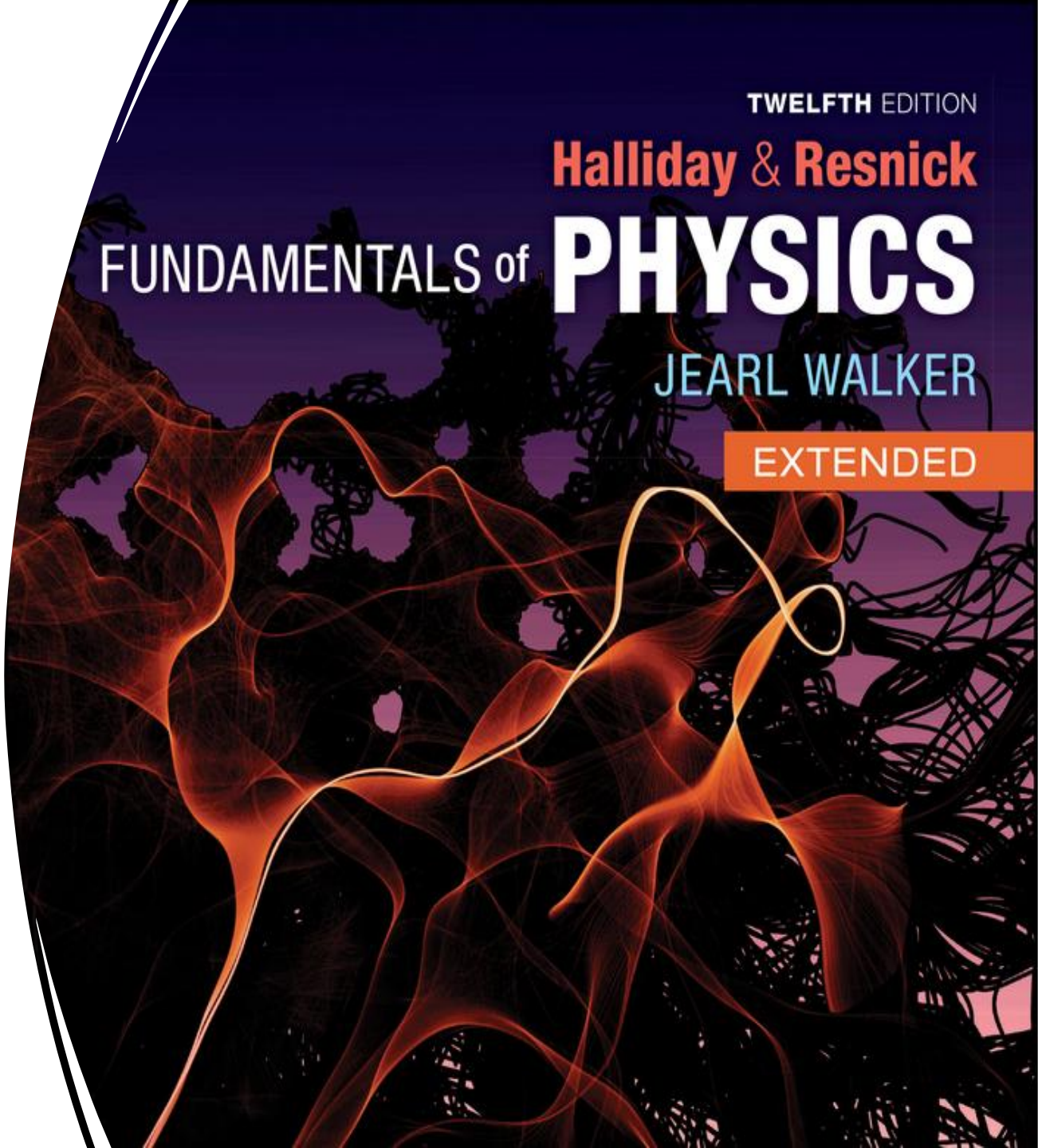
Chapter 3

Vectors



Units of Chapter 3

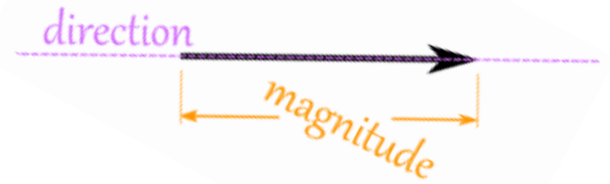
3.1 Vectors and their Components

3.2 Unit Vectors, Adding Vectors by Components

3.3 Multiplying Vectors

3-1 Vectors and their Components

- **Vector quantity** is a quantity that has both a **magnitude and a direction** and thus can be represented with a vector.
- Examples: position, displacement, velocity
  - $\vec{r}$ represents a vector of magnitude r (or $|\vec{r}|$).



- **Scalar quantity is a** quantity that can be described by a single pure number, accompanied by a unit number with units
  - Examples: time, temperature, mass

Displacement as a Vector

The simplest example is a **displacement vector**

If a particle changes position from *A* to *B*, we represent this by a vector arrow pointing from *A* to *B*

In Fig.(a) we see that all three arrows have the same magnitude and direction: they are identical displacement vectors.

In Fig.(b) we see that all three paths correspond to the same displacement vector. The vector tells us nothing about the actual path that was taken between *A* and *B*.

Displacement vectors represent only the overall effect of the motion, not the motion itself.

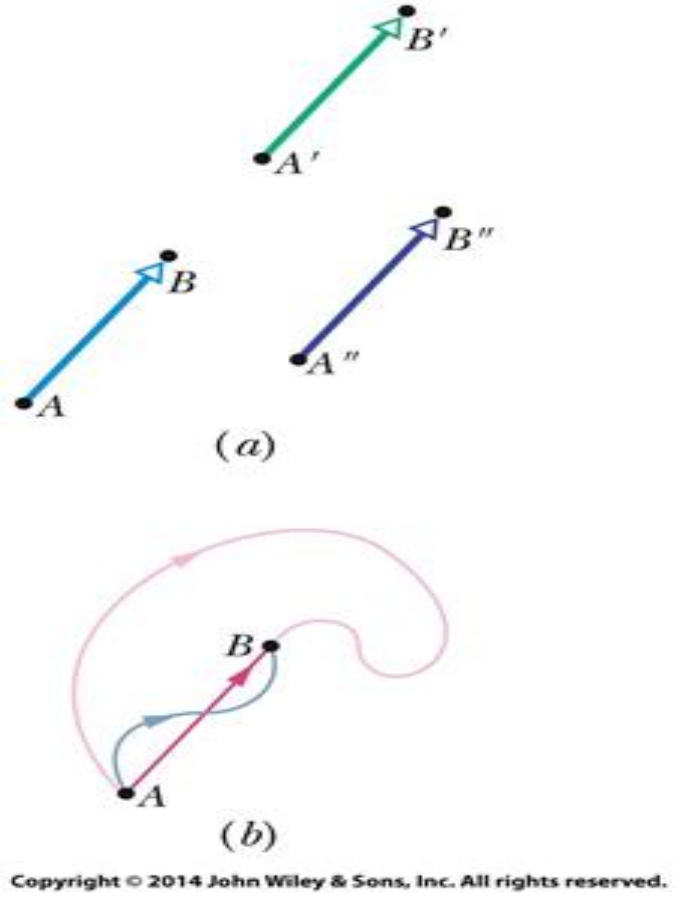


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Figure 3.1.1

Resultant Vector Equals Vector Sum

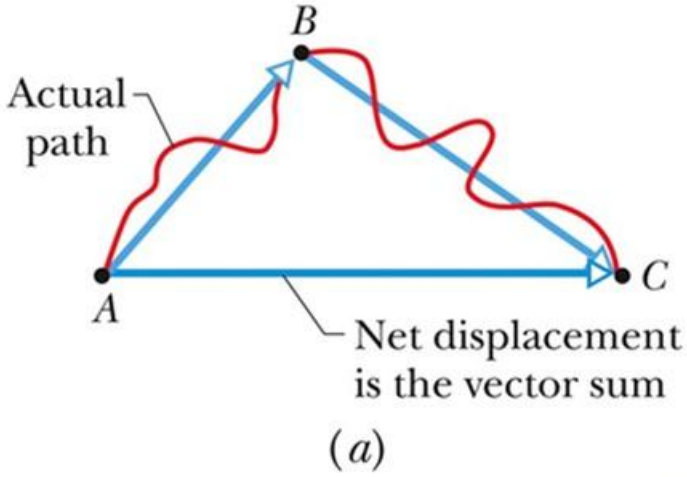
- The **vector sum,** or **resultant**
  - Is the result of performing vector addition
  - Represents the net displacement of two or more displacement vectors

