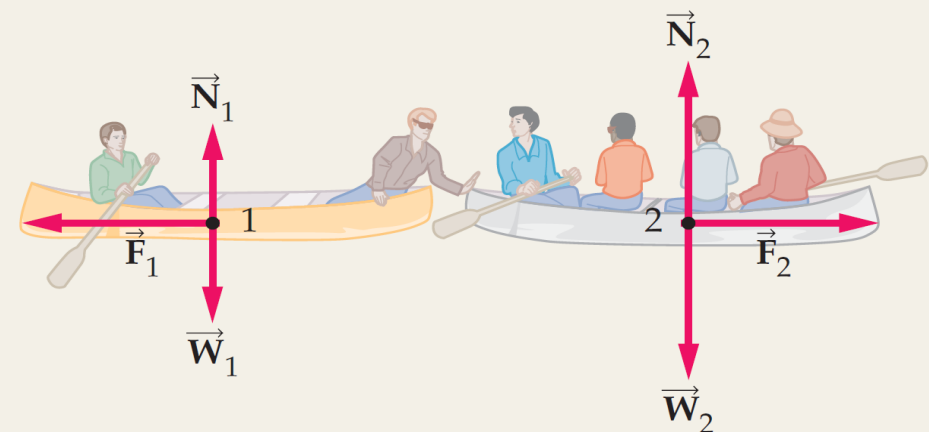
(a) find the acceleration the push gives to each canoe.

(b) What is the separation of the canoes after 1.2 s of pushing?

[Ans.: $a_{1,x} = -0.31$ m/s²; $a_{2,x} = 0.18$ m/s²; $x_2 = 0.13$ m; $x_1 = -0.22$ m; $x_2 - x_1 = 0.35$ m]



Physical picture



Free-body diagrams

SOLUTION

Part (a)

1. Use Newton's second law to find the acceleration of canoe 2:
2. Do the same calculation for canoe 1. Note that the acceleration of canoe 1 is in the negative direction:

$$a_{2,x} = \frac{\sum F_{2,x}}{m_2} = \frac{46 \text{ N}}{250 \text{ kg}} = 0.18 \text{ m/s}^2$$

$$a_{1,x} = \frac{\sum F_{1,x}}{m_1} = \frac{-46 \text{ N}}{150 \text{ kg}} = -0.31 \text{ m/s}^2$$

Part (b)

3. Use $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ to find the position of canoe 2 at $t = 1.2 \text{ s}$. From the problem statement, we know the canoes start at the origin ($x_0 = 0$) and at rest ($v_{0x} = 0$):
4. Repeat the calculation for canoe 1:
5. Subtract the two positions to find the separation of the canoes:

$$x_2 = \frac{1}{2}a_{2,x}t^2 = \frac{1}{2}(0.18 \text{ m/s}^2)(1.2 \text{ s})^2 = 0.13 \text{ m}$$

$$x_1 = \frac{1}{2}a_{1,x}t^2 = \frac{1}{2}(-0.31 \text{ m/s}^2)(1.2 \text{ s})^2 = -0.22 \text{ m}$$

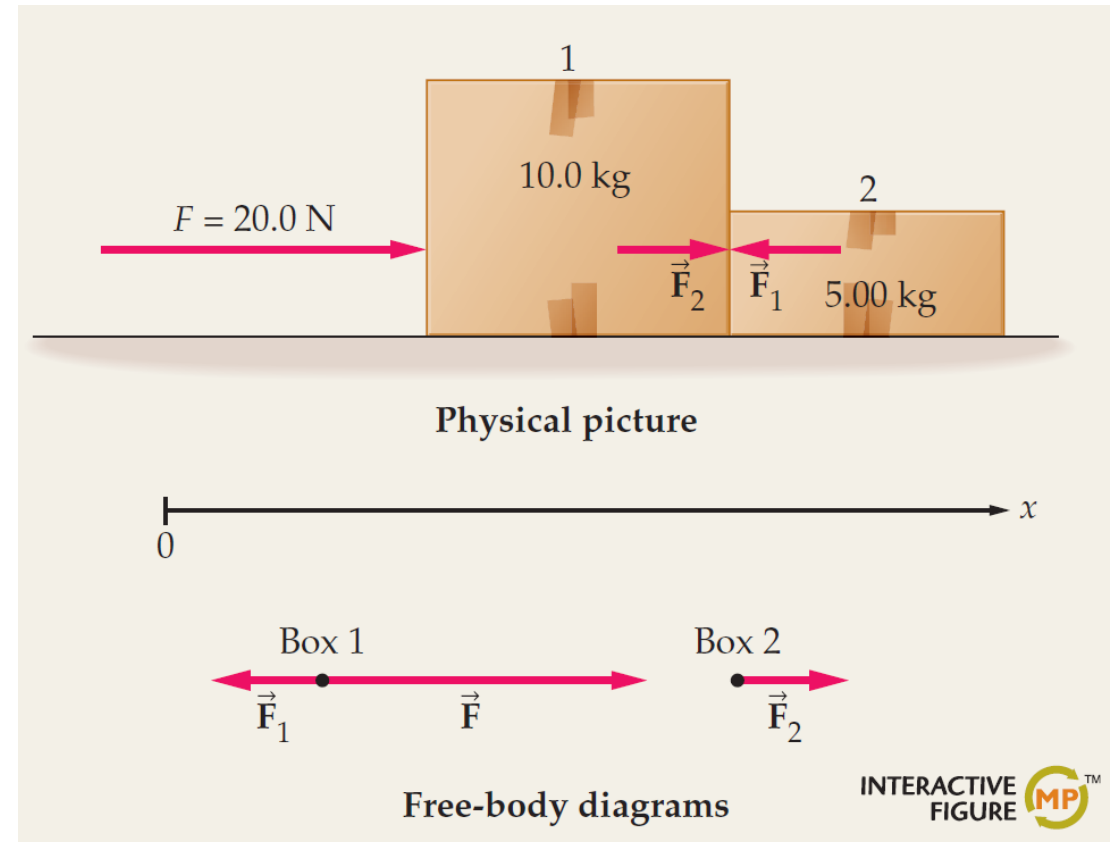
$$x_2 - x_1 = 0.13 \text{ m} - (-0.22 \text{ m}) = 0.35 \text{ m}$$

Example of an action-reaction pair: Contact forces

Example 12: a box of mass 10 kg rests on a smooth horizontal floor next to a box of mass 5 kg. If you push on box1 with a horizontal force of magnitude 20 N.

