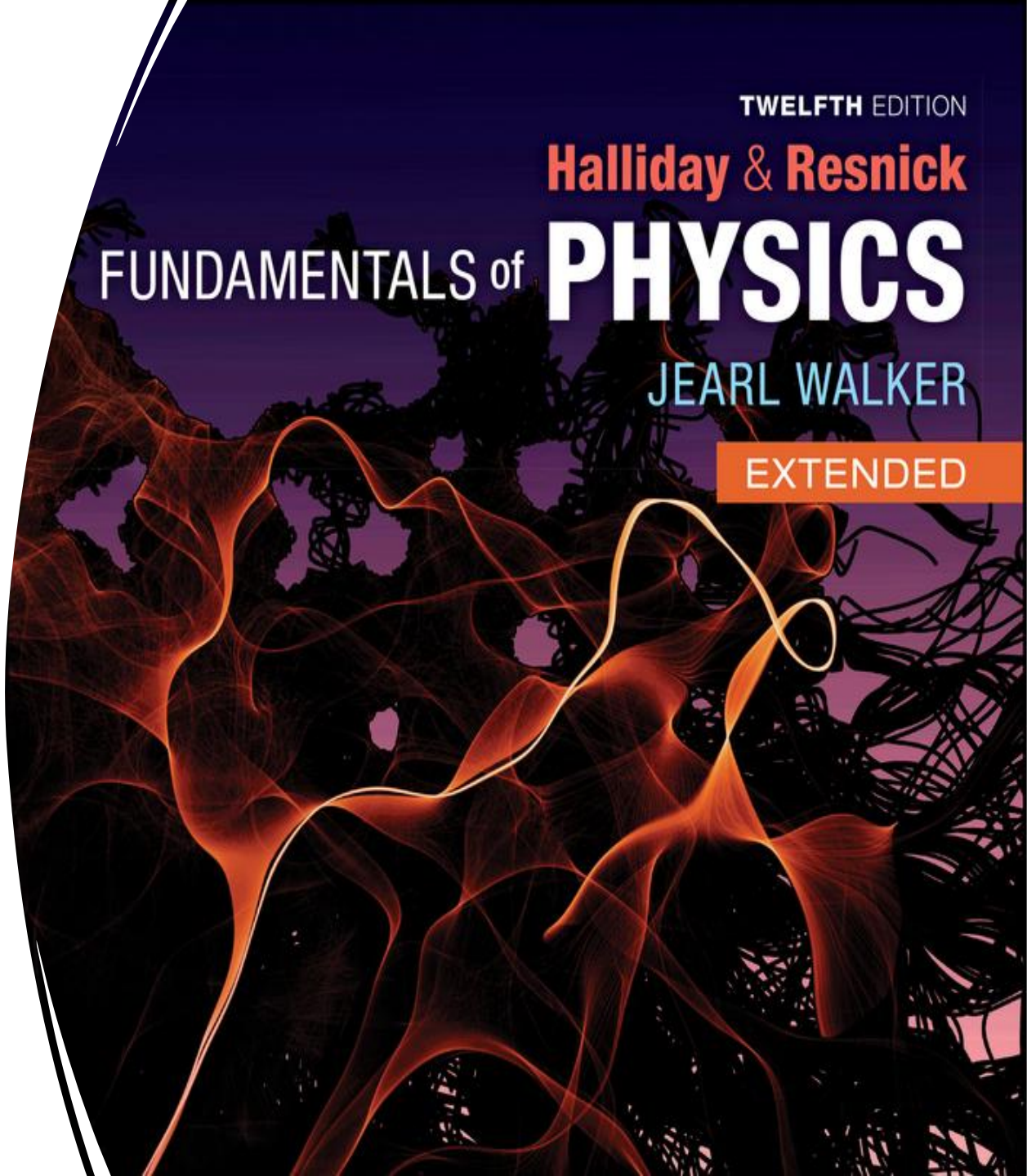
Chapter 4

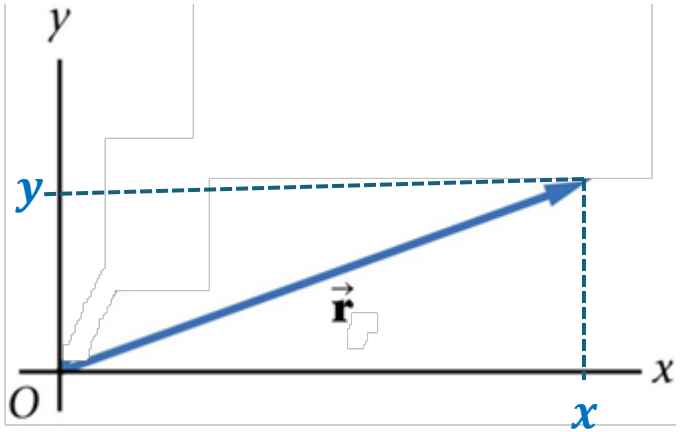
Motion in Two and Three Dimensions



Units of Chapter 4

- 4.1. Position and Displacement
- 4.2. Average Velocity and Instantaneous Velocity
- 4.3. Average Acceleration and Instantaneous Acceleration
- 4.4. Projectile motion

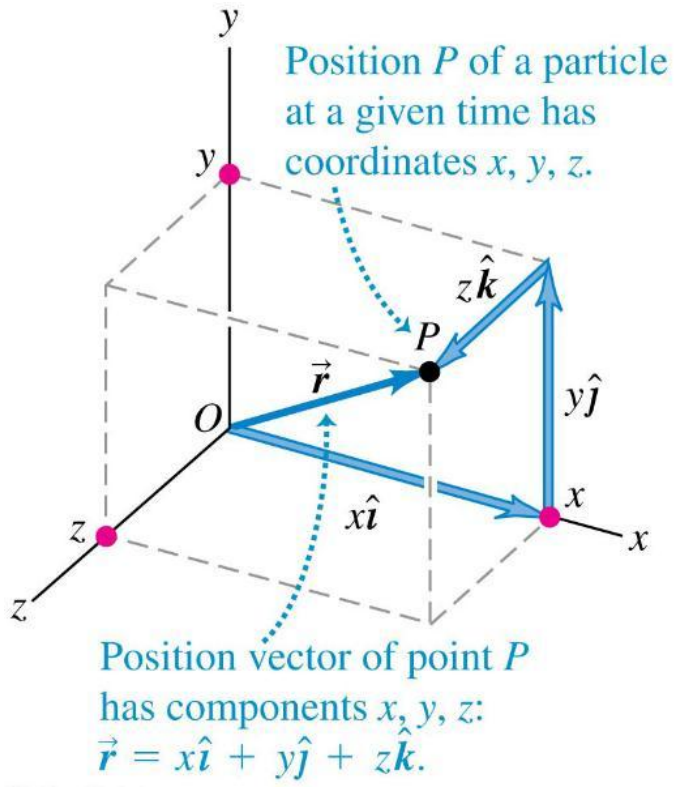
4.1 Position and Displacement



- A **position vector** locates a particle in space
  - Extends from a reference point (origin) to the particle

In 2 dimension $\vec{\mathbf{r}} = x\,\hat{\imath} + y\,\hat{\jmath}$

In 3 dimension $\vec{\mathbf{r}} = x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}$



Example

Position vector (−3m, 2m, 5m)

$$\vec{r} = (-3\ \mathrm{m})\hat{\mathrm{i}} + (2\,\mathrm{m})\hat{\mathrm{j}} + (5\,\mathrm{m})\hat{\mathrm{k}}$$

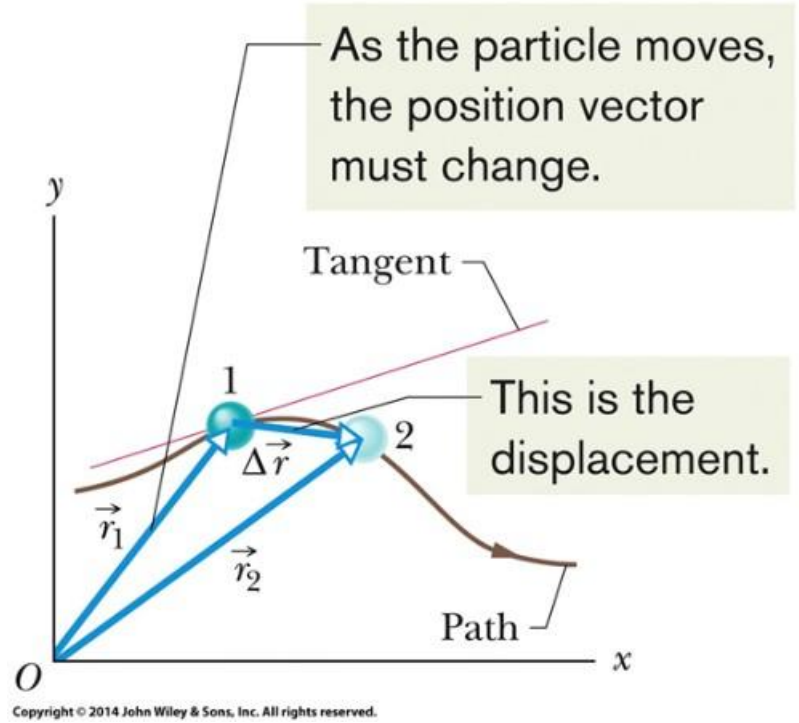
4.1 Position and Displacement

- Change in position vector is a **displacement**

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

or

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1. \qquad \textbf{Equation (4.1.2)}$$



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- We can rewrite this as:

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} \qquad \textbf{Equation (4.1.3)}$$

- Or express it in terms of changes in each coordinate:

$$\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k} \qquad \textbf{Equation (4.1.4)}$$

Example 1. A positron undergoes a displacement $\vec{r} = 2.0\,\hat{\imath} - 3.0\,\hat{j} + 6.0\,\hat{k}$, ending with the position vector $\vec{r} = 3.0\,\hat{\imath} - 4.0\,\hat{k}$, in meters.
What was the positron's initial position vector?

3. The initial position vector $\vec{r}_0$ satisfies $\vec{r} - \vec{r}_0 = \Delta\vec{r}$, which results in

$$\vec{r}_0 = \vec{r} - \Delta\vec{r} = (3.0\hat{\mathrm{j}} - 4.0\hat{\mathrm{k}})\ \mathrm{m} - (2.0\hat{\mathrm{i}} - 3.0\hat{\mathrm{j}} + 6.0\hat{\mathrm{k}})\ \mathrm{m} = (-2.0\ \mathrm{m})\hat{\mathrm{i}} + (6.0\ \mathrm{m})\hat{\mathrm{j}} + (-10\ \mathrm{m})\hat{\mathrm{k}}.$$

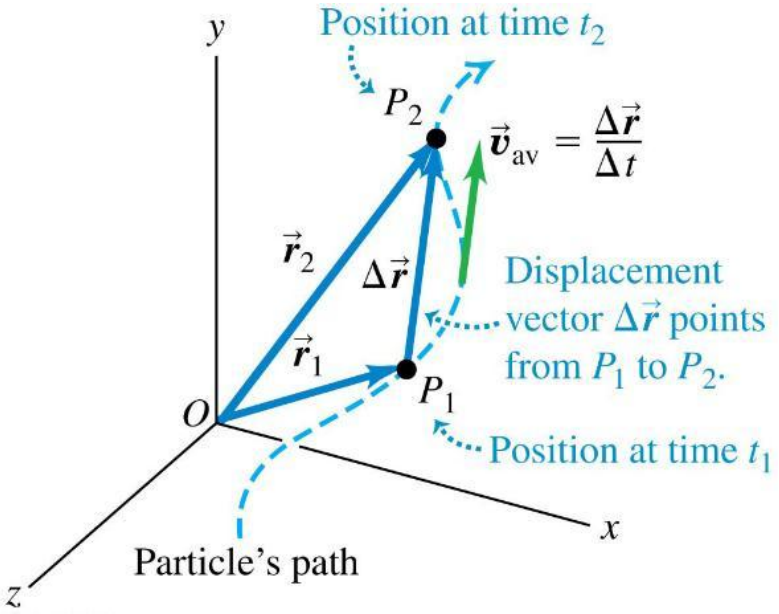
4.2 Average Velocity and Instantaneous Velocity

Average velocity is A displacement divided by its time interval

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} \qquad \textbf{Equation (4.2.1)}$$

- We can write this in component form:

$$\vec{v}_{\text{avg}} = \frac{\Delta x\hat{\text{i}} + \Delta y\hat{\text{j}} + \Delta z\hat{\text{k}}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{\text{i}} + \frac{\Delta y}{\Delta t}\hat{\text{j}} + \frac{\Delta z}{\Delta t}\hat{\text{k}} \qquad \textbf{Equation (4.2.2)}$$



A particle moves through displacement $(12\text{ m})\hat{\imath} + (3.0\text{ m})\,\hat{k}$ in 2.0 s:

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} = \frac{(12\,\text{m})\hat{\text{i}} + (3.0\text{ m})\hat{\text{k}}}{2.0\,\text{s}} = (6.0\text{ m/s})\hat{\text{i}} + (1.5\text{ m/s})\hat{\text{k}}$$

Instantaneous velocity is defined as: The velocity of a particle at a single point in time

$$\vec{v} = \frac{d\,\vec{r}}{dt} \qquad \textbf{Equation (4.2.3)}$$

Example 2: A dragonfly is observed initially at the position $\vec{r_i} = (2.00\ \mathrm{m})\,\hat{\imath} + (3.50\ \mathrm{m})\,\hat{\jmath}$. Three seconds later it is at the position $\vec{r_f} = (-3.00\ \mathrm{m})\,\hat{\imath} + (5.50\ \mathrm{m})\,\hat{\jmath}$. What was the dragonfly's average velocity during this time?