$$\vec{s} = \vec{a} + \vec{b}$$ **Equation (3.1.1)**

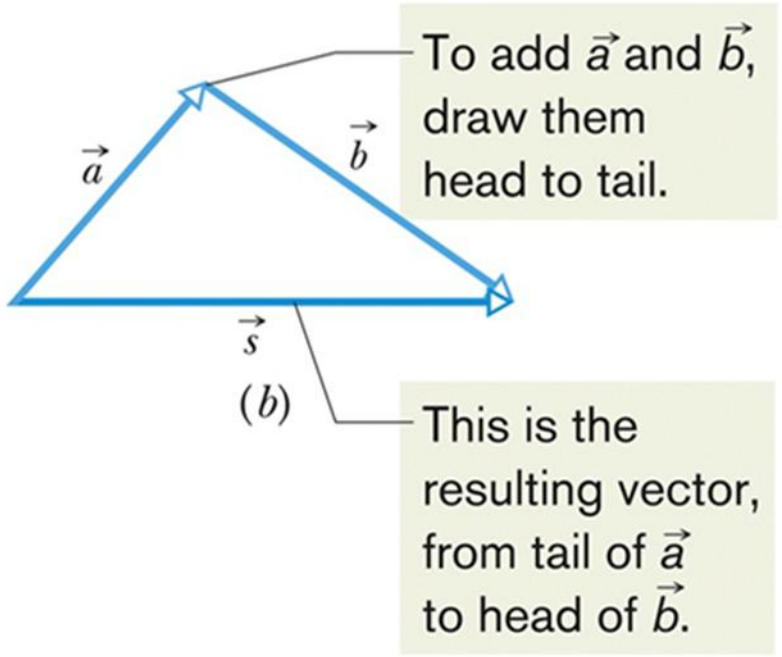
  - Can be added graphically as shown on next page:

Graphical Vector Addition: Head-to-Tail

(a) A particle moves from *A* to *B* and then later from *B* to *C*. We can represent its overall displacement (no matter what its actual path) with two successive displacement vectors, *AB* and *BC*. The *net* displacement of these two displacements is a single displacement from *A* to *C*. We call *AC* the **vector sum** (or **resultant**) of the vectors *AB* and *BC*. This sum is not the usual algebraic sum



(b) We redraw the vectors of Fig. (*a)* and relabel them in the way that we shall use from now on, namely, with an arrow over an italic symbol as, $\vec{a}, \vec{b}\ and \vec{s}$



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Vector Subtraction

- A negative sign reverses vector direction

$$\vec{b} + \left(-\vec{b}\right) = 0$$

- We use this to define vector subtraction

$$\vec{d} = \vec{a} - \vec{b} = \vec{a} + \left(-\vec{b}\right)$$ **Equation (3.1.4)**

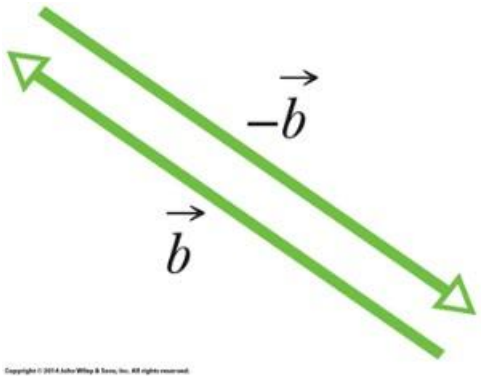


Figure (3.1.5)

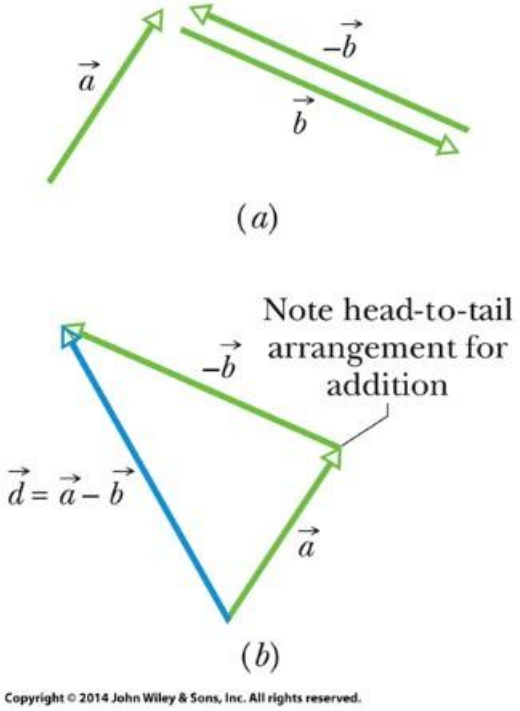
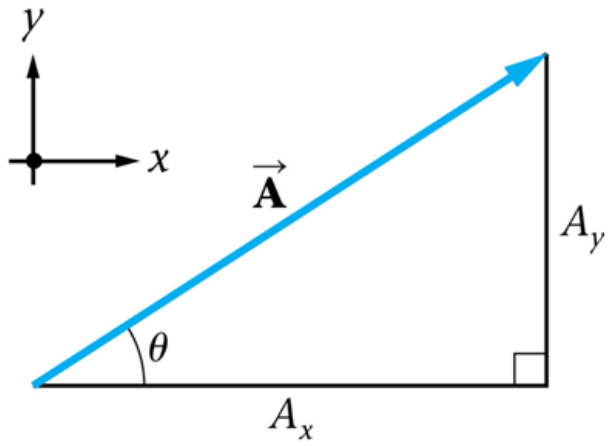


Figure (3.1.6)

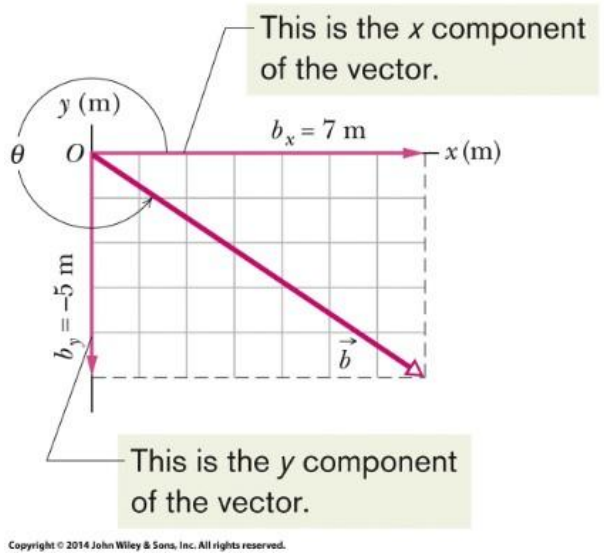
Components of Vectors

- Rather than using a graphical method, vectors can be added by **components**
  - A component is the projection of a vector on an axis



The process of finding components is called **resolving the vector**

- To describe a vector, one needs:
  - It's magnitude (e.g. length) and direction (angle), OR
  - It's components (here two components; along the x-axis and the y-axis).