- (a) What is the acceleration of the boxes?
- (b) What is the force of contact between the boxes.
- (c) If you push on box2 instead of box1 with the same force, what is the contact force between the boxes?

[Ans.: (a) 1.33 m/s^2 ; (b) 6.67 N; (c) 13.3 N]



SOLUTION

Part (a)

1. Find the net horizontal force acting on the two boxes.
Note that $\vec{F}_1$ and $\vec{F}_2$ are equal in magnitude but opposite in direction. Hence, they sum to zero; $\vec{F}_1 + \vec{F}_2 = 0$:

$$\sum_{\text{both boxes}} F_x = F = 20.0 \text{ N}$$

2. Divide the net force by the total mass, $m_1 + m_2$, to find the acceleration of the boxes:

$$a_x = \frac{\sum F_x}{m_1 + m_2} = \frac{20.0 \text{ N}}{(10.0 \text{ kg} + 5.00 \text{ kg})} = \frac{20.0 \text{ N}}{15.0 \text{ kg}} = 1.33 \text{ m/s}^2$$

Part (b)

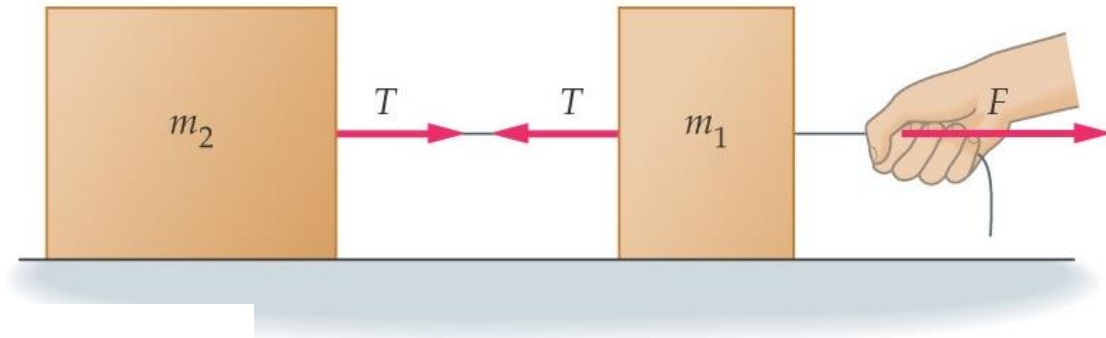
3. Find the net horizontal force acting on box 2, and set it equal to the mass of box 2 times its acceleration:
4. Determine the magnitude of the contact force, f , by substituting numerical values for m_2 and a_x :

$$\sum_{\text{box2}} F_x = F_{2,x} = f = m_2 a_x$$

$$f = m_2 a_x = (5.00 \text{ kg})(1.33 \text{ m/s}^2) = 6.67 \text{ N}$$

When forces are exerted on connected objects, their accelerations are the same.

If there are two objects connected by a string, and we know the force and the masses, we can find the acceleration and the tension:



$$\begin{aligned} F - T &= m_1 a_1 = m_1 a && \text{box 1} \\ T &= m_2 a_2 = m_2 a && \text{box 2} \end{aligned}$$

6-7

Since the boxes have the same acceleration, a , we have set $a_1 = a_2 = a$.

Next, we can eliminate the tension T by adding the two equations:

$$\begin{aligned} F - T &= m_1 a \\ T &= m_2 a \\ \hline F &= (m_1 + m_2) a \end{aligned}$$

With this result, it is straightforward to solve for the acceleration in terms of the applied force F :

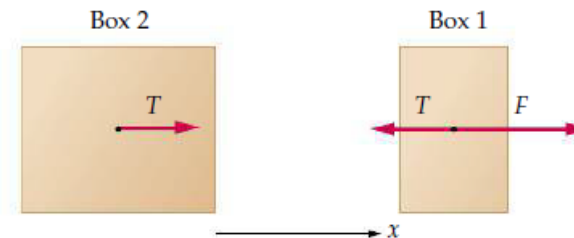
$$a = \frac{F}{m_1 + m_2}$$

6-8

$$T = m_2 a = \left(\frac{m_2}{m_1 + m_2} \right) F$$

Physical picture

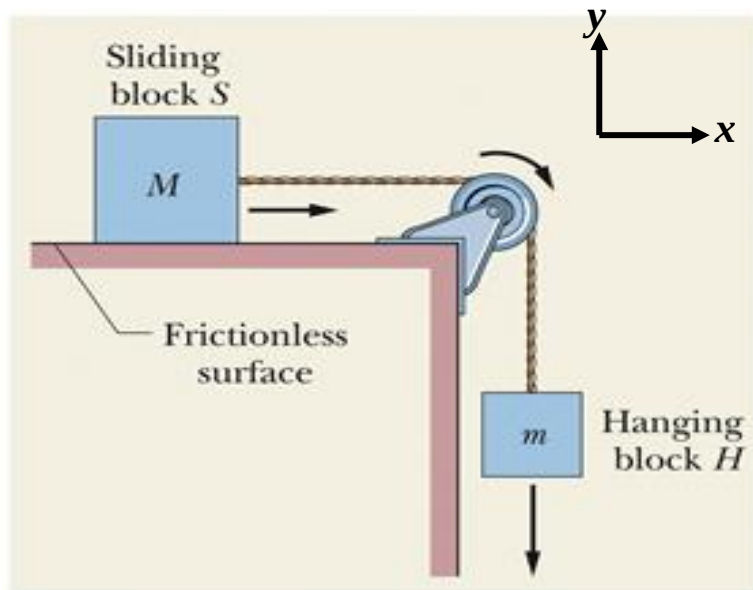
We treat each box as a separate system:



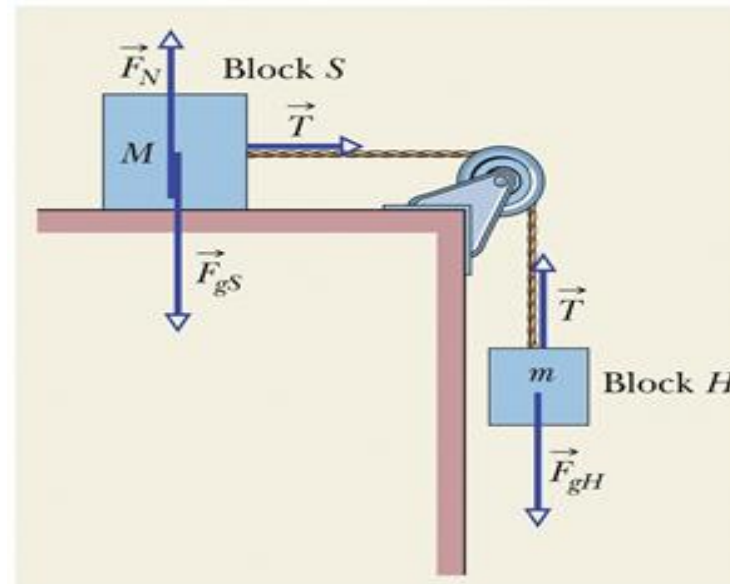
Free-body diagrams
(horizontal components only)

Example 13: A block of mass ($M = 3.3 \text{ kg}$ and $m = 2.1 \text{ kg}$) slides on a frictionless tabletop. It is connected to a string that passes over a pulley and suspends a mass. Find **(a)** the acceleration of the masses and **(b)** the tension in the string.

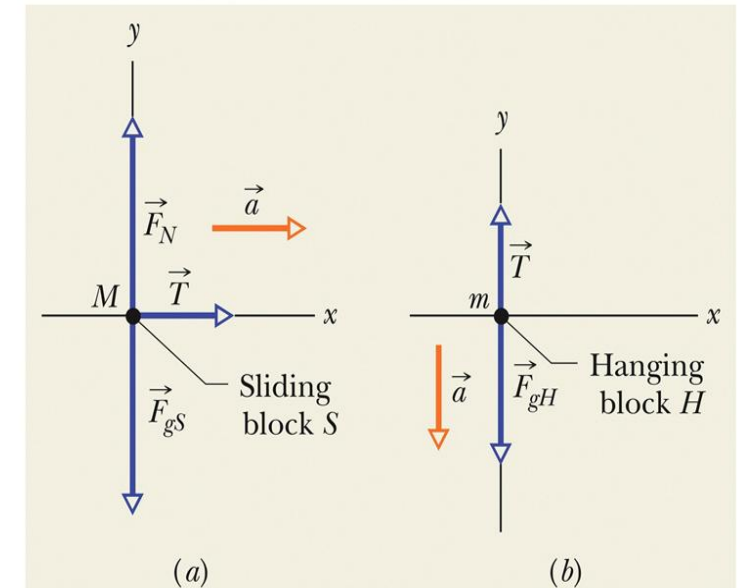
[Ans.: (a) 3.5 m/s^2 ; (b) 13 N]



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- For the sliding block: $T = Ma$.
- For the hanging block: $T - mg = -ma$.
- Combining we get: $a = \frac{m}{M + m} g$.

$$T = \frac{Mm}{M + m} g.$$

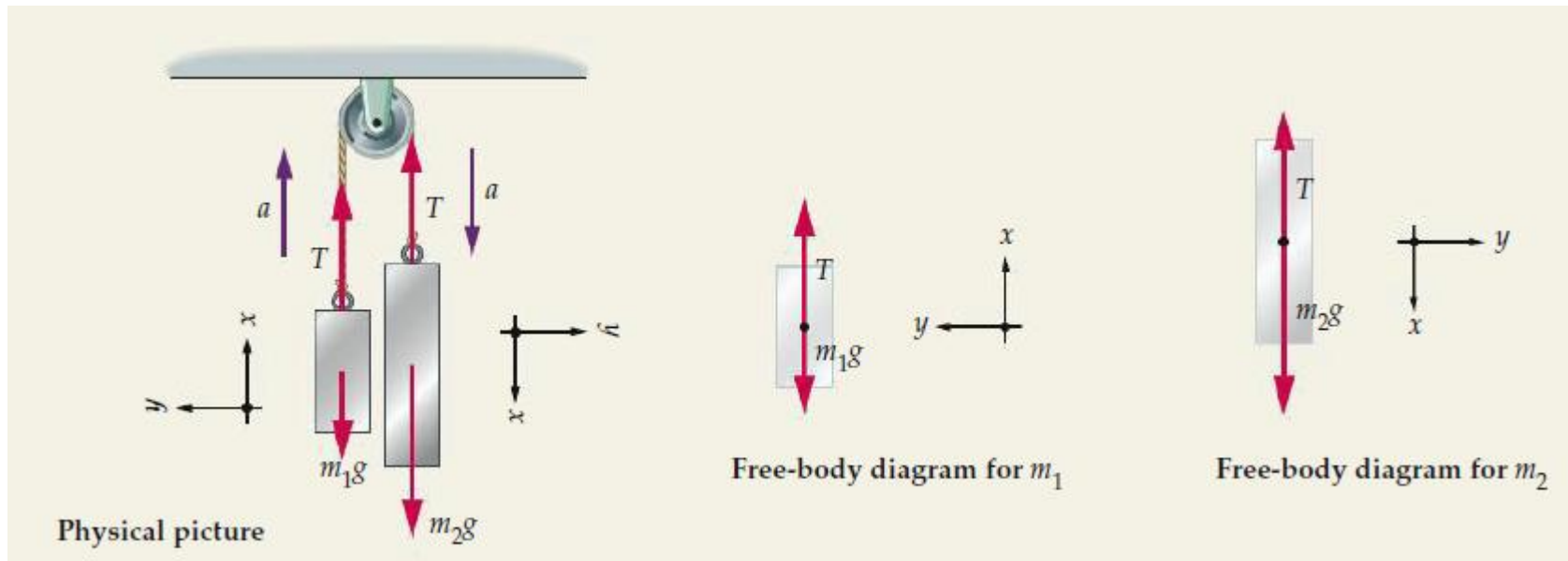
$$a = \frac{m}{M + m} g = \frac{2.1 \text{ kg}}{3.3 \text{ kg} + 2.1 \text{ kg}} (9.8 \text{ m/s}^2) \\ = 3.8 \text{ m/s}^2$$