SOLUTION

$$\vec{\mathbf{v}}_{av} = (\vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i)/\Delta t = [(-5.00\ \mathrm{m})\hat{\mathbf{x}} + (2.00\ \mathrm{m})\hat{\mathbf{y}}]/(3.00\ \mathrm{s})$$
$$= (-1.67\ \mathrm{m/s})\hat{\mathbf{x}} + (0.667\ \mathrm{m/s})\hat{\mathbf{y}}$$

Example 3: Find the speed and direction of motion for a rainbow trout whose velocity is $\vec{v} = (3.7\ \mathrm{m/s})\,\hat{\imath} + (-1.3\ \mathrm{m/s})\,\hat{\jmath}$.

SOLUTION

$$\text{speed} = v = \sqrt{(3.7\ \mathrm{m/s})^2 + (-1.3\ \mathrm{m/s})^2} = 3.9\ \mathrm{m/s}, \theta = \tan^{-1}\left(\frac{-1.3\ \mathrm{m/s}}{3.7\ \mathrm{m/s}}\right) = -19°,$$

that is, 19° below the x axis.

4.3 Average Acceleration and Instantaneous Acceleration

Average acceleration is a change in velocity divided by its time interval

$$\vec{a}_{avg} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t} \qquad \textbf{Equation (4.3.1)}$$

$v_2 = \text{final velocity},\ v_1 = \text{initial velocity}$

Instantaneous acceleration is again the limit $t \to 0$:

$$\vec{a} = \frac{d\vec{v}}{dt} \qquad \textbf{Equation (4.3.2)}$$

- We can write Eq. 4.3.2 in unit-vector form:

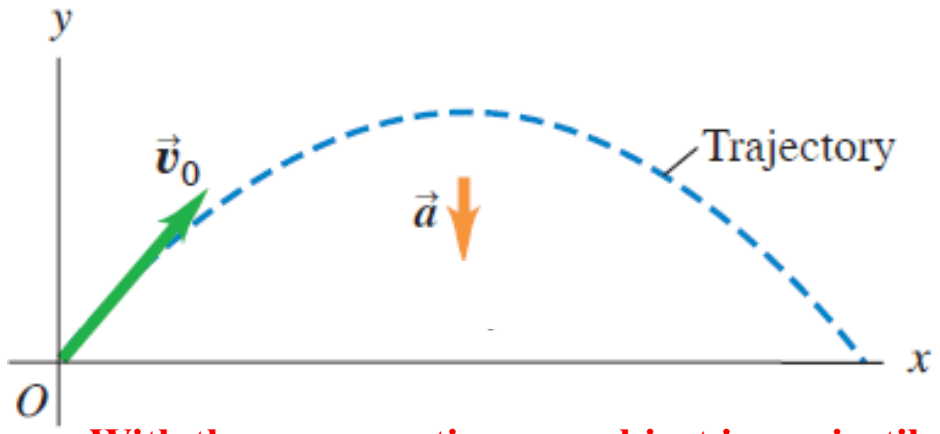
$$\vec{a} = \frac{d}{dt}\left(v_x \hat{\mathrm{i}} + v_y \hat{\mathrm{j}} + v_z \hat{\mathrm{k}}\right)$$

$$= \frac{dv_x}{dt}\hat{\mathrm{i}} + \frac{dv_y}{dt}\hat{\mathrm{j}} + \frac{dv_z}{dt}\hat{\mathrm{k}}$$

4.4 Projectile Motion

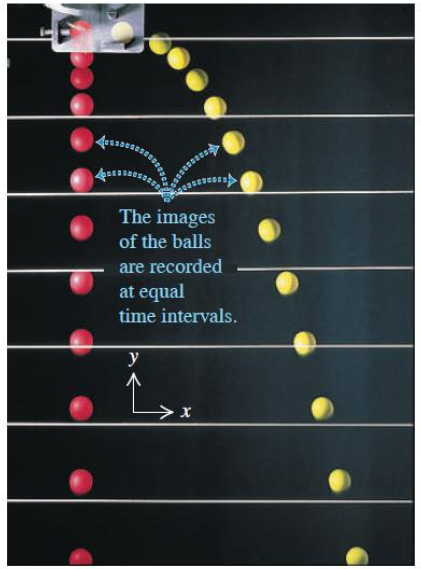
- A projectile is an object that is launched and moves solely under the influence of gravity.
- For example, a kicked ball becomes a projectile the moment it leaves your foot until just before it hits the ground.
- Its path, or trajectory, is influenced by gravity.
- Assumptions:
  - Ignore air resistance
  - The acceleration due to **gravity is constant**, **downward**, and has a magnitude $g = 9.81\ m/s^2$
  - Ignore Earth's rotation

If *y*-axis points upward, acceleration in *x*-direction is zero and acceleration in *y*-direction is $-9.81\ m/s^2$



With these assumptions, an object in projectile motion will follow a parabolic path

Figure 3.16 The red ball is dropped from rest, and the yellow ball is simultaneously projected horizontally.

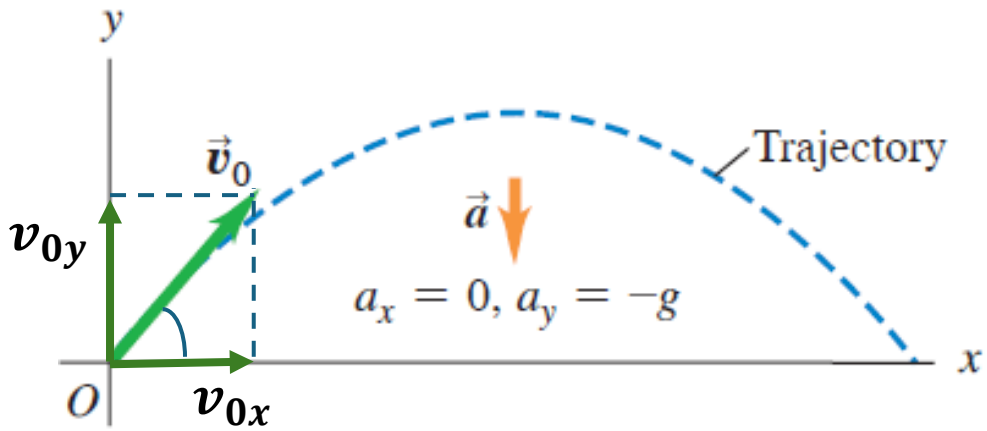


- At any time the two balls have different *x*-coordinates and *x*-velocities but the same *y*-coordinate, *y*-velocity, and *y*-acceleration.
- The horizontal motion of the yellow ball has no effect on its vertical motion.

4.4 Projectile Motion

- If the x-**axis is horizontal** and the y-**axis points upward**, then:
  - The acceleration in the x-direction is zero (constant x-velocity): $\boldsymbol{a_x = 0}$.
  - and the acceleration in the y-direction is $\boldsymbol{a_y = -g = -9.81\ \text{m/s}^2}$

- Setting $a_x = 0$ and $a_y = -g$, the equations of motion become:



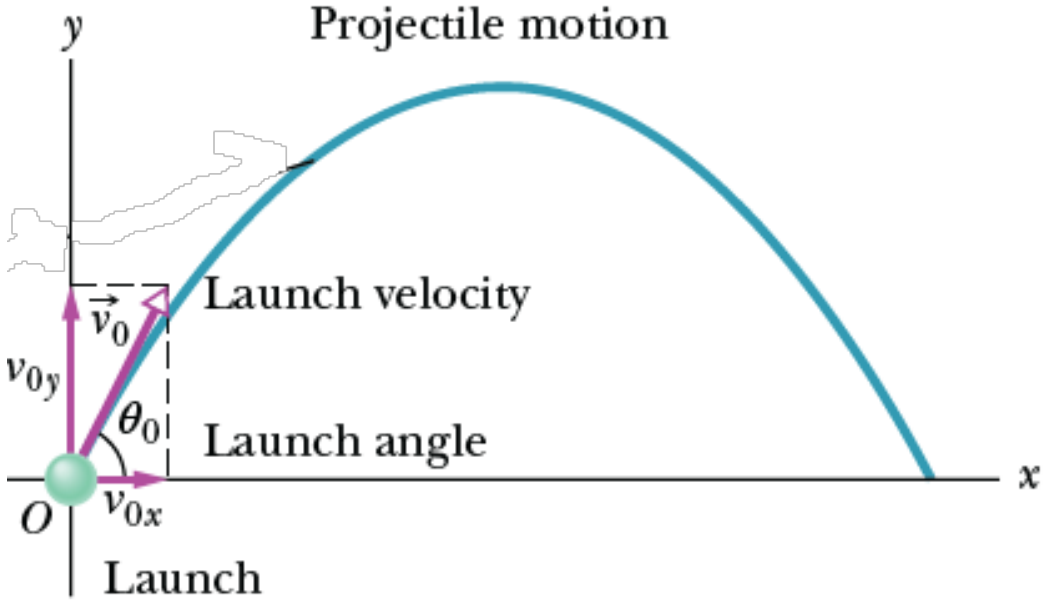
$x = x_0 + v_{0x}t$	$v_x = v_{0x}$	
$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$	$v_y = v_{0y} - gt$	$v_y{}^2 = v_{0y}{}^2 - 2g\Delta y$

4.4 Projectile Motion

The motion of a projectile is **projectile motion** launched with an initial velocity. The initial velocity $\overrightarrow{v_0}$ has two components.

$$\vec{v}_0 = v_{0x}\hat{\mathrm{i}} + v_{0y}\hat{\mathrm{j}}$$

$$v_{0x} = v_0 \cos\theta_0 \quad \text{and} \quad v_{0y} = v_0 \sin\theta_0$$

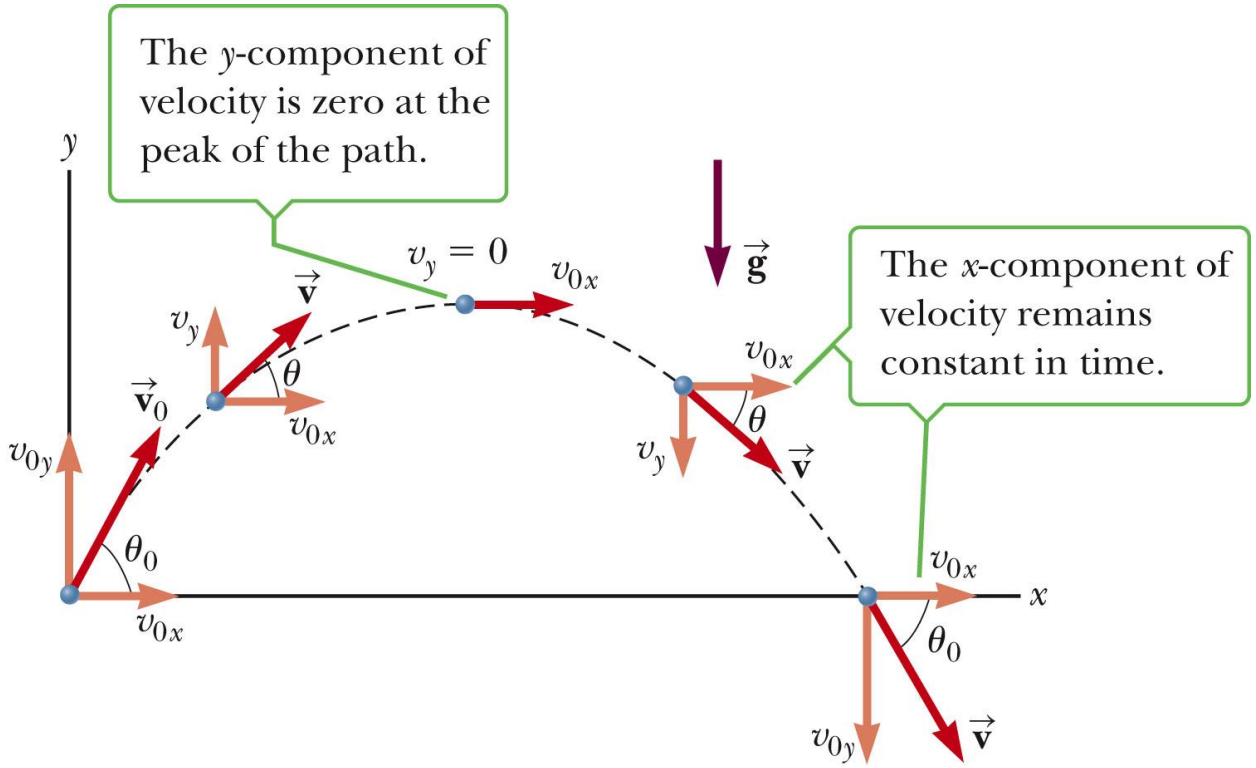


In projectile motion, the horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other.