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Calculating Vector Components

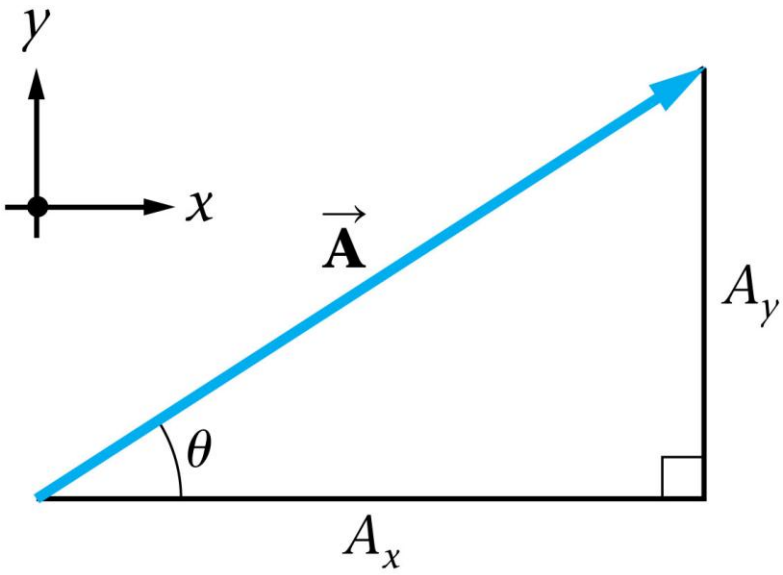
I. **Given the magnitude and the angle**, one can calculate **the components** of a vector using trigonometry. Since: $\cos\theta = A_x/A$ and $\sin\theta = A_y/A$, we get:

- $A_x = A\cos\theta$
- $A_y = A\sin\theta$

- Note that θ is the angle with the x-axis.

II. On the other hand, **given the components** one can calculate **the magnitude and the angle**:

$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$



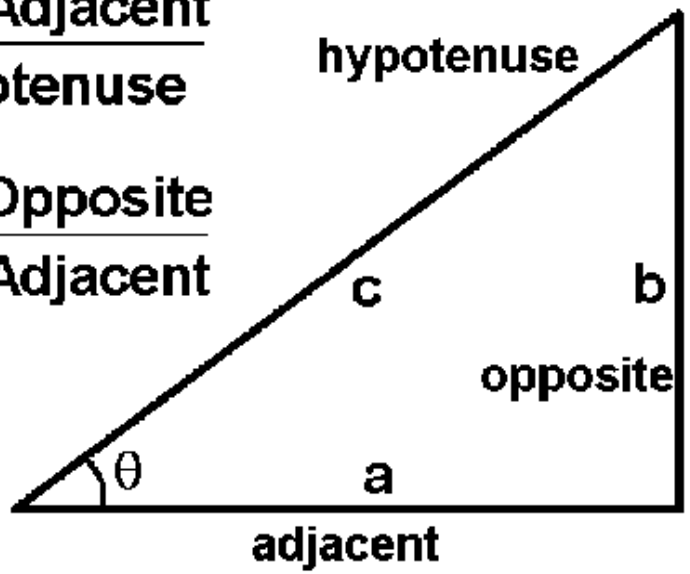
$A_x = A\cos\theta$
$A_y = A\sin\theta$

Trigonometry formula (reminder)

$$\text{SIN}\,\theta = \frac{\text{Side Opposite}}{\text{Hypotenuse}}$$

$$\text{COS}\,\theta = \frac{\text{Side Adjacent}}{\text{Hypotenuse}}$$

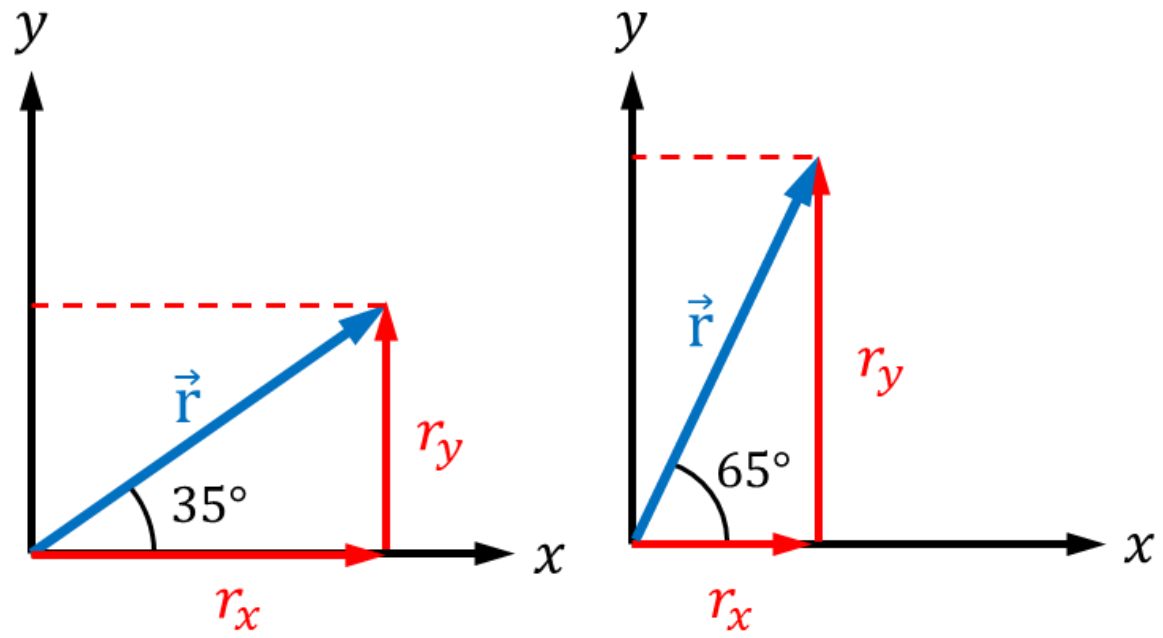
$$\text{TAN}\,\theta = \frac{\text{Side Opposite}}{\text{Side Adjacent}}$$



Pythagorean theorem $c^2 = a^2 + b^2$

Example 1: Find the x and y components of a position vector $\vec{r}$ of magnitude $r = 75$ m, if its angle relative to the x axis is (a) 35.0° and (b) 65.0°.

[Ans: (a) $r_x = 61$ m; $r_y = 43$ m
(b) $r_x = 32$ m; $r_y = 68$ m]



Solution: 1. (a) Find the x and y components: $r_x = r\cos\theta = (75\text{ m})\cos 35.0° = \boxed{61\text{ m}}$

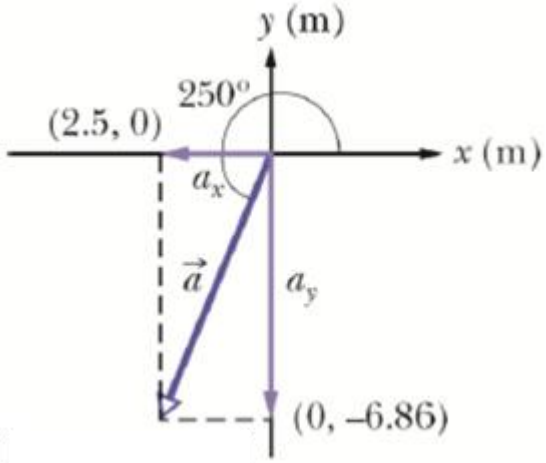
$r_y = r\sin\theta = (75\text{ m})\sin 35.0° = \boxed{43\text{ m}}$

2. (b) Find the x and y components: $r_x = r\cos\theta = (75\text{ m})\cos 65.0° = \boxed{32\text{ m}}$

$r_y = r\sin\theta = (75\text{ m})\sin 65.0° = \boxed{68\text{ m}}$

Example 2: What are (a) the *x* component and (b) the *y* component of a vector $\vec{a}$ in the *xy* plane if its direction is 250° counterclockwise from the positive direction of the *x* axis and its magnitude is 7.3 m?