and

$$T = \frac{Mm}{M + m} g = \frac{(3.3 \text{ kg})(2.1 \text{ kg})}{3.3 \text{ kg} + 2.1 \text{ kg}} (9.8 \text{ m/s}^2) \\ = 13 \text{ N}.$$

Example 14: Atwood's machine consists of two masses connected by a string that passes over a pulley, as shown below. Find the acceleration of the masses for general m_1 and m_2 , and evaluate for the specific case $m_1 = 3.1$ kg, $m_2 = 4.4$ kg.

[Ans.: (a) 1.7 m/s^2]



SOLUTION

1. Begin by writing out the expression $\Sigma F_{1,x} = m_1 a$. Note that two forces act in the x direction; T (positive direction) and $m_1 g$ (negative direction):

$$\Sigma F_{1,x} = T - m_1 g = m_1 a$$

2. Next, write out $\Sigma F_{2,x} = m_2 a$. The two forces acting in the x direction in this case are $m_2 g$ (positive direction) and T (negative direction):

$$\Sigma F_{2,x} = m_2 g - T = m_2 a$$

3. Sum the two relations obtained above to eliminate T :

$$\begin{array}{r} T - m_1 g = m_1 a \\ m_2 g - T = m_2 a \\ \hline (m_2 - m_1)g = (m_1 + m_2)a \end{array}$$

4. Solve for a :

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2} \right) g$$

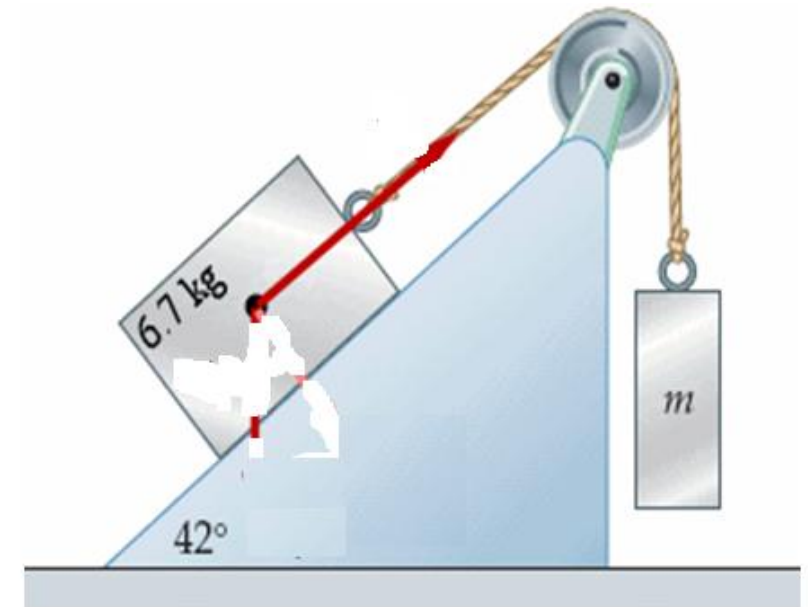
5. To evaluate the acceleration, substitute numerical values for the masses and for g :

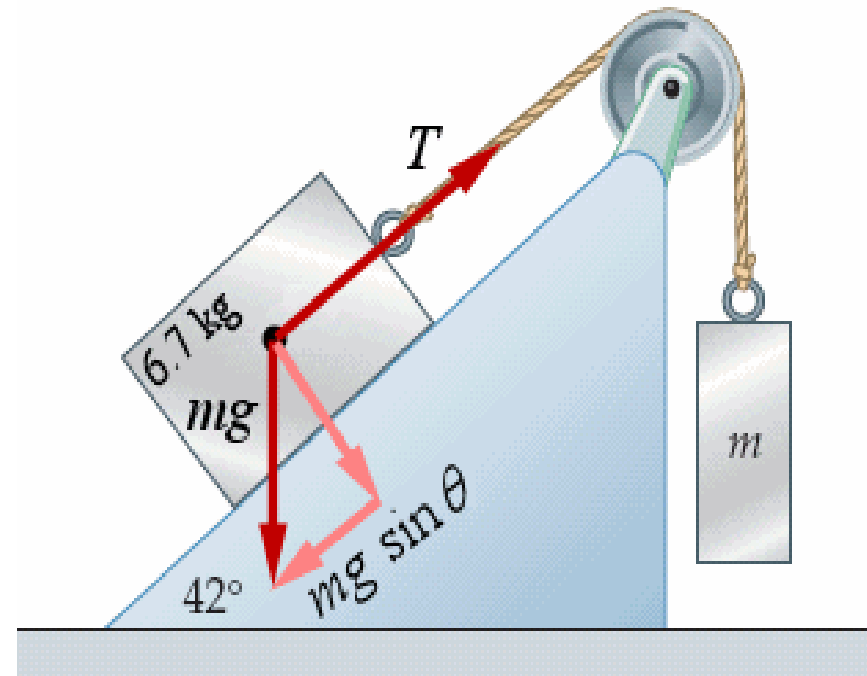
$$\begin{aligned} a &= \left(\frac{m_2 - m_1}{m_1 + m_2} \right) g \\ &= \left(\frac{4.4 \text{ kg} - 3.1 \text{ kg}}{3.1 \text{ kg} + 4.4 \text{ kg}} \right) (9.81 \text{ m/s}^2) = 1.7 \text{ m/s}^2 \end{aligned}$$