Horizontal Component of Projectile Motion

Horizontal motion: No acceleration ($a_x = 0$), so velocity is constant :

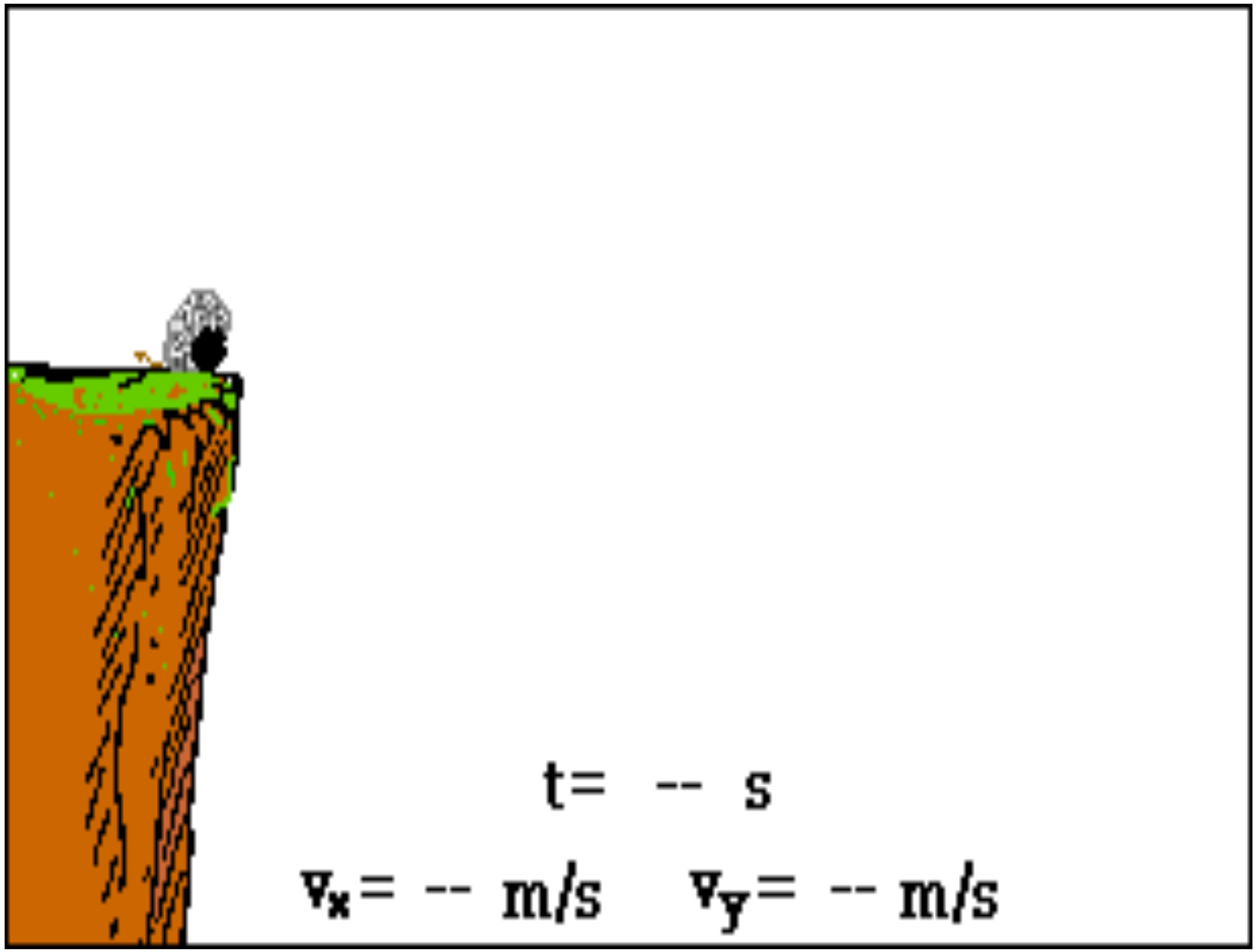
$$v_x = v_{0x} = v_0 \cos\theta_0$$



Vertical Component of Projectile Motion

Vertical motion: Acceleration is always $a_y = -g$:

$x - x_0 = (v_0 \cos\theta_0)t$	$v_x = v_0 \cos\theta_0$	
$y - y_0 = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2$	$v_y = v_0 \sin\theta_0 - gt$	$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0).$

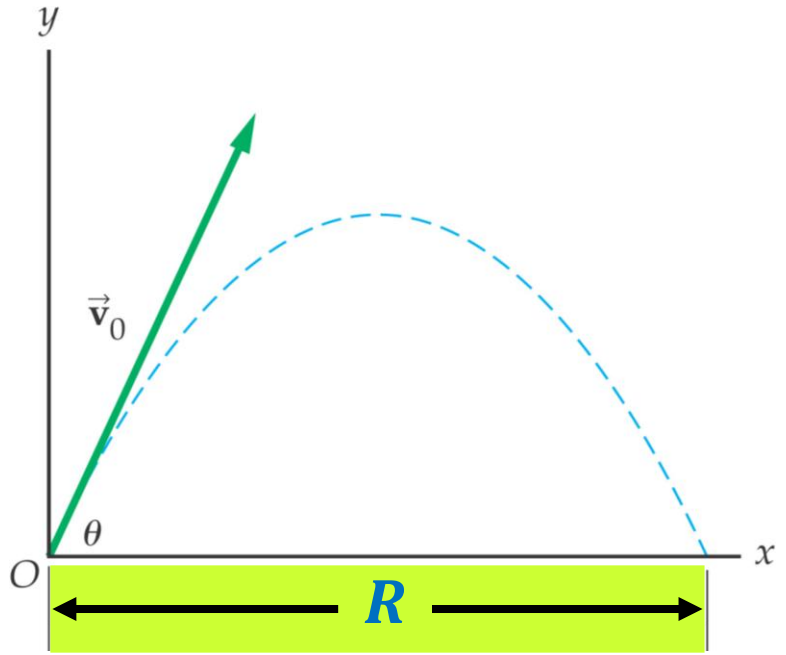

t= -- s
v_x= -- m/s
v_y= -- m/s

Horizontal Range

- The range (R): max. horizontal displacement.
- If the initial and final elevation are the same $(\Delta y = 0)$:

$$R = \left(\frac{{v_0}^2}{g}\right)\sin 2\theta \quad (\textit{same initial and final elevation})$$

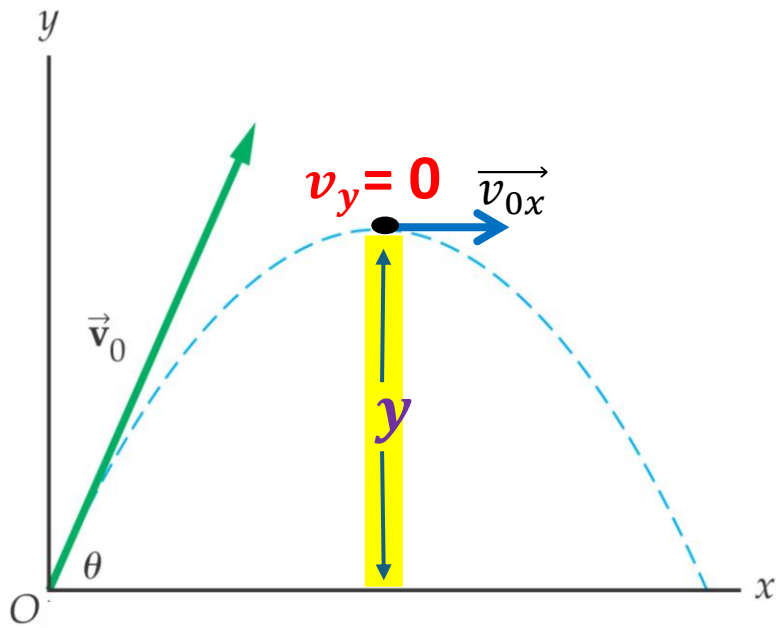
- Note that the range is maximum when the launch angle is 45°.