[Ans: (a) $a_x = -2.50$ m; $(b)\ a_y = -6.86$ m]



(a) The x component of $\vec{a}$ is

$$a_x = a\cos\theta = (7.3\text{ m})\cos 250° = -2.5\text{ m}.$$

(b) The y component is

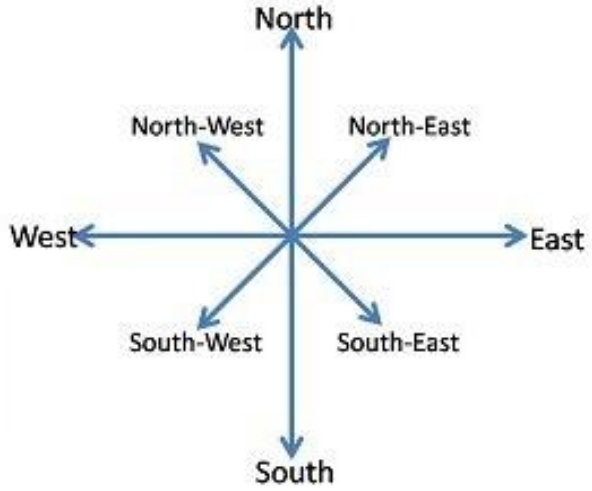
$$a_y = a\sin\theta = (7.3\text{ m})\sin 250° = -6.86\text{ m} \approx -6.9\text{ m}.$$

The results are depicted in the figure below:

$$a_x = -(7.3\text{ m})\cos 70° = -2.50\text{ m}, \quad a_y = -(7.3\text{ m})\sin 70° = -6.86\text{ m}.$$

Example 3: You drive a car 680 ft to the east, then 340 ft to the north. (a) What is the magnitude of your displacement? (b) What is the direction of your displacement?

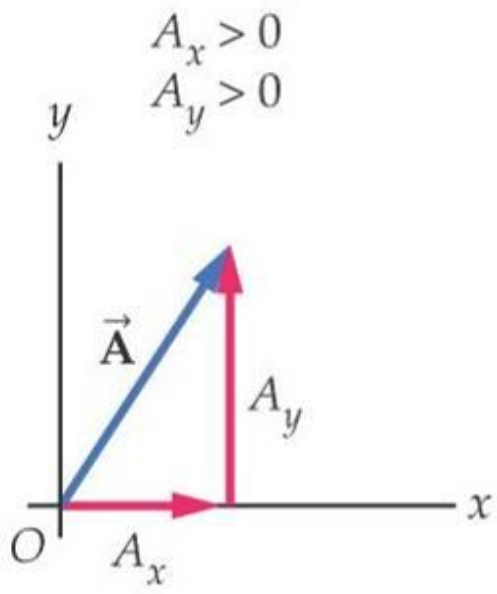
[Ans.: (a) $r = 760$ ft; (b) $\theta = 27^0$ north of east]



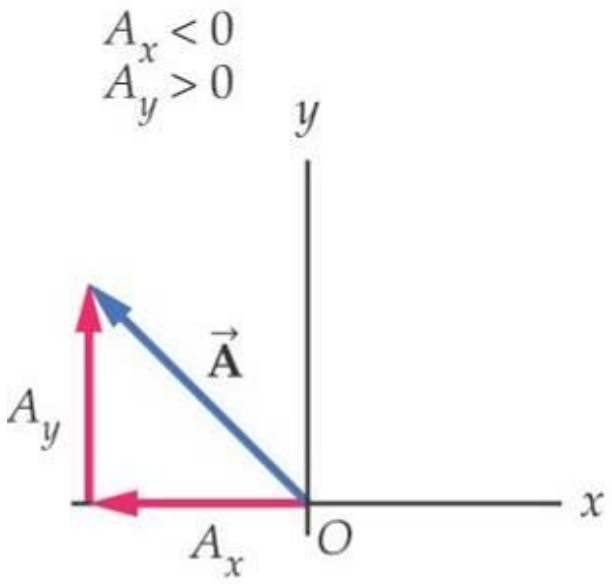
Solution: 1. (a) Find the magnitude of $\vec{\mathbf{r}}$: $r = \sqrt{(680\text{ ft})^2 + (340\text{ ft})^2} = \boxed{760\text{ ft}}$

(c) Use the inverse tangent function to find θ : $\theta = \tan^{-1}\left(\frac{340\text{ ft}}{680\text{ ft}}\right) = \boxed{27°}$ north of east

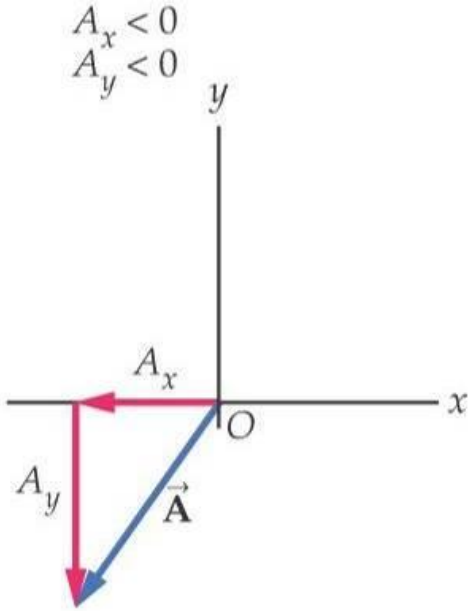
- Direction angle: The full angle measured from the positive x-axis counterclockwise. If used, the signs of the components will be determined automatically.
- We often use the acute angle with the x-axis and set the signs of the components manually. Sings of components:



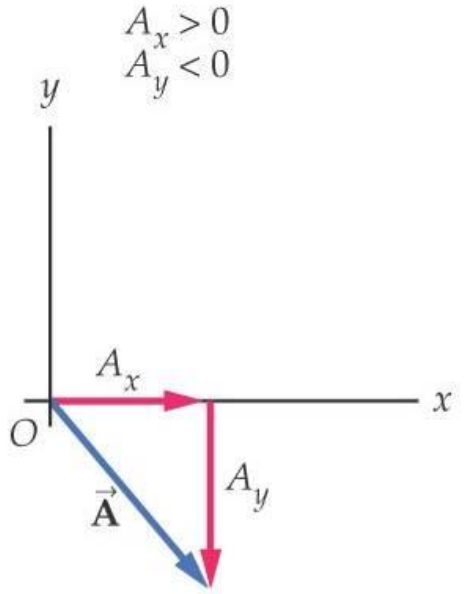
1st quadrant



2nd quadrant



3rd quadrant

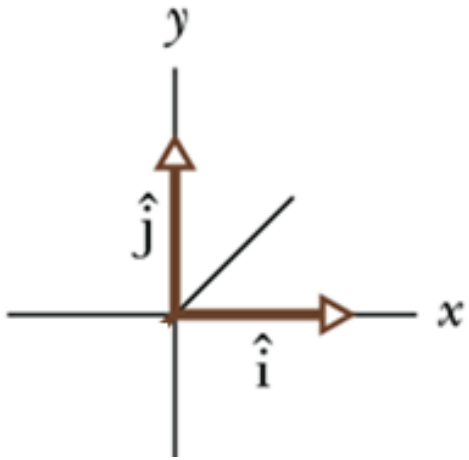


4th quadrant

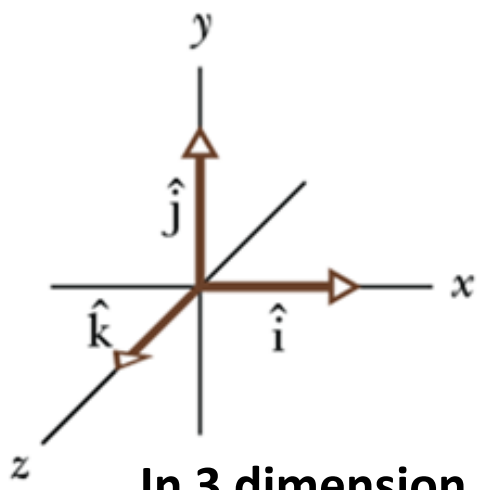
3.2 Units vectors and Adding Vectors by components