Example 15. Two blocks are connected by a string as shown. The smooth inclined surface makes an angle of 42° with the horizontal, and the block on the incline has a mass of 6.7 kg. Find the mass of the hanging block that will cause the system to be in equilibrium.

(The pulley is assumed to be ideal)

Ans: $m = 4.5 \text{ kg}$





Solution: 1. Write Newton's Second Law along the direction parallel to the incline:

2. Substitute $T = mg$ into the resulting equation to find m .

$$\sum F_x = T - Mg \sin \theta = 0$$

$$T = mg = Mg \sin \theta$$

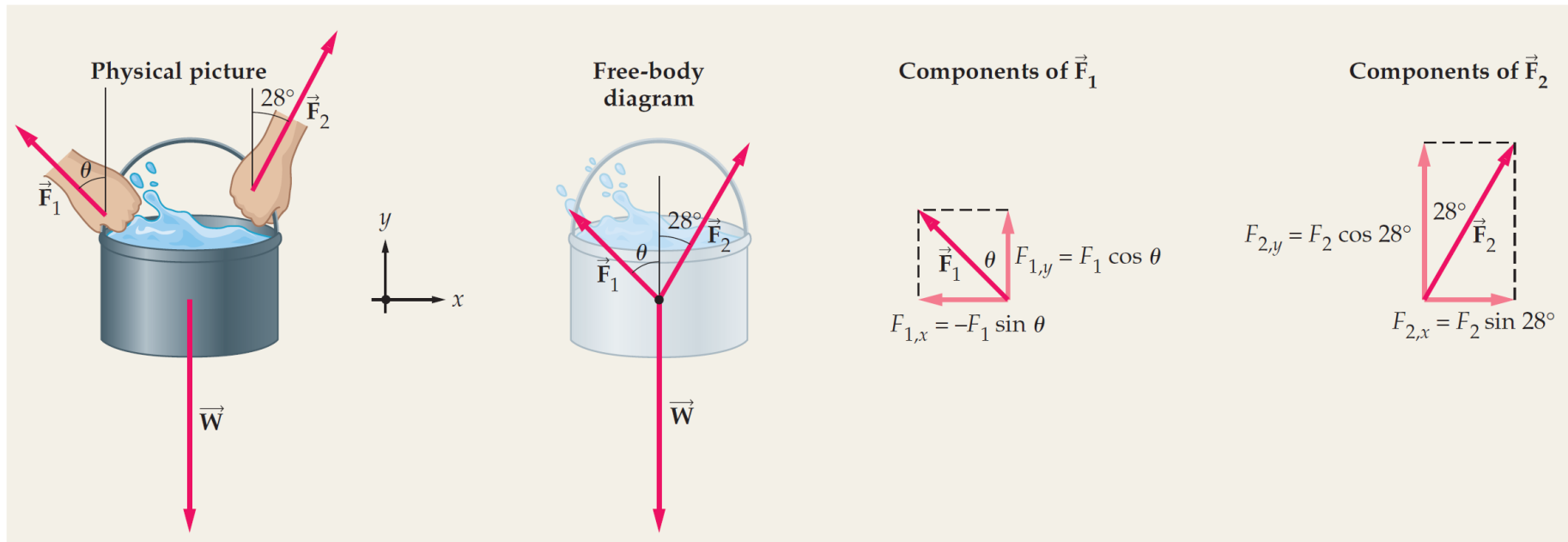
$$m = M \sin \theta = (6.7\text{ kg}) \sin 42^\circ = \boxed{4.5\text{ kg}}$$

Example 16: Jack and Jill lift upward on a 1.3 kg pail of water, with Jack exerting a force $\vec{F}_1$ of magnitude 7 N and Jill exerting a force $\vec{F}_2$ of magnitude 11 N. Jill's force is exerted at an angle of 28 degrees with the vertical, as shown below.

(a) At what angle with respect to the vertical should Jack exert his force if the pail is to accelerate straight upward?

(b) Determine the acceleration of the pail given its weight = 12.8 N.

[Ans.: (a) $\theta = 48^\circ$, $F_{1,y} = 4.7$ N, $F_{2,y} = 9.7$ N. (b) $a = 1.2$ m/s²]



SOLUTION

Part (a)

1. Begin by writing out the x component of each force.
Note that $\vec{W}$ has no x component and that the x component of $\vec{F}_1$ points in the negative x direction:
2. Sum the x components of force and set equal to zero.
Note that θ is the only unknown in this equation:
3. Solve for $\sin \theta$ and then for θ :

Part (b)