Maximum height

- The maximum height: max. vertical displacement.
- At maximum height $v_y = 0$

$$y = \frac{(v_0 \sin\theta)^2}{2g}$$

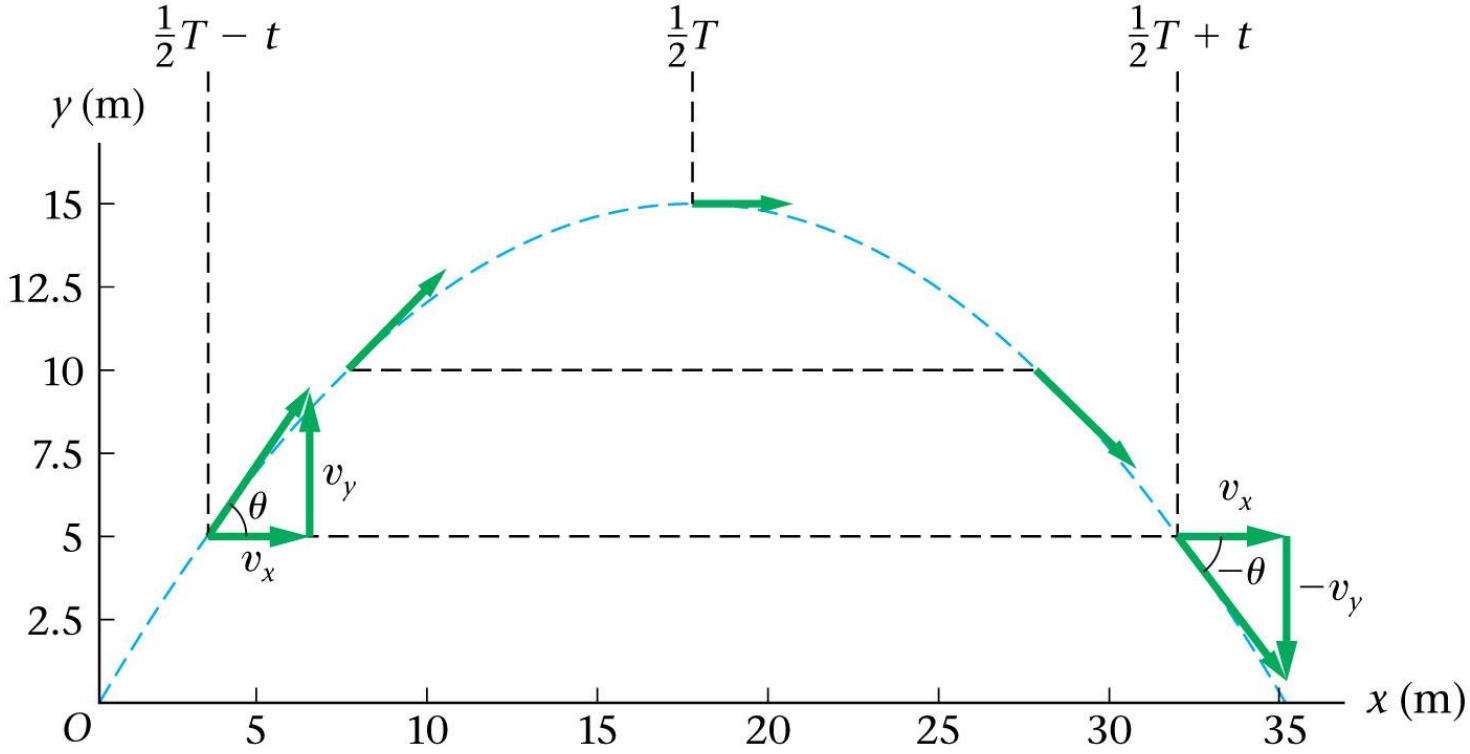


Time of Flight

The time taken by the projectile from when launched and when it lands is

$$t = \frac{(2v_0 \sin\theta)}{g}$$

Symmetry in projectile motion

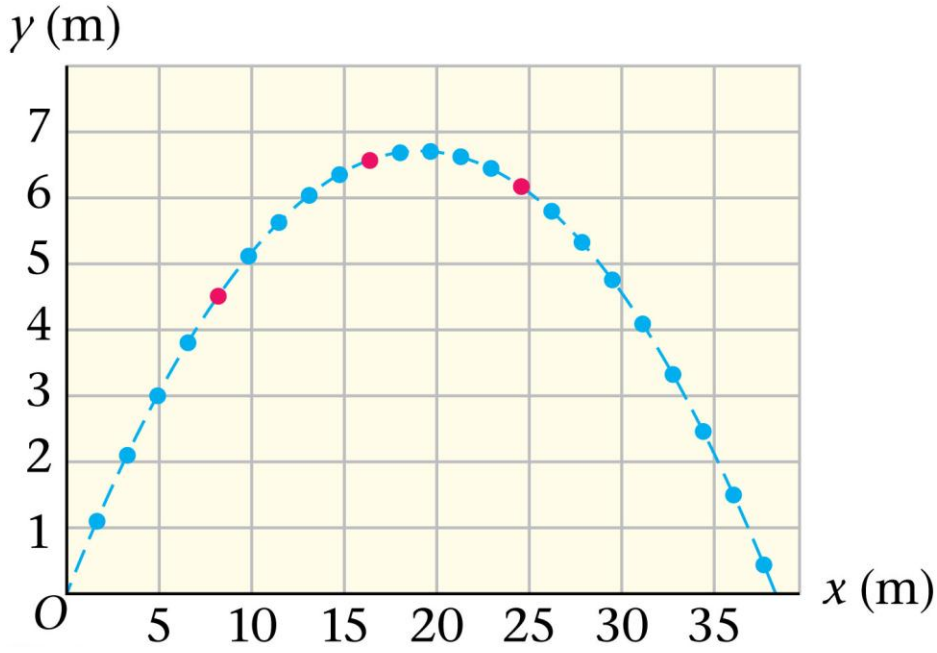


Example 4: A projectile is launch from the origin with an initial speed of 20.0 m/s at an angle of $35.0°$ above the horizontal.

(a) Find the x and y positions of the projectile at $t = 1.50$ s.

(b) find the **velocity of the projectile** at $t = 1.5$ s

[Ans.: (a) x = 24.6 m, y = 6.17 m; (b) $\vec{v} = (16.4\ \text{m/s})\hat{x} + (-3.24\ \text{m/s})\hat{y}$]



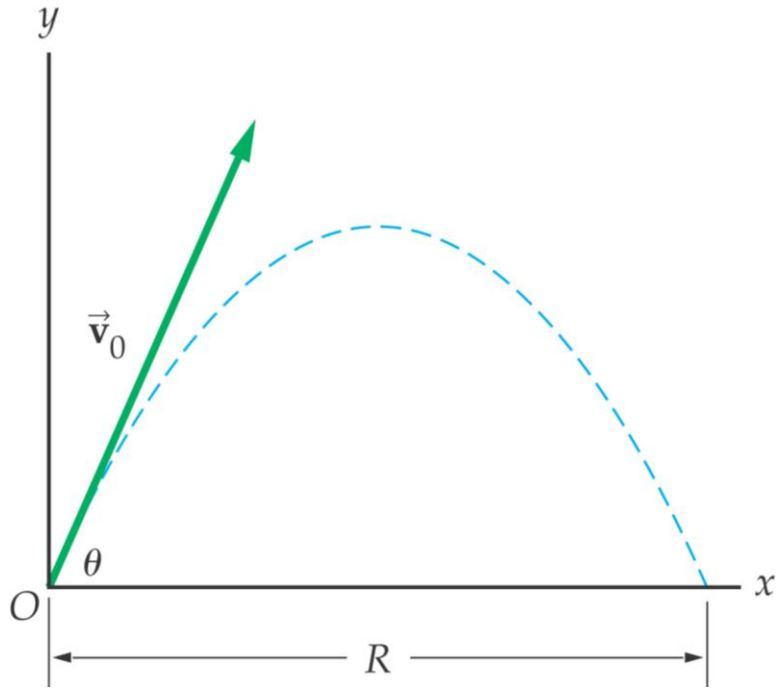
Example 5: An object is launched with an initial velocity of 20 m/s at an angle of 50° above the horizontal. If the object lands at the same launch height, find:

(a) The maximum height reached.

(b) The time of flight.

(c) The horizontal range of the projectile.

[Ans.: (a) 11.96 m, (b) 3.12 s, (c) 40.16 m]

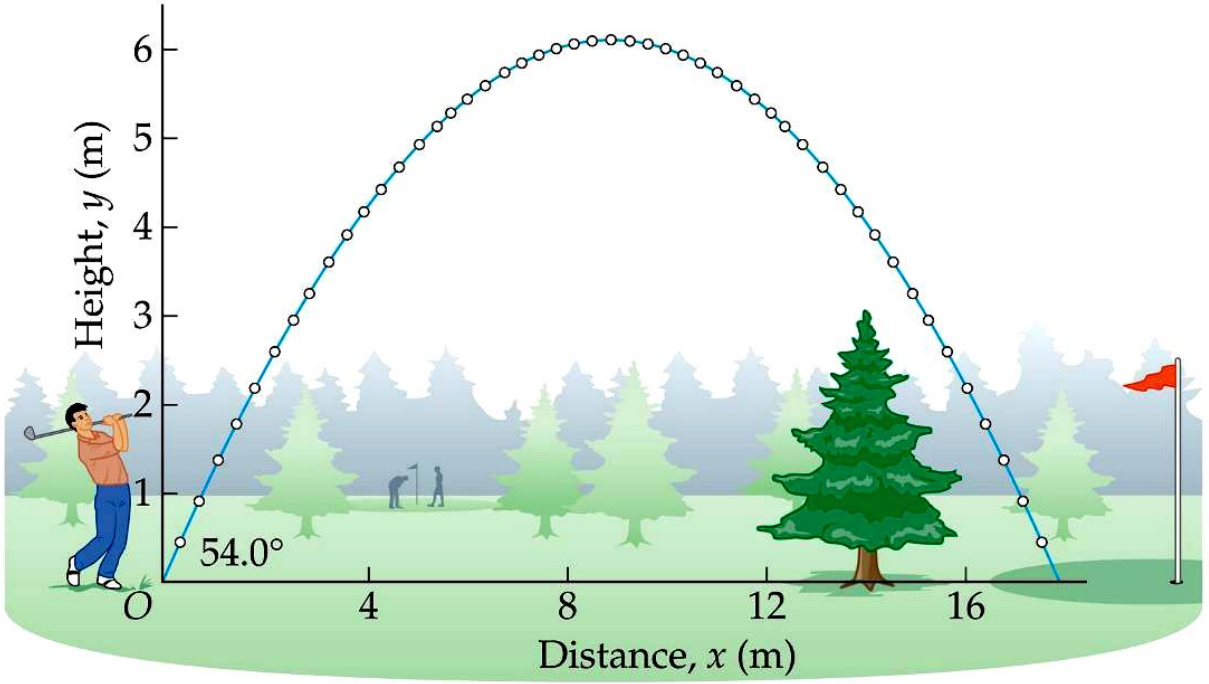


Example 6: A golfer sends the ball over a high tree that is 14.0 m away, as shown. The ball lands on the green at the same level from which it was struck after traveling a horizontal distance of 17.8 m.

(a) If the ball left the club at 54.0° above the horizontal and landed 2.24 s later, what was its initial speed v_0?

(b) How high was the ball when it passed over the tree?

[Ans.: (a)13.5 m/s, (b) t =1.76 s, Δy = 4.03 m]



SOLUTION

Part (a)

1. Divide the horizontal distance, d, by the time of flight, t, to obtain v_x:

$$v_x = \frac{d}{t} = \frac{17.8\ \text{m}}{2.24\ \text{s}} = 7.95\ \text{m/s}$$

2. Use $v_x = v_0 \cos\theta$ to find v_0, the initial speed:

$$v_x = v_0 \cos\theta \quad \text{or} \quad v_0 = \frac{v_x}{\cos\theta} = \frac{7.95\ \text{m/s}}{\cos 54.0^\circ} = 13.5\ \text{m/s}$$

Part (b)

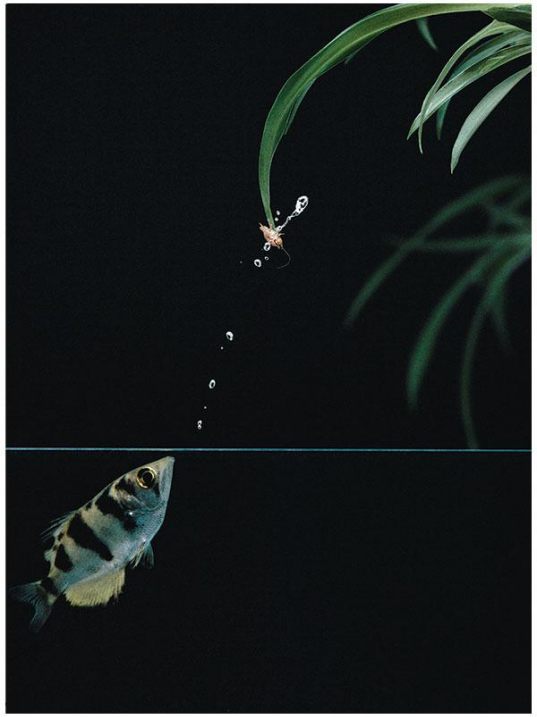
3. Use $x = (v_0 \cos\theta)t$ to find the time when $x = 14.0$ m. Recall that $x_0 = 0$:

$$x = (v_0 \cos\theta)t \quad \text{or} \quad t = \frac{x}{v_0 \cos\theta} = \frac{14.0\ \text{m}}{7.95\ \text{m/s}} = 1.76\ \text{s}$$

4. Evaluate $y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$ at the time found in Step 3. Recall that $y_0 = 0$:

$$\begin{aligned} y &= (v_0 \sin\theta)t - \tfrac{1}{2}gt^2 \\ &= [(13.5\ \text{m/s}) \sin 54.0^\circ](1.76\ \text{s}) - \tfrac{1}{2}(9.81\ \text{m/s}^2)(1.76\ \text{s})^2 \\ &= 4.03\ \text{m} \end{aligned}$$

Example 7: An Archerfish expels water from its mouth to hunt an insect on a branch. Suppose it expels the water with an initial speed of 2.3 m/s and at an angle of 19.5° above the horizontal. When the water reaches the insect at a height h above water's surface, it is moving horizontally.

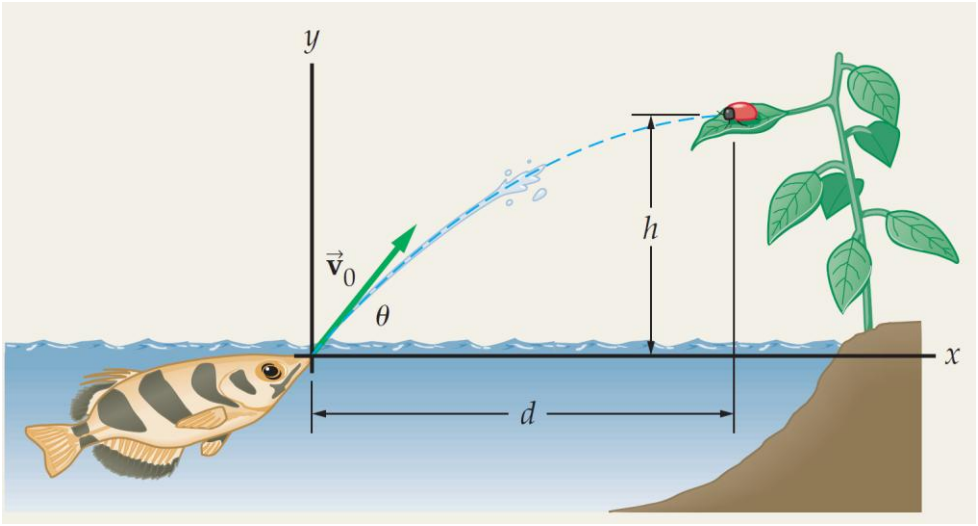


a) How much time does the insect have to react?

b) What is the height of the insect?

c) What is the horizontal distance d between the fish and the insect when the water is launched?