Unit Vector

- A **unit vector**
  - Has magnitude 1
  - Has a particular direction
  - Lacks both dimension and unit
  - Is labeled with a hat: ^
- We use a **right-handed coordinate system**
  - Remains right-handed when rotated



In 2 dimension



In 3 dimension

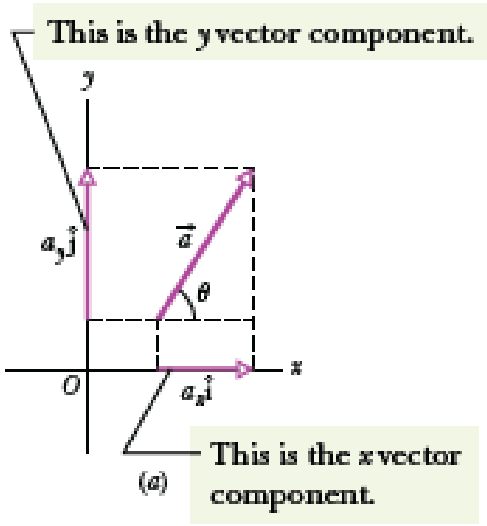
The unit vectors point along axes

- The quantities $a_x\hat{i}$ *and* $a_y\hat{j}$ are **vector components (in 2D)**

$$\vec{a} = a_x\hat{i} + a_y\hat{j} \qquad \textbf{Equation (3.2.1)}$$

$$\vec{b} = b_x\hat{i} + b_y\hat{j} \qquad \textbf{Equation (3.2.2)}$$

- The quantities a_x and a_y alone are **scalar components**
  - Or just "components" as before



Adding Vectors by Components

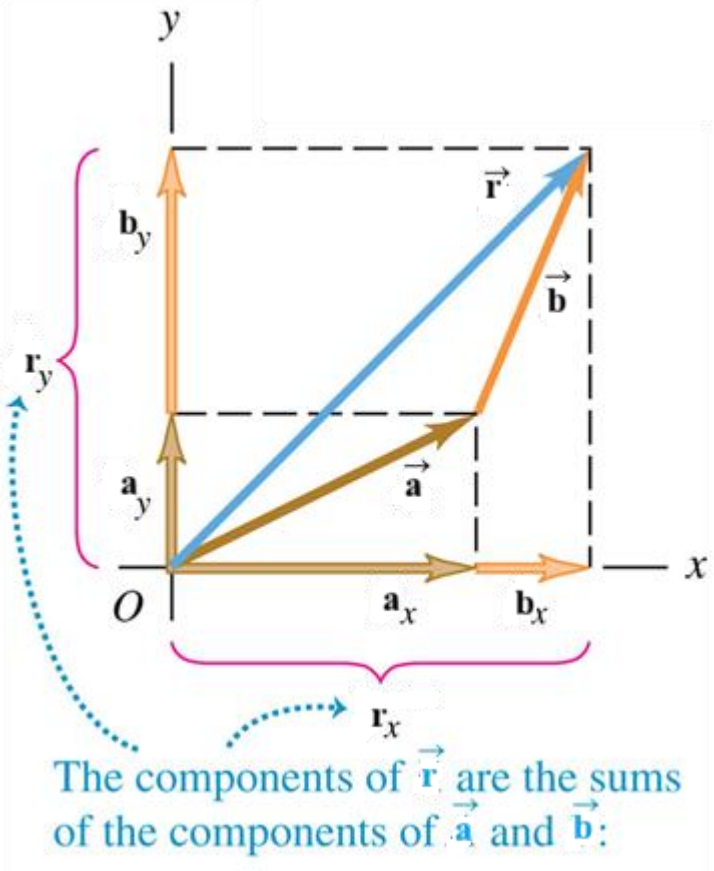
- Vectors can be added using components parallel to each coordinate axis.

In 2 dimension

$$\vec{r} = \vec{a} + \vec{b} \rightarrow \quad r_x = a_x + b_x$$
$$r_y = a_y + b_y$$

In 3 dimension

$$\vec{r} = \vec{a} + \vec{b} \rightarrow \quad r_x = a_x + b_x$$
$$r_y = a_y + b_y$$
$$r_z = a_z + b_z .$$



The components of $\vec{r}$ are the sums of the components of $\vec{a}$ and $\vec{b}$:

- To subtract two vectors, we subtract components

$$d_x = a_x - b_x, \quad d_y = a_y - b_y, \quad \text{and} \quad d_z = a_z - b_z,$$

$$\vec{d} = d_x \hat{\mathrm{i}} + d_y \hat{\mathrm{j}} + d_z \hat{\mathrm{k}} \quad \textbf{Equation (3.2.7)}$$

Vector Addition Checkpoint #3

(a) In the figure here, what are the signs of the *x* components of $\vec{d}_1$ and $\vec{d}_2$?

(b) What are the signs of the *y* components of $\vec{d}_1$ and $\vec{d}_2$?

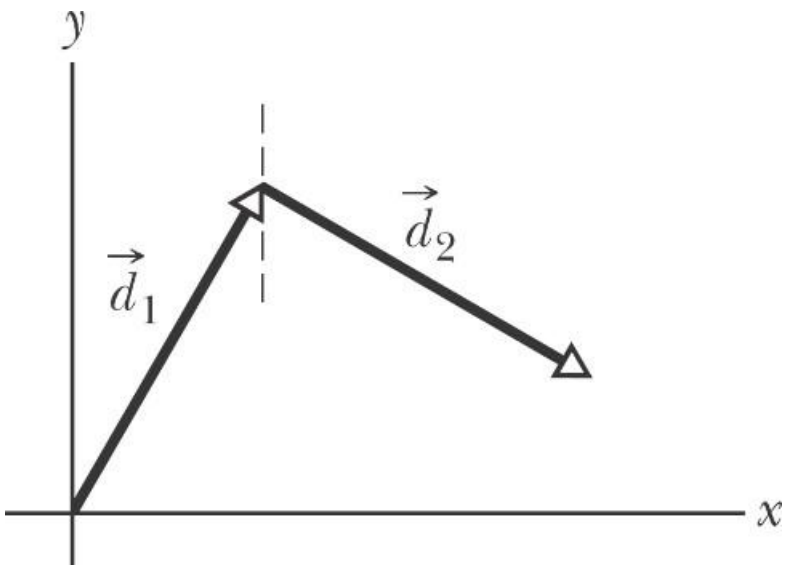
(c) What are the signs of the *x* and *y* components of $\vec{d}_1 + \vec{d}_2$?

Answer:

(a) positive, positive

(b) positive, negative

(c) positive, positive



Example 4: Given the vector $\vec{A} = -3\hat{\imath} + 4\hat{\jmath}$ m, determine its magnitude and direction angle.

[Ans.: magnitude: A = 5 m. Direction: $\theta = \tan^{-1}\left(\frac{4}{-3}\right) = -53.1°$; direction angle $= -53.1 + 180 = 126.9°$)

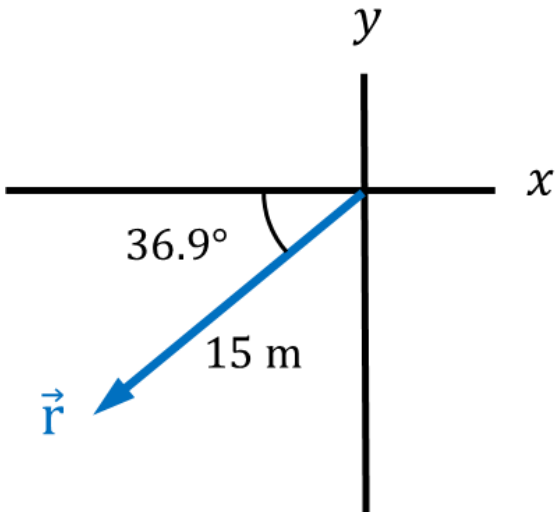
Example 5: Vector $\vec{\mathrm{r}}$ has a magnitude of 15 m and makes an angle of 36.9° below the negative x-axis. Calculate its components and express the vector in unit vector notation. [Ans.: $r_x = -12$ m; $r_y = -9$ m; $\vec{\mathrm{r}} = -12\,\hat{\imath} - 9\hat{j}$ m]

direction angle $= 36.9° + 180 = 216.9°$

$r_x = (15\mathrm{m})\cos(216.9°) = -12$ m

$r_y = (15\mathrm{m})\sin(216.9°) = -9$ m

$\vec{\mathrm{r}} = -12\,\hat{\imath} - 9\hat{j}$ m



Example 6 : For the displacement vectors

$$\vec{a} = (3.0\ m)\ \hat{\imath} + (4.0\ m)\ \hat{\jmath} \text{ and, } \vec{b} = (5.0\ m)\hat{\imath} + (-2.0\ m)\hat{\jmath},$$

give $\vec{a} + \vec{b}$ in (a) unit-vector notation, and as (b) a magnitude and (c) an angle (relative to $\hat{\imath}$). Now give $\vec{b} - \vec{a}$ in (d) unit-vector notation, and as (e) a magnitude and (f) an angle.