4. First, determine the y component of each force.
Note that $\vec{W}$ points in the negative y direction and that the y components of both $\vec{F}_1$ and $\vec{F}_2$ are positive:
5. Sum the y components of force and divide by the mass m to obtain the acceleration of the pail of water:

$$F_{1,x} = -F_1 \sin \theta \quad F_{2,x} = F_2 \sin 28^\circ \quad W_x = 0$$

$$\sum F_x = -F_1 \sin \theta + F_2 \sin 28^\circ + 0 = ma_x = 0 \text{ or} \\ F_1 \sin \theta = F_2 \sin 28^\circ$$

$$\sin \theta = \frac{F_2 \sin 28^\circ}{F_1} = \frac{(11 \text{ N}) \sin 28^\circ}{7.0 \text{ N}} = 0.74 \\ \theta = \sin^{-1}(0.74) = 48^\circ$$

$$F_{1,y} = F_1 \cos \theta = (7.0 \text{ N}) \cos 48^\circ = 4.7 \text{ N}$$

$$F_{2,y} = F_2 \cos 28^\circ = (11 \text{ N}) \cos 28^\circ = 9.7 \text{ N}$$

$$W_y = -W = -12.8 \text{ N}$$

$$\sum F_y = F_1 \cos \theta + F_2 \cos 28^\circ - W \\ = 4.7 \text{ N} + 9.7 \text{ N} - 12.8 \text{ N} = 1.6 \text{ N} \\ a_y = (\sum F_y)/m = (1.6 \text{ N})/(1.3 \text{ kg}) = 1.2 \text{ m/s}^2$$

Summary of Chapter 5

- Force: a push or pull
- Mass: measures the difficulty in accelerating an object
- Newton's first law: if the net force on an object is zero, its velocity is constant
- Inertial frame of reference: one in which the first law holds
- Newton's second law: $\sum \vec{F} = m\vec{a}$
- Free-body diagram: a sketch showing all the forces on an object
- Newton's third law: If object 1 exerts a force $\vec{F}_1$ on object 2, then object 2 exerts a force $-\vec{F}_1$ on object 1.

Summary of Chapter 5

- Contact forces: an action—reaction pair of forces produced by two objects in physical contact
- Forces are vectors
- Newton's laws can be applied to each component of the forces independently
- Weight: gravitational force exerted by the Earth on an object
- On the surface of the Earth, $W = mg$
- Apparent weight: force felt from contact with a floor or scale
- Normal force: force exerted perpendicular to a surface by that surface
- Normal force may be equal to, lesser than, or greater than the object's weight
- Friction along a surface
- Tension in a cord

Additional Exercises

1. On a planet far, far away, an astronaut picks up a rock that has a mass of 5.00 kg, and on this particular planet its weight is 40.0 N. If the astronaut exerts an upward force of 46.2 N on the rock, what is its acceleration.
2. In a grocery store, you push a 12.3 kg shopping cart with a force of 10.1 N. If the cart starts at rest, how far does it move in 2.50 s?
3. You are pulling your little sister on her sled across an icy (frictionless) surface. When you exert a constant horizontal force of 120 N, the sled has an acceleration of 2.5 m/s^2 . If the sled has a mass of 7.4 kg, what is the mass of your little sister?
4. A 0.53 kg billiard ball initially at rest is given a speed of 12 m/s during a time interval of 4.0 ms. What average force acted on the ball during this time?
5. A 92 kg water skier floating in a lake is pulled from rest to a speed of 12 m/s in a distance of 25 m. What is the net force exerted on the skier, assuming his acceleration is constant?

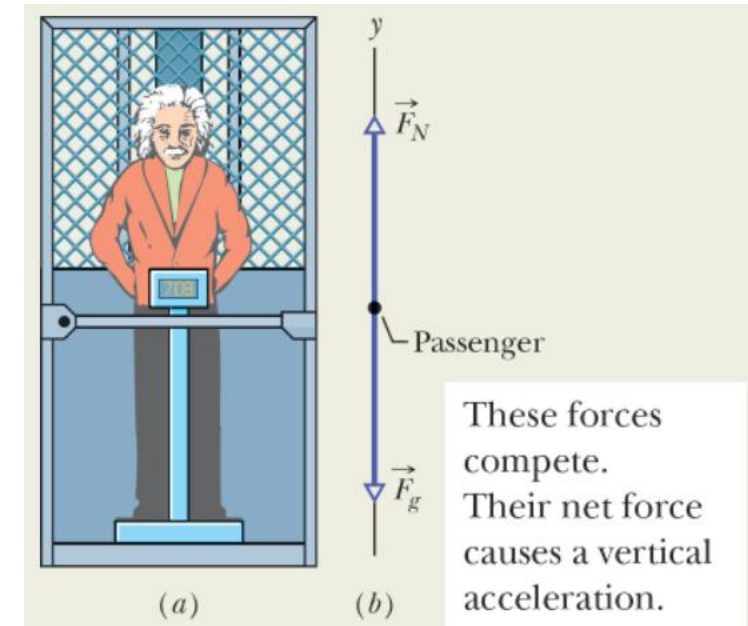
Additional Exercises

Example In figure, a passenger of mass $m = 72.2$ kg stands on a platform scale in an elevator cab. We are concerned with the scale readings when the cab is stationary and when it is moving up or down.

Find a general solution for the scale reading (or apparent weight), whatever the vertical motion of the cab.

What does the scale read if the cab is stationary or moving upward at a constant 0.50 m/s?

What does the scale read if the cab accelerates upward at 3.20 m/s² and downward at 3.20 m/s²?



Because the two forces on the passenger and his acceleration are all directed vertically, along the y axis in Fig. 5.3.8b, we can use Newton's second law written for y components

$$F_{net,y} = ma \quad \text{To get} \quad F_N - F_g = ma \quad F_N = F_g + ma$$

This tells us that the scale reading, which is equal to normal force magnitude F_N , depends on the vertical acceleration. Substituting mg for F_g gives us $F_N = m(g + a)$

For any constant velocity (zero or otherwise), the acceleration a of the passenger is zero.

$$F_N = (72.2 \text{ kg})(9.8 \text{ m/s}^2 + 0) = 708 \text{ N.}$$

This is the weight of the passenger and is equal to the magnitude F_g of the gravitational force on him.

(c) What does the scale read if the cab accelerates upward at 3.20 m/s^2 and downward at 3.20 m/s^2 ?