[Ans.: (a) t=0.0783 s, (b) h =0.0300 m, d = 0.170 m]



SOLUTION

Part (a)

1. Set $v_y = v_0 \sin\theta - gt$ equal to zero and solve for the corresponding time t:

$$v_y = v_{0y} - gt = v_0 \sin\theta - gt = 0$$
$$t = \frac{v_0 \sin\theta}{g}$$

2. Substitute numerical values to determine the reaction time:

$$t = \frac{v_0 \sin\theta}{g} = \frac{(2.30\ \text{m/s}) \sin 19.5°}{9.81\ \text{m/s}^2} = 0.0783\ \text{s}$$

Part (b)

3. To calculate the height, we substitute $t = (v_0 \sin\theta)/g$ into $y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$:

$$y = (v_0 \sin\theta)\left(\frac{v_0 \sin\theta}{g}\right) - \frac{1}{2}g\left(\frac{v_0 \sin\theta}{g}\right)^2 = \frac{(v_0 \sin\theta)^2}{2g}$$

4. Substitute numerical values to find the height h:

$$h = \frac{(v_0 \sin\theta)^2}{2g} = \frac{[(2.30\ \text{m/s}) \sin 19.5°]^2}{2(9.81\ \text{m/s}^2)} = 0.0300\ \text{m}$$

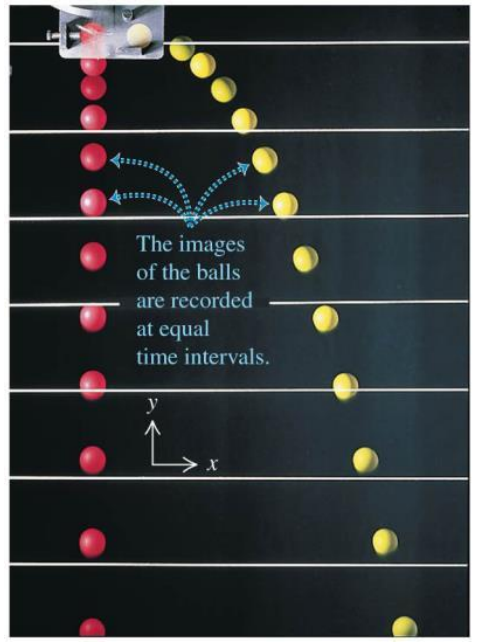
Part (c)

5. We can find the horizontal distance d using x as a function of time, $x = (v_0 \cos\theta)t$:

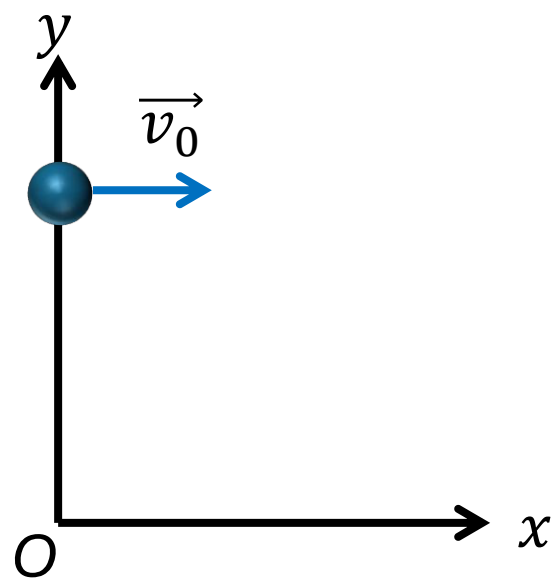
$$x = (v_0 \cos\theta)t$$
$$d = [(2.30\ \text{m/s}) \cos 19.5°](0.0783\ \text{s}) = 0.170\ \text{m}$$

Horizontally Launched Projectile

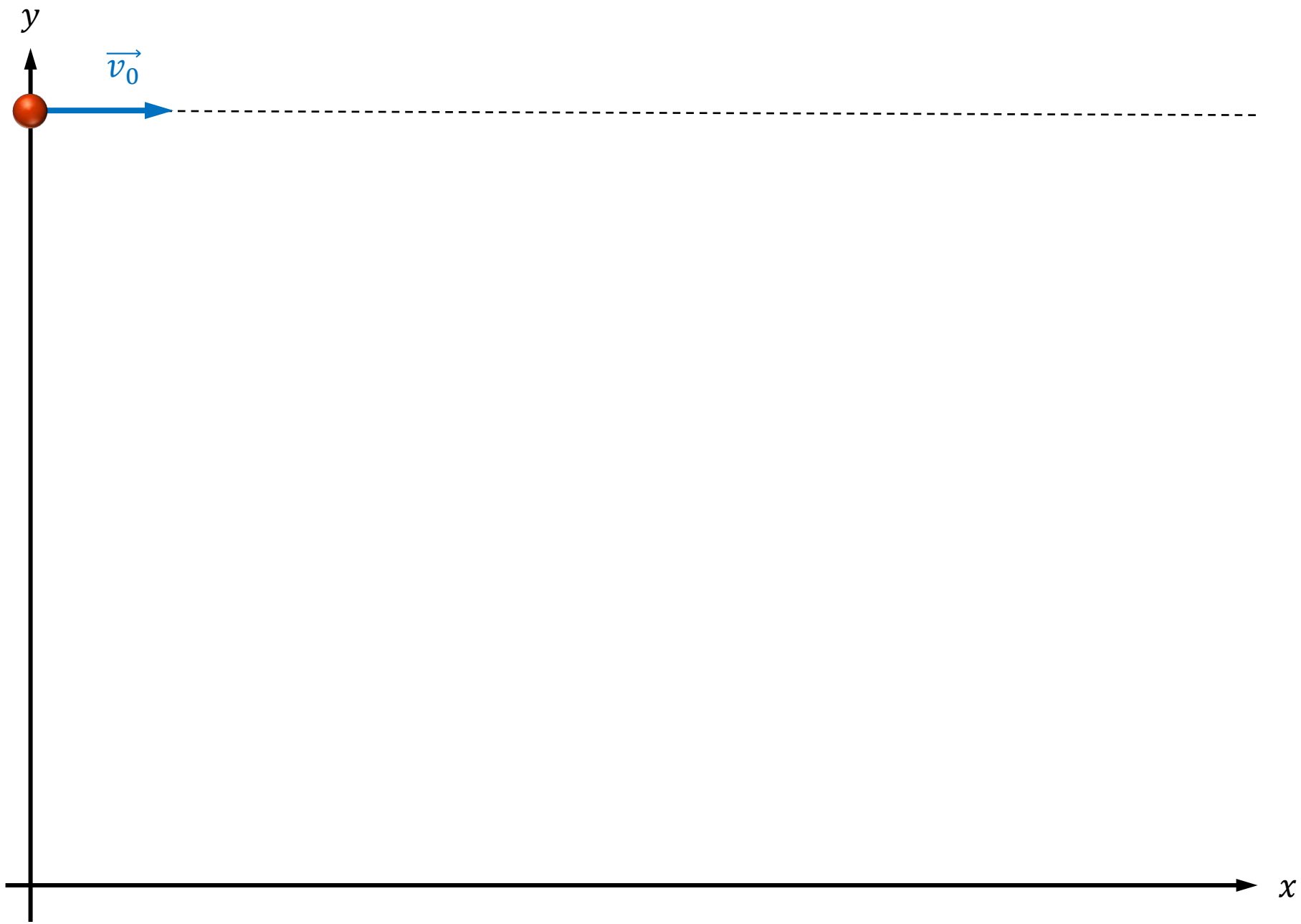
The red ball is dropped at the same time that the yellow ball is launched horizontally. We can analyze projectile motion as horizontal motion with constant velocity and vertical motion with constant acceleration: $a_x = 0$ and $a_y = -g$