[Ans.: (a) $\vec{a} + \vec{b} = (8.0\ m)\ \hat{\imath} + (2.0\ m)\ \hat{\jmath}$, (b) $|\vec{a} + \vec{b}| = 8.2\ m$, (c) $\Theta = 14°$, (d) $\vec{b} - \vec{a} = (2.0\ m)\hat{\imath} - (6.0\ m)\ \hat{\jmath}$,(e) $|\vec{b} - \vec{a}| = 6.3\ m$, (f) $\Theta = -72°$]

16. (a) $\vec{a}+\vec{b} = (3.0\hat{\mathrm{i}}+4.0\hat{\mathrm{j}})\ \mathrm{m} + (5.0\hat{\mathrm{i}}-2.0\hat{\mathrm{j}})\ \mathrm{m} = (8.0\ \mathrm{m})\hat{\mathrm{i}} + (2.0\ \mathrm{m})\hat{\mathrm{j}}.$

(b) The magnitude of $\vec{a}+\vec{b}$ is

$$|\vec{a}+\vec{b}| = \sqrt{(8.0\ \mathrm{m})^2 + (2.0\ \mathrm{m})^2} = 8.2\ \mathrm{m}.$$

(c) The angle between this vector and the $+x$ direction is

$$\tan^{-1}[(2.0\ \mathrm{m})/(8.0\ \mathrm{m})] = 14°.$$

(d) $\vec{b}-\vec{a} = (5.0\hat{\mathrm{i}}-2.0\hat{\mathrm{j}})\ \mathrm{m} - (3.0\hat{\mathrm{i}}+4.0\hat{\mathrm{j}})\ \mathrm{m} = (2.0\ \mathrm{m})\hat{\mathrm{i}} - (6.0\ \mathrm{m})\hat{\mathrm{j}}.$

(e) The magnitude of the difference vector $\vec{b}-\vec{a}$ is

$$|\vec{b}-\vec{a}| = \sqrt{(2.0\ \mathrm{m})^2 + (-6.0\ \mathrm{m})^2} = 6.3\ \mathrm{m}.$$

(f) The angle between this vector and the $+x$ direction is $\tan^{-1}[(-6.0\ \mathrm{m})/(2.0\ \mathrm{m})] = -72°$. The vector is 72° *clockwise* from the axis defined by $\hat{\mathrm{i}}$.

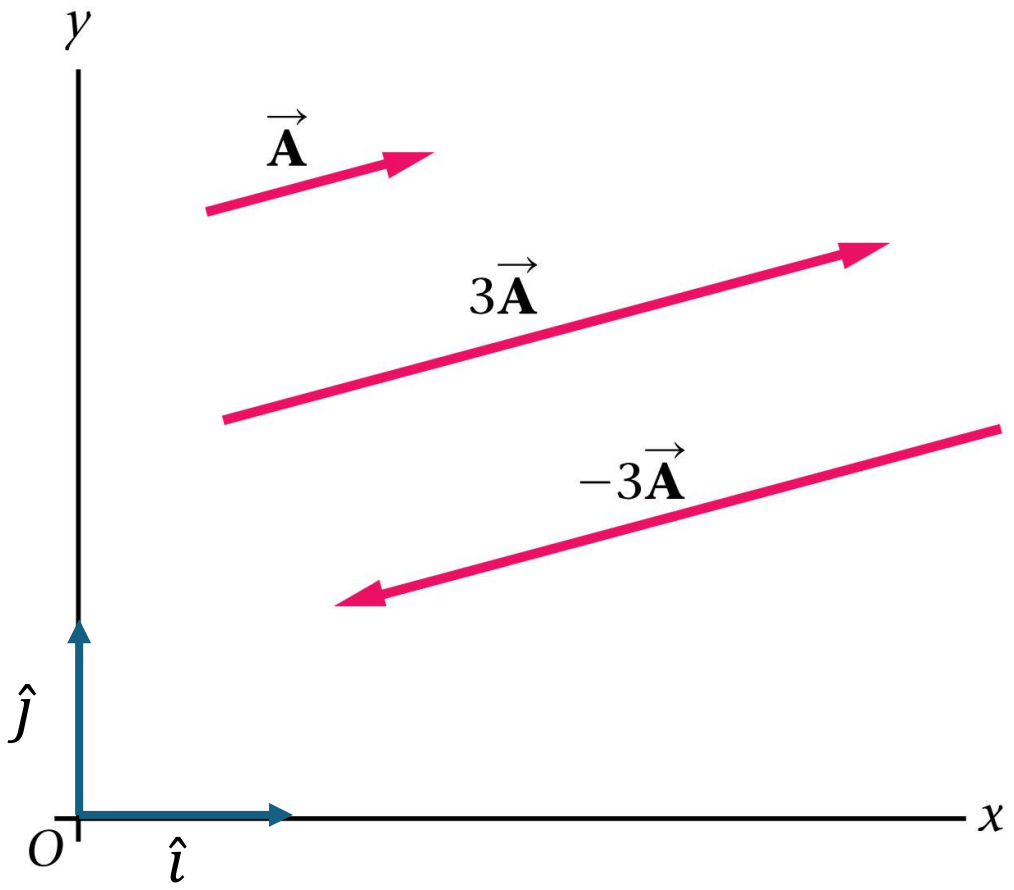
3.3 Multiplying Vectors

Multiplying a vector by scalar

- The multiplier changes the length, and the sign indicates the direction.
- In the figure:
  - The magnitude of each of $3\vec{A}$ and $-3\vec{A}$ is three times that of $\vec{A}$.
  - $3\vec{A}$ is in the same direction of $\vec{A}$ while $-3\vec{A}$ is in the opposite direction.
  - In unit vector notation:

$$3\vec{A} = 3A_x\,\hat{\imath} + 3A_y\,\hat{\jmath}$$

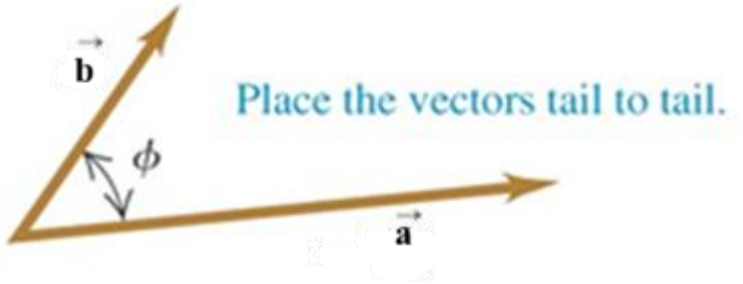
$$-3\vec{A} = -3A_x\,\hat{\imath} - 3A_y\,\hat{\jmath}$$



Scalar (or dot)Product

A scalar (or dot) product is: the product of the magnitude of one vector times the scalar component of the other vector in the direction of the first vector

$$\vec{a} \cdot \vec{b} = a\, b \cos\phi$$



- θ is the angle between the two vectors.
- The result is a scalar.
- Note that: $\hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = 1, \quad \hat{\imath} \cdot \hat{\jmath} = 0$, ……
- The following relations are valid:

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

$$\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$$

we can do the dot product in component form

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z.$$

Vector (or Cross) Product

The **cross product** of two vectors with magnitudes *a* & *b*, separated by angle θ, produces a vector with magnitude:

$$\left|\vec{a} \times \vec{b}\right| = ab \sin\theta$$

- θ is the angle between the two vectors.
- The result is a vector, perpendicular to the plane in which the two vectors exist.

$$\vec{a} \times \vec{b} = ab \sin\theta \ \hat{n}$$

- $\hat{n}$ is the unit vector perpendicular to the plane containing $\vec{a}$ and $\vec{b}$, and its direction is determined by the right-hand rule.
- For example: $\hat{\imath} \times \hat{\jmath} = \hat{k}$, $\hat{\imath} \times \hat{\imath} = 0$.