Calculations: For $a = 3.20 \text{ m/s}^2$, $F_N = (72.2 \text{ kg})(9.8 \text{ m/s}^2 + 3.20 \text{ m/s}^2)$
 $= 939 \text{ N,}$

and for $a = -3.20 \text{ m/s}^2$, it gives $F_N = (72.2 \text{ kg})(9.8 \text{ m/s}^2 - 3.20 \text{ m/s}^2)$
 $= 477 \text{ N.}$

For an upward acceleration (either the cab's upward speed is increasing or its downward speed is decreasing), the scale reading is greater than the passenger's weight. That reading is a measurement of an apparent weight, because it is made in a noninertial frame. For a downward acceleration (either decreasing upward speed or increasing downward speed), the scale reading is less than the passenger's weight.

Additional Exercises

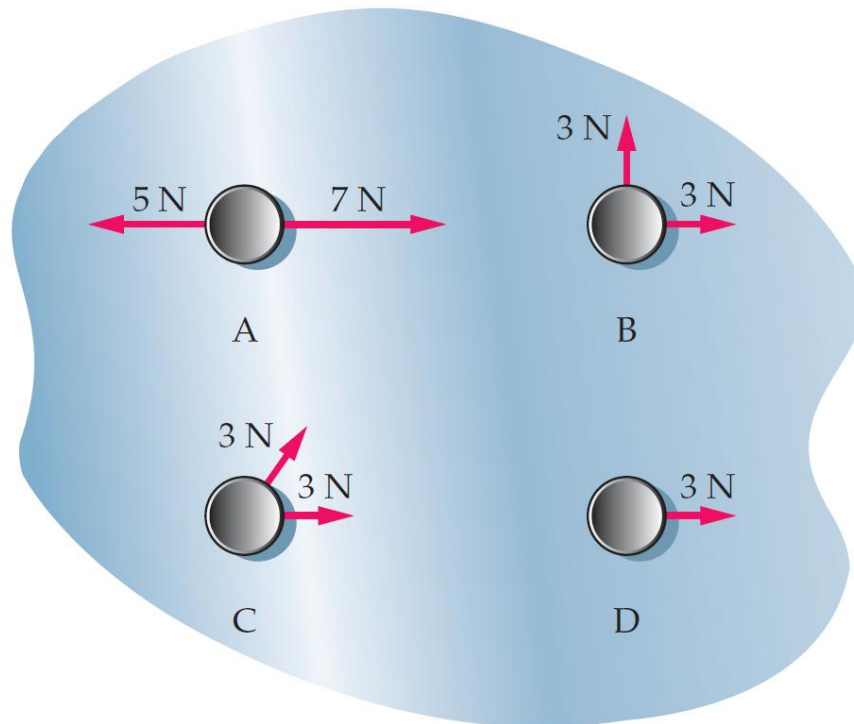
A 65-kg skier speeds down a trail, as shown in **Figure**. The surface is smooth and inclined at an angle of 22° with the horizontal.

- (a) Draw a free-body diagram for the skier
- (b) Find the direction and magnitude of the net force acting on the skier.
- (c) Find the acceleration of the skier
- (d) Determine the normal force acting on the skier.

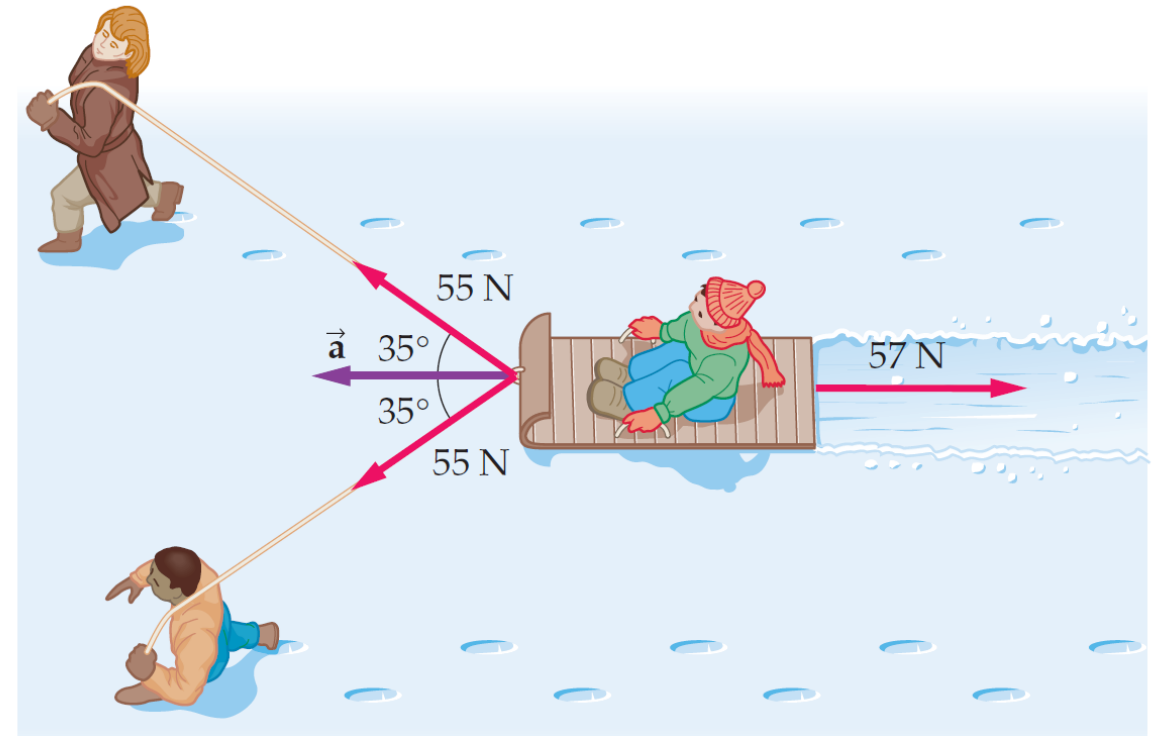


Additional Exercises

29. •• **CE** A hockey puck is acted on by one or more forces, as shown in **Figure 5–24**. Rank the four cases, A, B, C, and D, in order of the magnitude of the puck's acceleration, starting with the smallest. Indicate ties where appropriate.