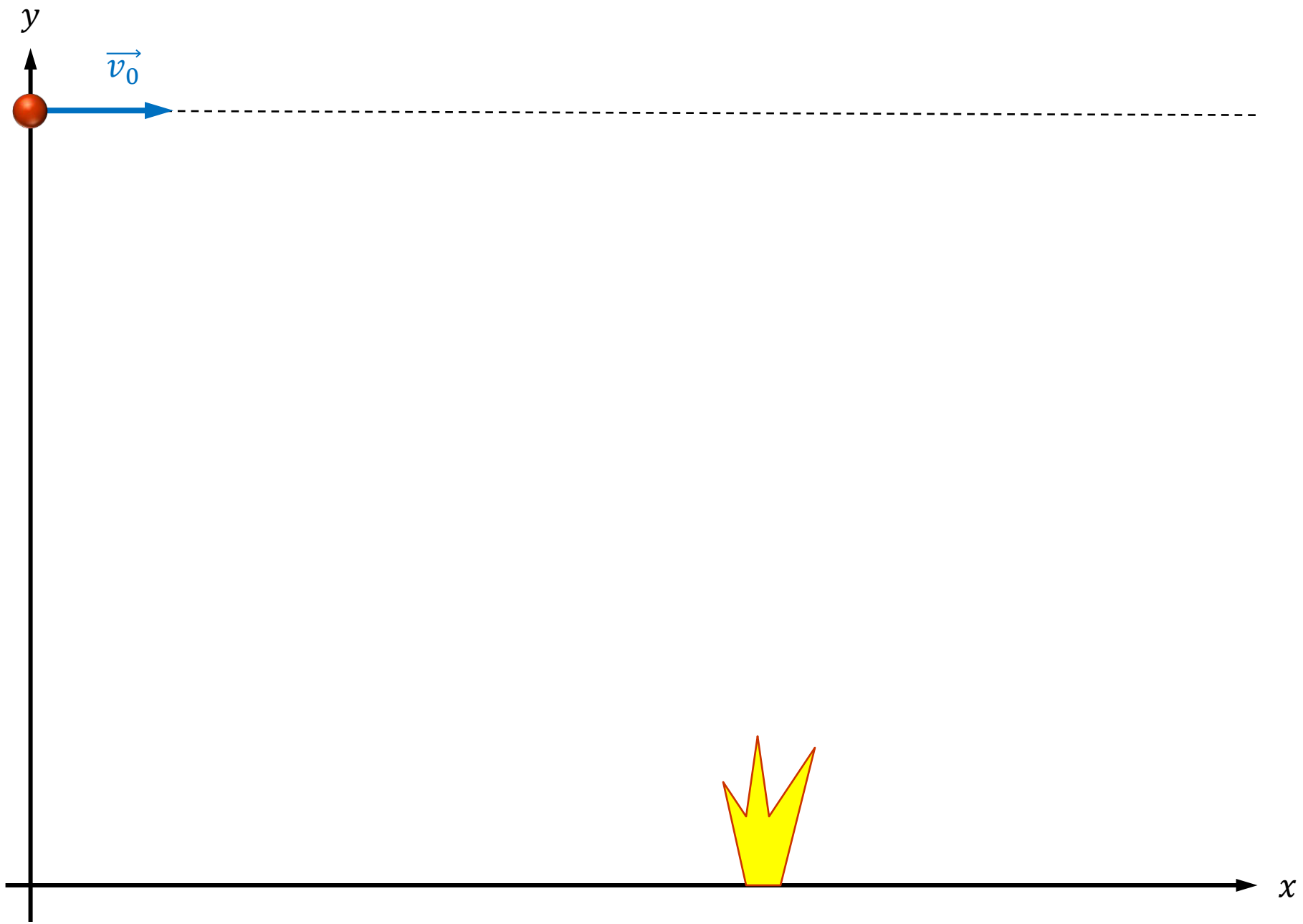


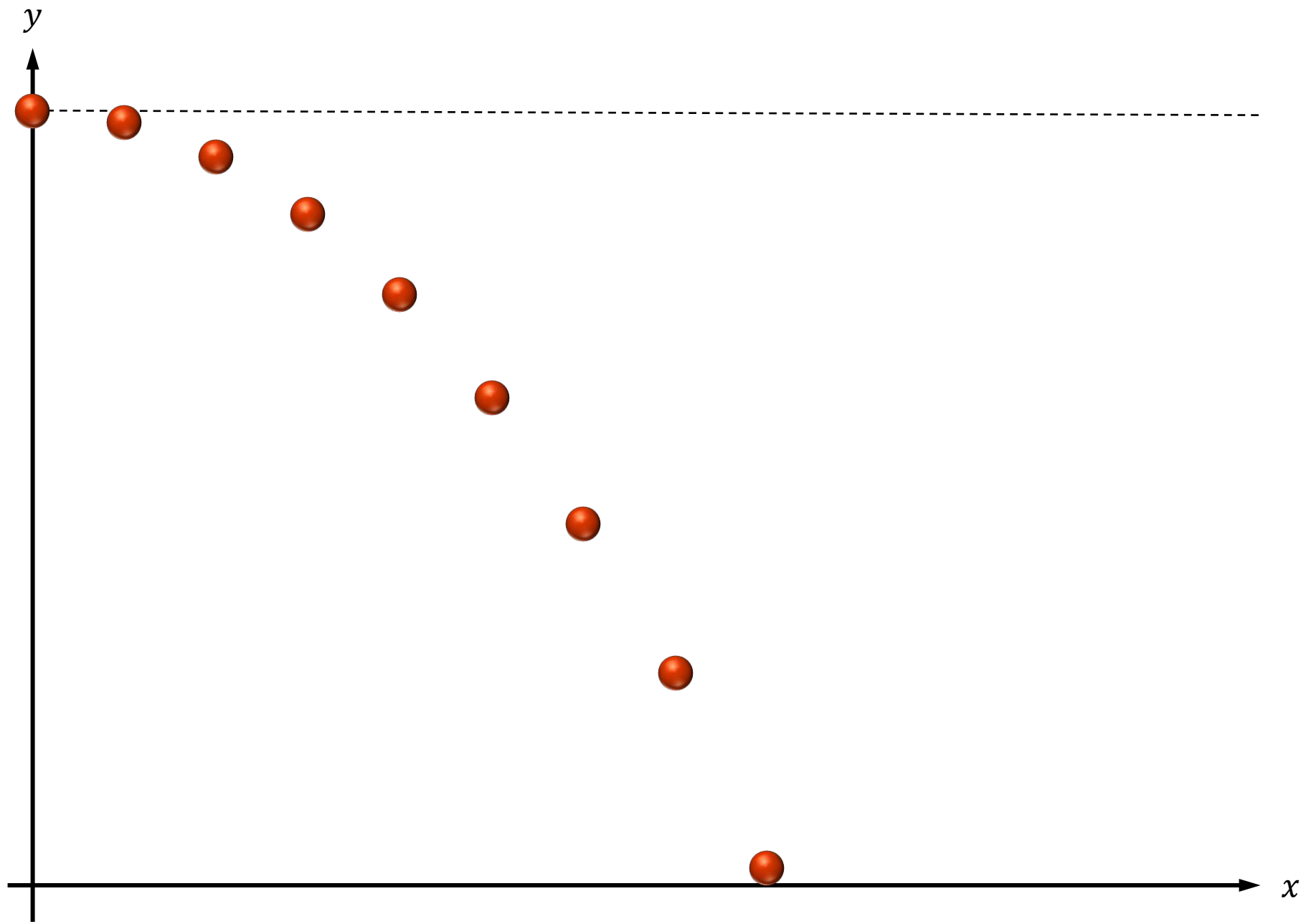
- In this case, the initial velocity is horizontal. This means that the vertical component of the initial velocity is zero ($v_{0y} = 0$).
- The equations of motion apply with $v_{0y} = 0$ and $v_{0x} = v_0$

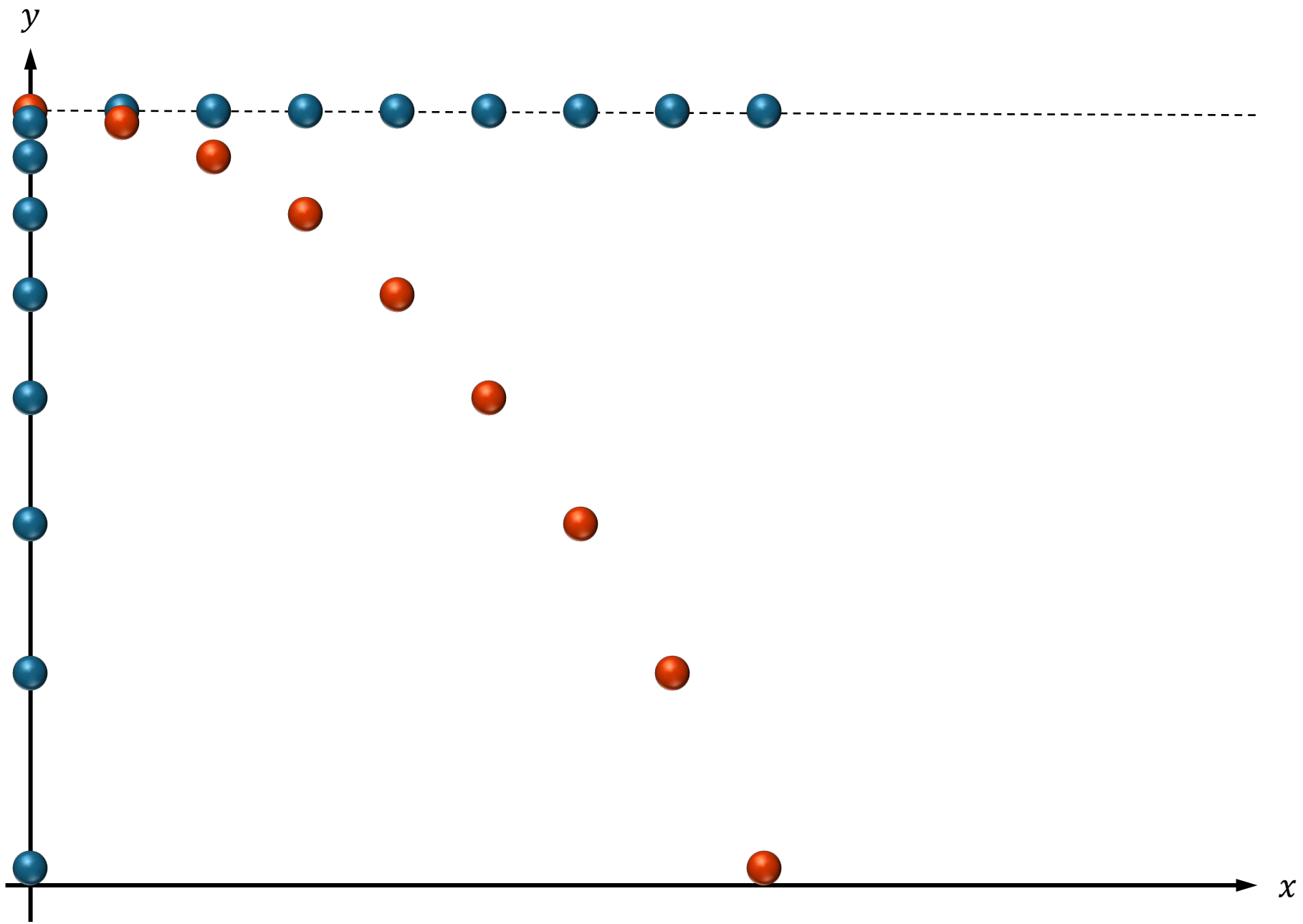


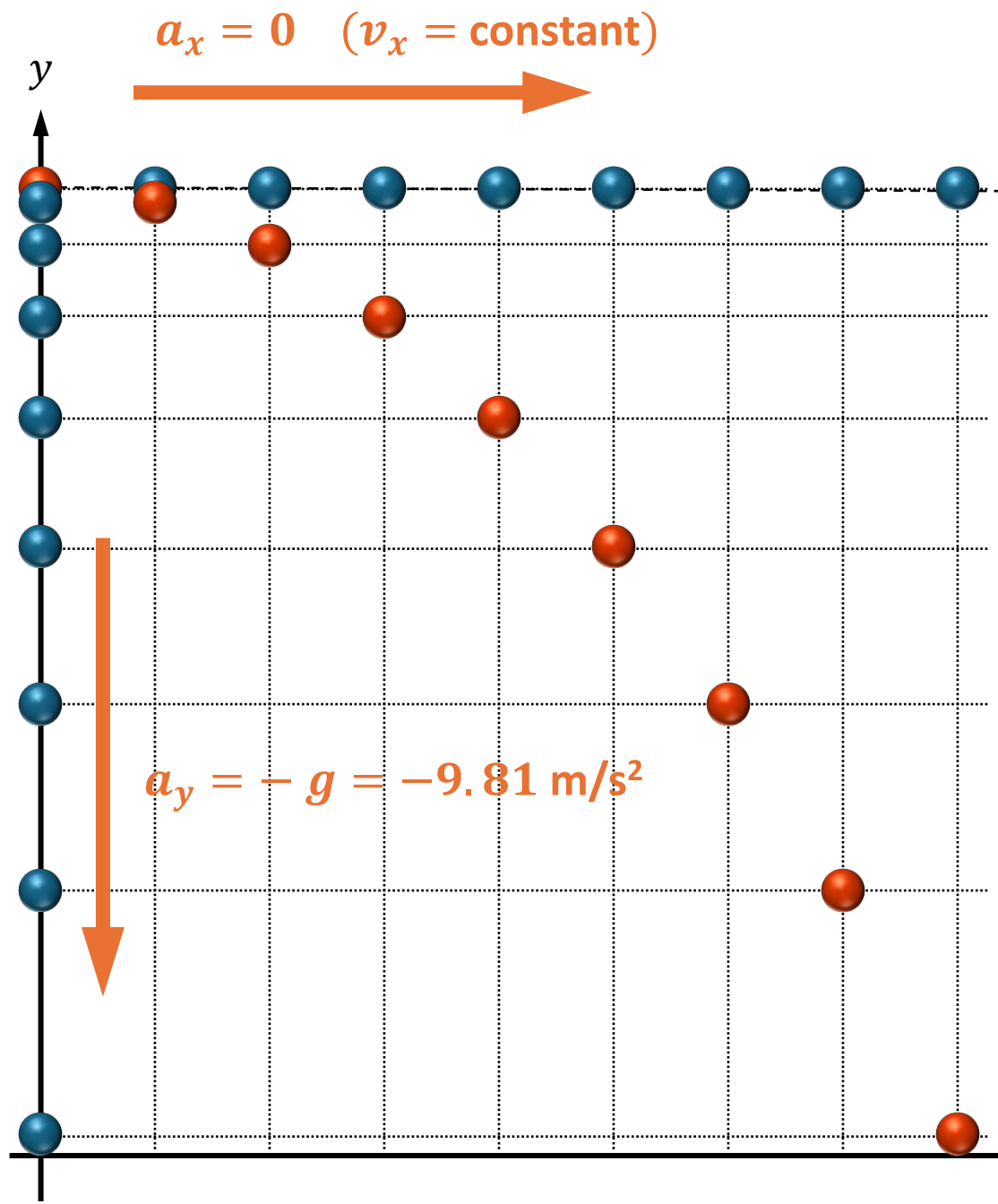
$x = v_0 t$	$v_x = v_0 = \text{constant}$	${v_x}^2 = {v_0}^2 = \text{constant}$
$y = h - \frac{1}{2}gt^2$	$v_y = -gt$	${v_y}^2 = -2g\Delta y$