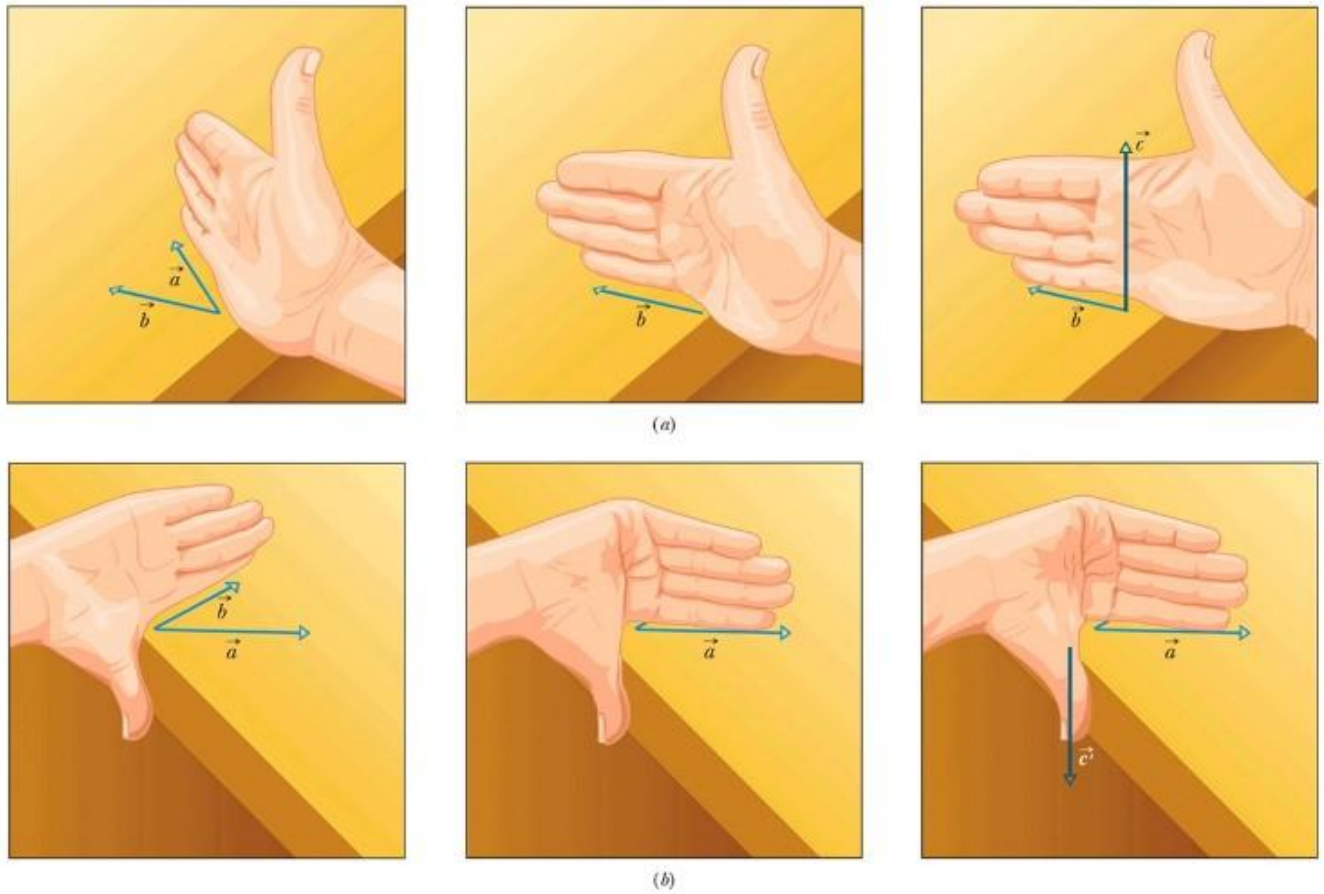
- The following relations are valid:

$$\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

$$\vec{a} \times \vec{b} = (a_y b_z - b_y a_z)\hat{\imath} + (a_z b_x - b_z a_x)\hat{\jmath} + (a_x b_y - b_x a_y)\hat{k}$$

Cross Product Direction by Right Hand Rule



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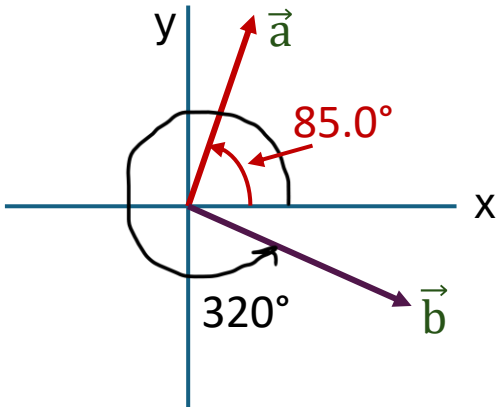
The upper shows vector $\vec{a}$ cross vector $\vec{b}$, the lower shows vector $\vec{b}$ cross vector $\vec{a}$

Example 7 : Two vectors, $\vec{a}$ and $\vec{b}$, lie in the *xy* plane. Their magnitudes are 4.50 and 7.30 units, respectively, and their directions are 320° and 85.0°, respectively, as measured counterclockwise from the positive *x* axis. What are the values of (a) $\vec{a} \cdot \vec{b}$ and (b) $\vec{a} \times \vec{b}$?

[Ans.: (a) $\vec{a} \cdot \vec{b} = -18.8$, (b) $|\vec{a} \times \vec{b}| = 26.9$]

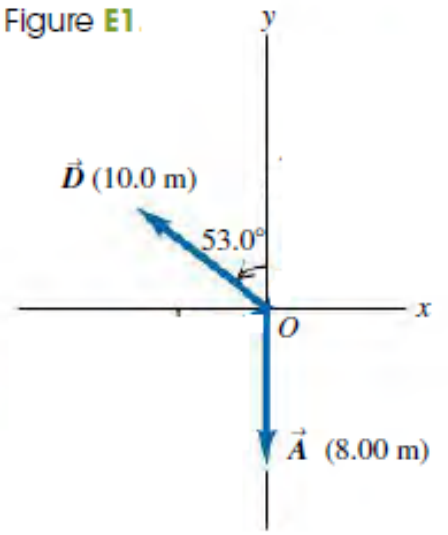


35. (a) The scalar or dot product is $(4.50)(7.30)\cos(320° - 85.0°) = -18.8$.

(b) The vector or cross product is in the $\hat{k}$ direction (by the right-hand rule) with magnitude $|(4.50)(7.30)\sin(320° - 85.0°)| = 26.9$.

Example 8 : For the two vectors $\vec{A}$ and $\vec{D}$ n Fig. E1, find the magnitude and direction of (a) the vector product $\vec{A} \times \vec{D}$

Figure E1



$\vec{A} \times \vec{D}$ has magnitude $AD\sin\phi$. Its direction is given by the right-hand rule.