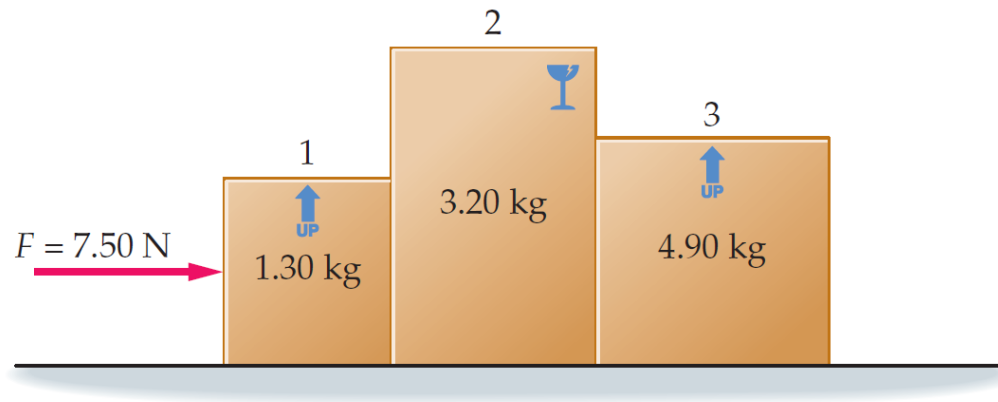


30. •• To give a 19-kg child a ride, two teenagers pull on a 3.7-kg sled with ropes, as indicated in **Figure 5–25**. Both teenagers pull with a force of 55 N at an angle of 35° relative to the forward direction, which is the direction of motion. In addition, the snow exerts a retarding force on the sled that points opposite to the direction of motion, and has a magnitude of 57 N. Find the acceleration of the sled and child.



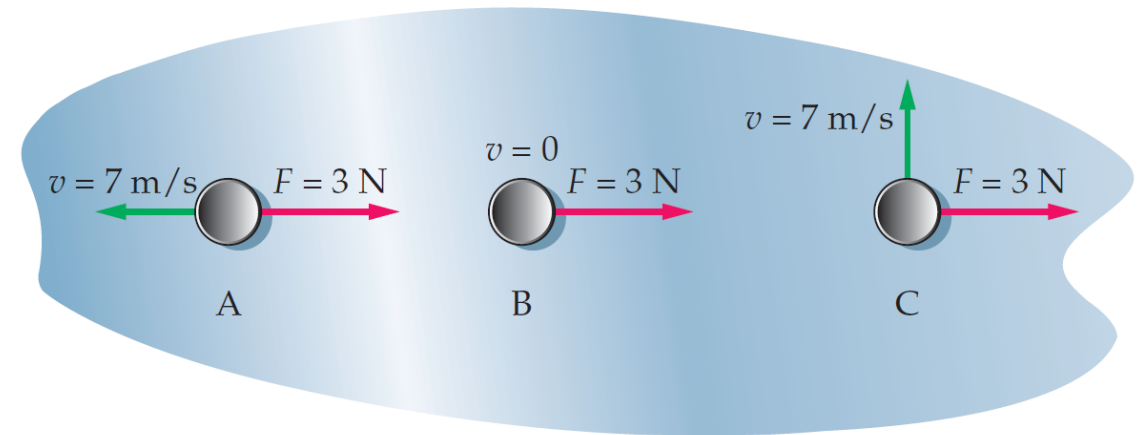
Additional Exercises

20. •• A force of magnitude 7.50 N pushes three boxes with masses $m_1 = 1.30$ kg, $m_2 = 3.20$ kg, and $m_3 = 4.90$ kg, as shown in **Figure 5–19**. Find the magnitude of the contact force (a) between boxes 1 and 2, and (b) between boxes 2 and 3.



38. • Suppose a rocket launches with an acceleration of 30.5 m/s^2 . What is the apparent weight of an 92-kg astronaut aboard this rocket?

58. •• **CE** Each of the three identical hockey pucks shown in **Figure 5–31** is acted on by a 3-N force. Puck A moves with a speed of 7 m/s in a direction opposite to the force; puck B is instantaneously at rest; puck C moves with a speed of 7 m/s at right angles to the force. Rank the three pucks in order of the magnitude of their acceleration, starting with the smallest. Indicate ties with an equal sign.



60. •• An object of mass $m = 5.95$ kg has an acceleration $\vec{a} = (1.17 \text{ m/s}^2)\hat{x} + (-0.664 \text{ m/s}^2)\hat{y}$. Three forces act on this object: $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$. Given that $\vec{F}_1 = (3.22 \text{ N})\hat{x}$ and $\vec{F}_2 = (-1.55 \text{ N})\hat{x} + (2.05 \text{ N})\hat{y}$, find $\vec{F}_3$.

51 **M** **GO** Figure 5.30 shows two blocks connected by a cord (of negligible mass) that passes over a frictionless pulley (also of negligible mass). The arrangement is known as *Atwood's machine*. One block has mass $m_1 = 1.30$ kg; the other has mass $m_2 = 2.80$ kg. What are (a) the magnitude of the blocks' acceleration and (b) the tension in the cord?

$$T - m_1g = m_1a$$
$$m_2g - T = m_2a,$$

which can be solved to yield

$$a = \left(\frac{m_2 - m_1}{m_2 + m_1} \right) g.$$

Substituting the result back, we have

$$T = \left(\frac{2m_1m_2}{m_1 + m_2} \right) g.$$