y
$\overrightarrow{v_0}$
x

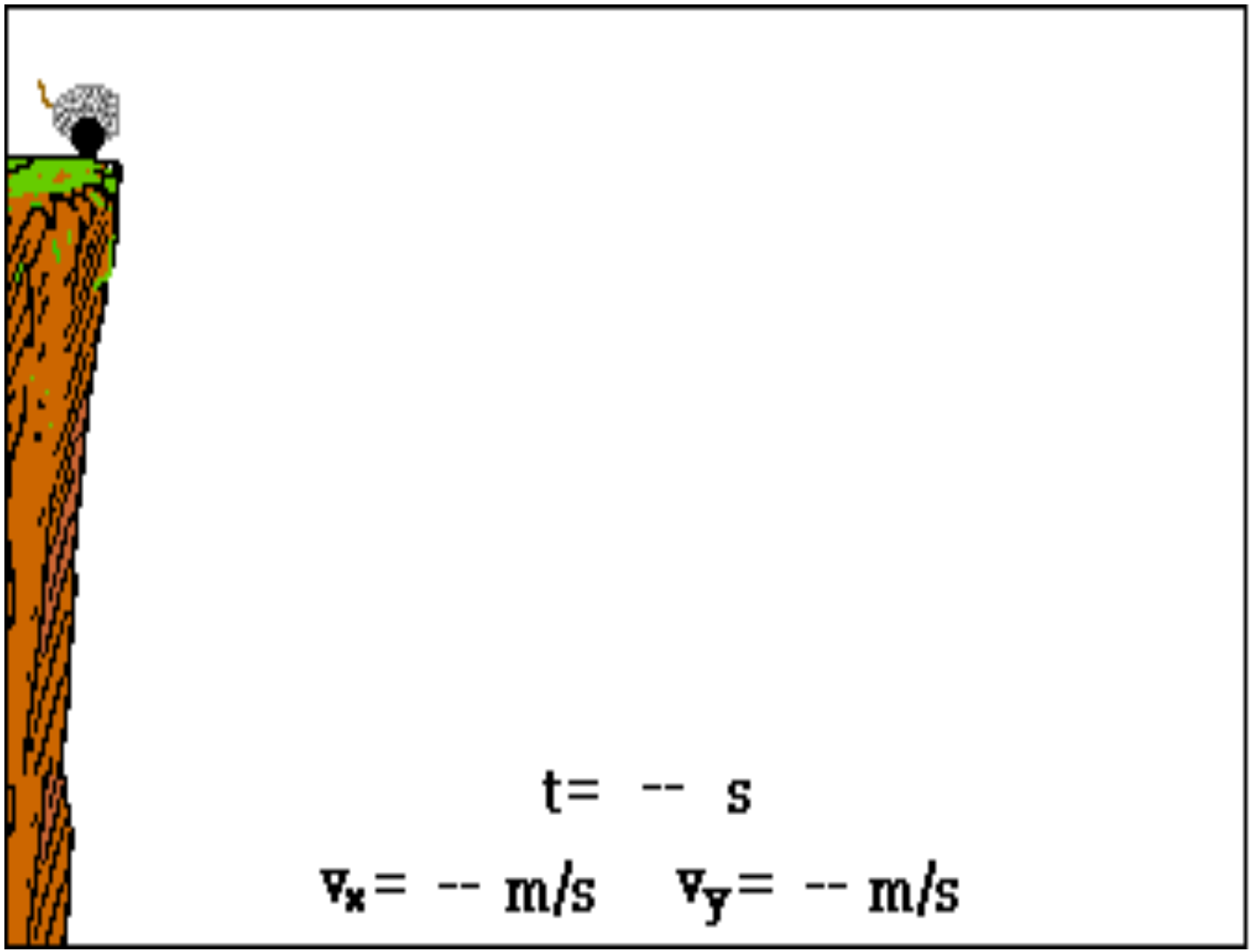

y
$\overrightarrow{v_0}$
x


y
x


y
x



- Motion is accelerated
- Acceleration is constant, and downward $a_y = -g = -9.81\ \text{m/s}^2$
- The horizontal (x) component of velocity is constant ($a_x = 0$)
- The horizontal and vertical motions are independent of each other, but they have a common time.


t= -- s
v_x= -- m/s
v_y= -- m/s

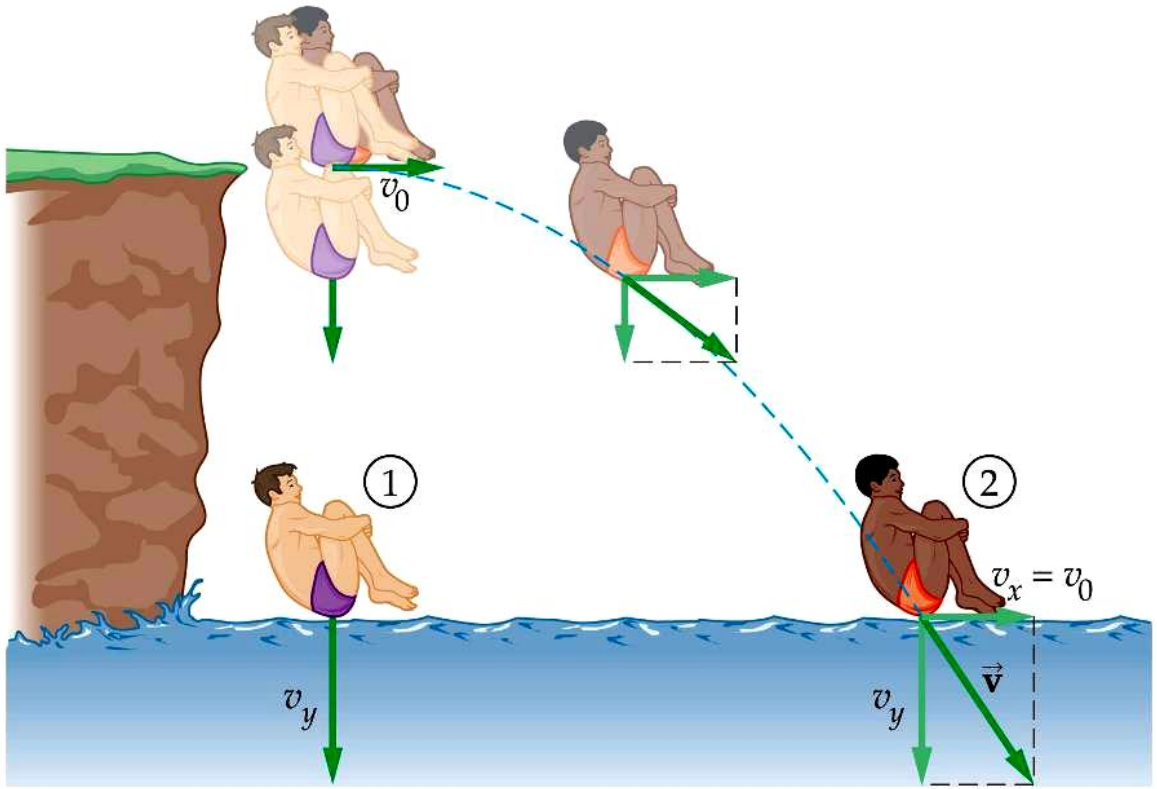
Question : Two boys, George and Sam, dive from a high overhanging cliff into a lake below. George (1) drops straight down.

Sam (2) runs horizontally and dives outward.

If they leave the cliff at the same time:

a) which boy reaches the water first?

b) Which boy hits the water with the greater speed?



(a) Since the vertical motion determines the time required to reach the water, both boys reach the water at the same (they have the same vertical velocity (y)).

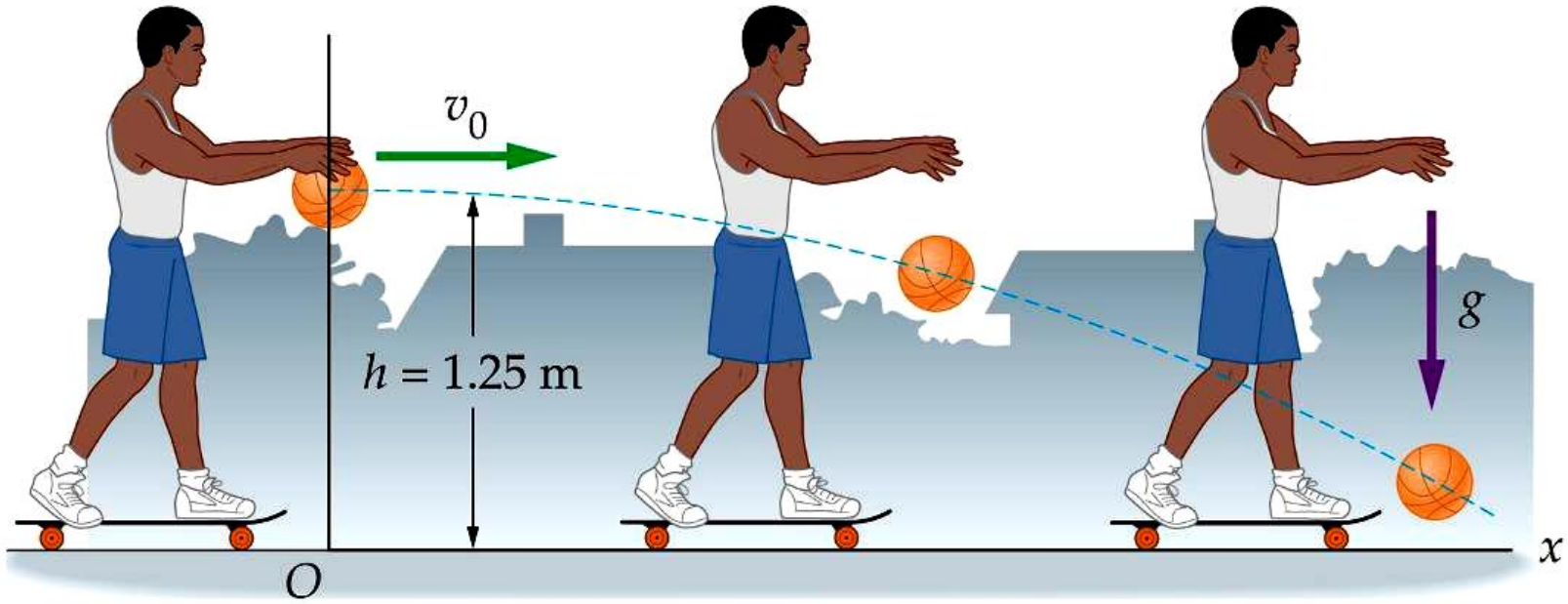
(b) George (1) reaches the water with only a vertical velocity, but Sam (2) reaches the water with both horizontal and vertical velocity components. The vertical velocities are the same for both, so Sam's speed on entering the water is greater than that of George

Example 8: A person skateboarding with a constant speed of 1.30 m/s releases a ball from a height of 1.25 m above the ground. Use $x_0 = 0$ and $y_0 = h = 1.25$ m.

Find x and y for (a) $t = 0.25$ s and (b) $t = 0.5$ s.

(c) Find the velocity, speed, and direction of motion of the ball at $t = 0.5$ s.

[Ans.: (a) 0.325 m, 0.943 m (b) 0.650 m, 0.0238 m (c) v_x = 1.3 m/s, v_y = −4.91 m/s, v = 5.08 m/s, $\theta = -75.2°$]



SOLUTION

Part (a)

1. Substitute $t = 0.250$ s into the x and y equations of motion:

$$x = v_0 t = (1.30\ \text{m/s})(0.250\ \text{s}) = 0.325\ \text{m}$$
$$y = h - \tfrac{1}{2} g t^2$$
$$= 1.25\ \text{m} - \tfrac{1}{2}(9.81\ \text{m/s}^2)(0.250\ \text{s})^2 = 0.943\ \text{m}$$

Part (b)

2. Substitute $t = 0.500$ s into the x and y equations of motion: Note that the ball is only about an inch above the ground at this time:

$$x = v_0 t = (1.30\ \text{m/s})(0.500\ \text{s}) = 0.650\ \text{m}$$
$$y = h - \tfrac{1}{2} g t^2$$
$$= 1.25\ \text{m} - \tfrac{1}{2}(9.81\ \text{m/s}^2)(0.500\ \text{s})^2 = 0.0238\ \text{m}$$

Part (c)

3. First, calculate the x and y components of the velocity at $t = 0.500$ s using $v_x = v_0$ and $v_y = -gt$:

$$v_x = v_0 = 1.30\ \text{m/s}$$
$$v_y = -gt = -(9.81\ \text{m/s}^2)(0.500\ \text{s}) = -4.91\ \text{m/s}$$

4. Use these components to determine $\vec{\mathbf{v}}$, v, and θ:

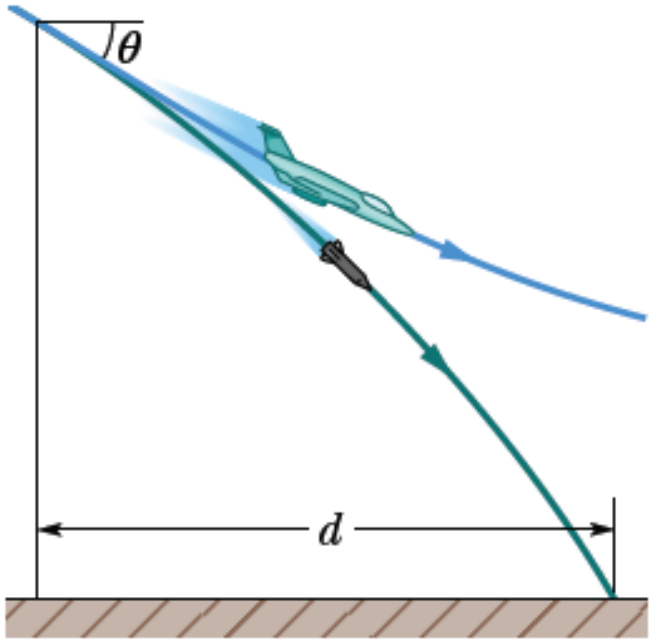
$$\vec{\mathbf{v}} = (1.30\ \text{m/s})\hat{\mathbf{x}} + (-4.91\ \text{m/s})\hat{\mathbf{y}}$$
$$v = \sqrt{v_x^2 + v_y^2}$$
$$= \sqrt{(1.30\ \text{m/s})^2 + (-4.91\ \text{m/s})^2} = 5.08\ \text{m/s}$$
$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{(-4.91\ \text{m/s})}{1.30\ \text{m/s}} = -75.2°$$

Example 9 A certain airplane has a speed of 290.0 km/h and is diving at an angle of ϑ = 30.0° below the horizontal when the pilot releases a radar decoy (see figure). The horizontal distance between the release point and the point where the decoy strikes the ground is d = 700 m.

(a) How long is the decoy in the air?

(b) How high was the release point.

[Ans.: (a) 10.0 s, (b) 897 m]



(a) We use Eq. 4.2.5 to solve for the time:

$$\Delta x = (v_0 \cos\theta_0)t \quad \Rightarrow \quad t = \frac{700 \text{ m}}{(80.6 \text{ m/s})\cos(-30.0°)} = 10.0 \text{ s}.$$

(b) And we use Eq. 4.4.4 to solve for the initial height y_0:

$$y - y_0 = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 \quad \Rightarrow \quad 0 - y_0 = (-40.3 \text{ m/s})(10.0 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(10.0 \text{ s})^2,$$

which yields y_0 = 897 m.

Summary of Chapter 4

Position Vector: Locates a particle in 3-space $\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}$

Displacement : Change in position vector $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

Average and Instantaneous Velocity $\vec{v}_{\text{avg}} = \dfrac{\Delta\vec{r}}{\Delta t}$ $\vec{v} = \dfrac{d\,\vec{r}}{dt}$

Average and Instantaneous Accel. $\vec{a}_{\text{avg}} = \dfrac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \dfrac{\Delta\vec{v}}{\Delta t}$ $\vec{a} = \dfrac{d\vec{v}}{dt}.$

Summary of Chapter 4

- Components of motion in the *x*- and *y*-directions can be treated independently
- In projectile motion, the acceleration is $-g$
- If the launch angle is zero, the initial velocity has only an *x*-component
- The path followed by a projectile is a parabola
- The range is the horizontal distance the projectile travels

Example : An electron's position is given by $\vec{\mathbf{r}} = 3.00\,t\,\hat{\imath} - 4.00\,t^2\,\hat{\jmath} + 2.00\,\hat{k}$, with t in seconds and $\vec{\mathbf{r}}$ in meters.

(a)In unit-vector notation, what is the instantaneous velocity of electron at t = 2.00 s,

(b) Calculate the magnitude and direction of the instantaneous velocity at t = 2.00 s relative to the positive direction of the x axis

6. To emphasize the fact that the velocity is a function of time, we adopt the notation $v(t)$ for dx/dt.

(a) Equation 4.2.3 leads to

$$v(t) = \frac{d}{dt}\,(3.00t\hat{\mathrm{i}} - 4.00t^2\hat{\mathrm{j}} + 2.00\hat{\mathrm{k}}) = (3.00\ \mathrm{m/s})\hat{\mathrm{i}} - (8.00\ \mathrm{m/s^2})t\hat{\mathrm{j}}$$

(b) Evaluating this result at t = 2.00 s produces $\vec{v}$ = (3.00$\hat{\mathrm{i}}$ − 16.0$\hat{\mathrm{j}}$) m/s.

(c) The speed at t = 2.00 s is $v = |\vec{v}| = \sqrt{(3.00\ \mathrm{m/s})^2 + (-16.0\ \mathrm{m/s})^2} = 16.3$ m/s.

(d) The angle of $\vec{v}$ at that moment is

$$\tan^{-1}\left(\frac{-16.0\ \mathrm{m/s}}{3.00\ \mathrm{m/s}}\right) = -79.4^\circ \text{ or } 101^\circ,$$

Additional Examples

Example 4 (31): A ball is launched at an angle of 35.0° above the horizontal. If the ball travels a horizontal distance of 1.30 m in 1.25 s, what was its initial speed.

[Ans.: v_x = 1.04 m/s; v_0 = 1.27 m/s]

$$v_x = \frac{x}{t} = \frac{1.30\ \text{m}}{1.25\ \text{s}} = 1.04\ \text{m/s} = v_{0x}$$

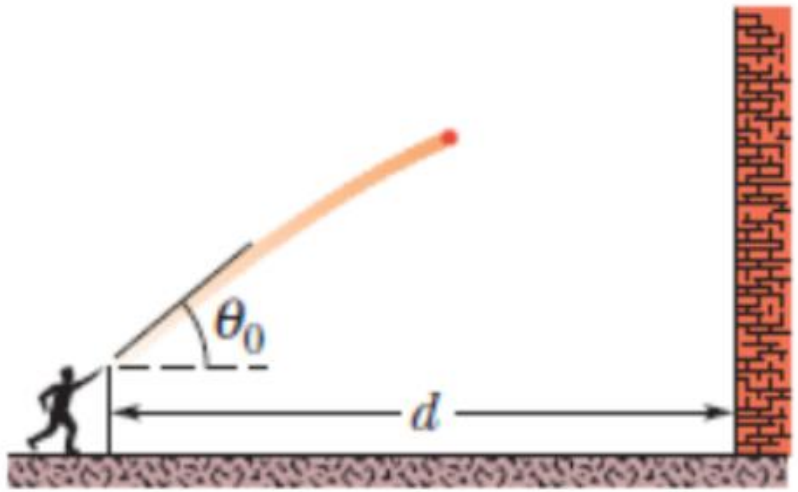
$$v_0 = \frac{v_{0x}}{\cos\theta} = \frac{1.04\ \text{m/s}}{\cos 35.0^\circ} = \boxed{1.27\ \text{m/s}}$$

Additional Examples

Example: A ball is thrown with initial velocity of 14 m/s at an angle of 34° to hit a wall 3.8 m away.

(a)How far above the release point does the ball hit the wall?

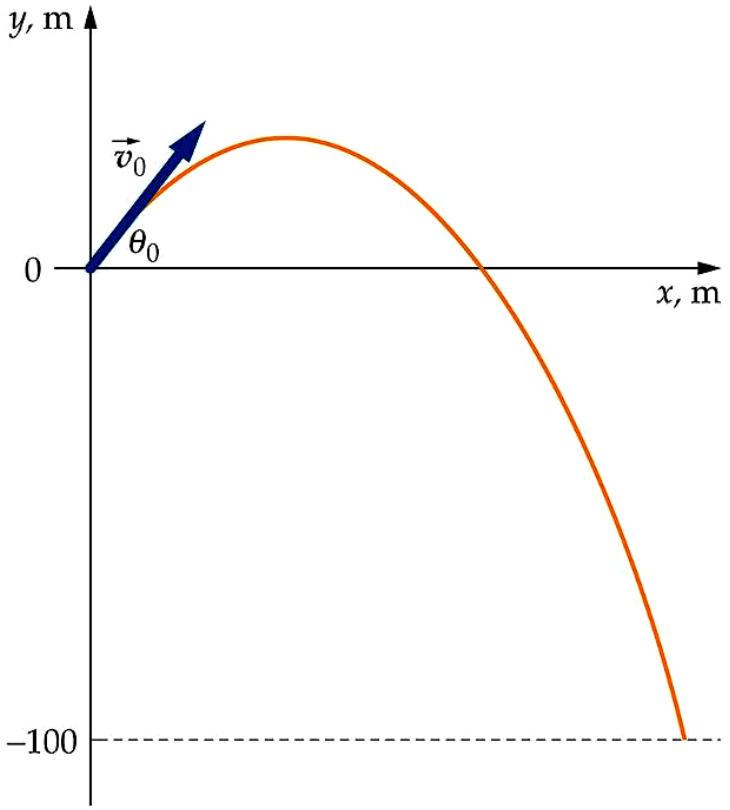
(b)What is velocity of the ball just before hitting the wall?



Additional Examples

Example: A helicopter drops a supply package to flood victims on a raft in a swollen lake. When the package is released, the helicopter is 100 m directly above the raft and flying at a velocity of 25.0 m/s at an angle of 36.9° above the horizontal. [Sin(36.9°) = $^3/_5$; Cos(36.9°) = $^4/_5$].

(a) How long is the package in the air?
(b) How far from the raft does the package land?
(c) If the helicopter continues at constant speed, where is it when the package lands?



$$y(t) = v_{0y}t - \tfrac{1}{2}gt^2 \qquad 0 = \tfrac{1}{2}gt^2 - v_{0y}t + y$$

$$v_{0y} = v_0 \sin\theta_0 = (25.0\ \text{m/s})(\sin 36.9°) = 15.0\ \text{m/s}; \quad v_{0x} = v_0 \cos\theta_0 = 20.0\ \text{m/s}$$

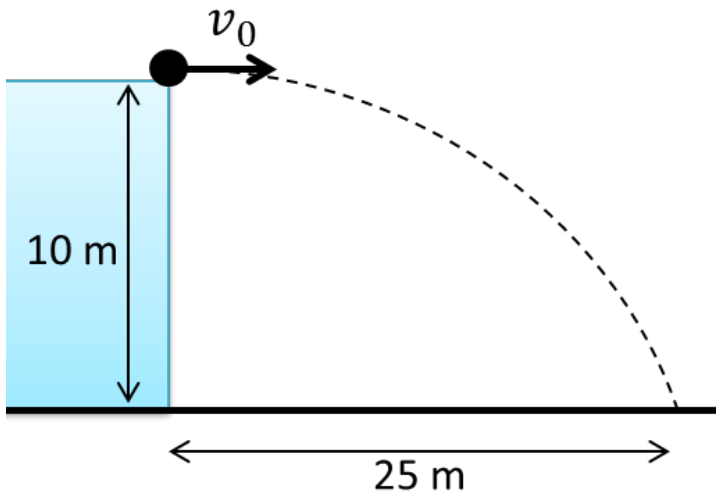
$$t = \frac{v_{0y} \pm \sqrt{v_{0y}^2 - 2gy}}{g} = \frac{(15.0\ \text{m/s}) \pm \sqrt{(15.0\ \text{m/s})^2 - 2(9.81\ \text{m/s}^2)(-100\ \text{m})}}{(9.81\ \text{m/s}^2)}$$

$$t = -3.24\ \text{s and } t = 6.30\ \text{s} \qquad x = x_h = v_{0x}t = (20.0\ \text{m/s})(6.30\ \text{s}) = 126\ \text{m}$$

Additional Examples

Example: A stone is thrown horizontally from a 10 m high building. The stone covers a horizontal distance of 25 m before hitting the ground.

(a) How long does it take the stone to reach the ground?
(b) At what speed v_0 was the stone thrown?
(c) What is the velocity vector of the stone as it hits the ground?
(d) What are the speed and direction of motion of the stone as it hits the ground?



Example: A diver runs horizontally off the end of diving board with an initial speed of 1.85 m/s. If the diving board id 3.00 m above the water, what is the divers speed just before she enters the water? [Ans.: $\vec{v} = 1.85\,\hat{x} - 7.67\,\hat{y}$ m/s, $v = 7.89$ m/s]

Solution: 1. Use equation 4-6 to find v_y

$$v_y^2 = v_{0y}^2 - 2g\Delta y = 0 - 2\left(9.81\ \text{m/s}^2\right)\left(0\ \text{m} - 3.00\ \text{m}\right) = \underline{\underline{58.9\ \text{m}^2/\text{s}^2}}$$

v_x and v_y

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(1.85\ \text{m/s}\right)^2 + 58.9\ \text{m}^2/\text{s}^2} = \boxed{7.89\ \text{m/s}}$$