$\phi = 180° - 53° = 127°$

$|\vec{A} \times \vec{D}| = (8.00\text{ m})(10.0\text{ m})\sin 127° = 63.9\text{ m}^2$. The right-hand rule says $\vec{A} \times \vec{D}$ is in the $-z$-direction (into the page).

Summary of Chapter 3

- Scalar: number, with appropriate units.
- Vector: quantity with magnitude and direction.
- Vector components: $A_x = A \cos \theta$, $B_y = B \sin \theta$
- Magnitude: $A = \sqrt{A_x^2 + Ay^2}$
- Direction: $\theta = \tan^{-1} (A_y / A_x)$
- To add vectors, add components of individual vectors, then find magnitude and direction.

Summary of Chapter 3

- Obeys commutative and associative laws

$$\vec{a}+\vec{b}=\vec{b}+\vec{a} \quad (\text{commutative law})$$

- **Equation (3.1.2)**

$$\left(\vec{a}+\vec{b}\right)+\vec{c}=\vec{a}+\left(\vec{b}+\vec{c}\right) \quad (\text{associative law})$$

Equation (3.1.3)

Unit Vector Notation: We can write vectors in terms of unit vectors

$$\vec{a}=a_x\hat{\mathrm{i}}+a_y\hat{\mathrm{j}}+a_z\hat{\mathrm{k}}$$

Equation (3.2.1)

Summary of Chapter 3

Adding by Components:

$$\vec{r} = \vec{a} + \vec{b} \rightarrow \quad r_x = a_x + b_x \qquad \textbf{Equation (3.2.4)}$$

$$r_y = a_y + b_y \qquad \textbf{Equation (3.2.5)}$$

$$r_z = a_z + b_z. \qquad \textbf{Equation (3.2.6)}$$

Dot (Scalar) Product:

$$\vec{a} \cdot \vec{b} = ab\cos\phi \qquad \textbf{Equation (3.3.1)}$$

$$\vec{a} \cdot \vec{b} = \left(a_x \hat{\mathrm{i}} + a_y \hat{\mathrm{j}} + a_z \hat{\mathrm{k}}\right) \cdot \left(b_x \hat{\mathrm{i}} + b_y \hat{\mathrm{j}} + b_z \hat{\mathrm{k}}\right) \qquad \textbf{Equation (3.3.3)}$$

Cross (Vector) Product:

$$c = ab\sin\phi,$$

Additional Exercises

EXERCISE 3–3

The vector $\vec{\mathbf{B}}$ has components $B_x = -2.10\ \text{m}$ and $B_y = -1.70\ \text{m}$. Find the direction angle, θ, for this vector.

SOLUTION

$$\tan^{-1}[(-1.70\ \text{m})/(-2.10\ \text{m})] = \tan^{-1}(1.70/2.10) = 39.0°,\ \theta = 39.0 + 180° = 219°$$

EXERCISE 3–4

If a vector's direction angle relative to the x axis is 35°, then its direction angle relative to the y axis is 55°. Find the components of a vector $\vec{\mathbf{A}}$ of magnitude 5.2 m in terms of

a. its direction relative to the x axis, and

b. its direction relative to the y axis.

SOLUTION

a. $A_x = (5.2\ \text{m})\cos 35° = 4.3\ \text{m},\ A_y = (5.2\ \text{m})\sin 35° = 3.0\ \text{m}$

b. $A_x = (5.2\ \text{m})\sin 55° = 4.3\ \text{m},\ A_y = (5.2\ \text{m})\cos 55° = 3.0\ \text{m}$

Additional Exercises

32. • Find the direction and magnitude of the vectors.

(a) $\vec{A} = (25\text{ m})\hat{x} + (-12\text{ m})\hat{y}$,

(b) $\vec{B} = (2.0\text{ m})\hat{x} + (15\text{ m})\hat{y}$, and **(c)** $\vec{A} + \vec{B}$.

33. • For the vectors given in Problem 32, express **(a)** $\vec{A} - \vec{B}$ and **(b)** $\vec{B} - \vec{A}$ in unit vector notation.

34. • Express each of the vectors in **Figure 3–38** in unit vector notation.

35. •• Referring to the vectors in Figure 3–38, express the sum $\vec{A} + \vec{B} + \vec{C}$ in unit vector notation.

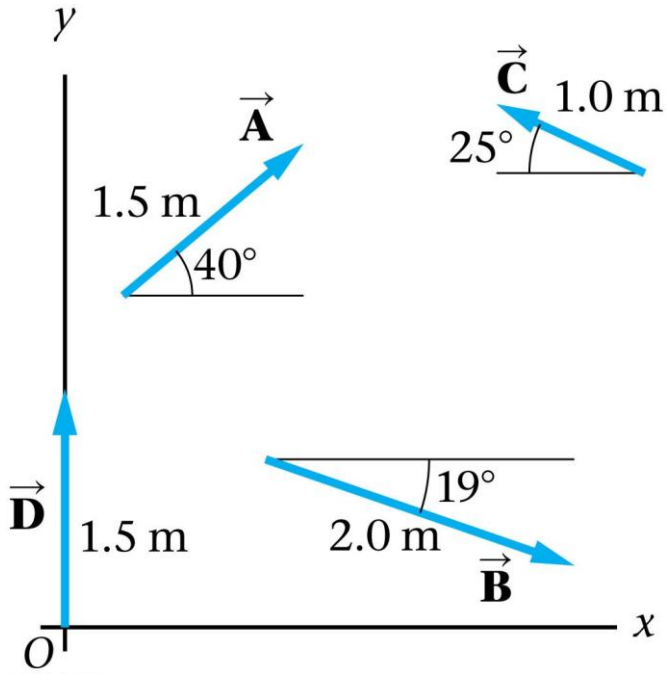
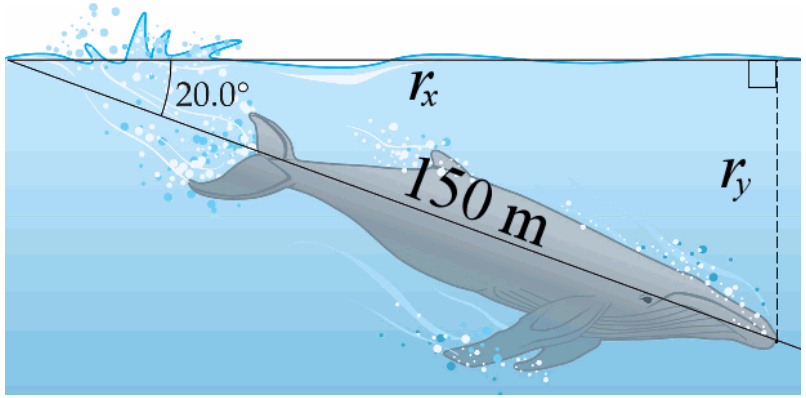


Figure 3—38

Example 4: A whale comes to the surface to breathe and then dives at an angle of 20.0° below the horizontal. If the whale continues in a straight line for 150 m, (a) how deep is it, and (b) how far has it travelled horizontally?



[Ans.: (a) 51 m; (b) 141 m]

30 M GO Here are two vectors:

$$\vec{a} = (4.0\ \text{m})\hat{i} - (3.0\ \text{m})\hat{j} \text{ and } \vec{b} = (6.0\ \text{m})\hat{i} + (8.0\ \text{m})\hat{j}.$$

What are (a) the magnitude and (b) the angle (relative to $\hat{i}$) of $\vec{a}$? What are (c) the magnitude and (d) the angle of $\vec{b}$? What are (e) the magnitude and (f) the angle of $\vec{a} + \vec{b}$; (g) the magnitude and (h) the angle of $\vec{b} - \vec{a}$; and (i) the magnitude and (j) the angle of $\vec{a} - \vec{b}$? (k) What is the angle between the directions of $\vec{b} - \vec{a}$ and $\vec{a} - \vec{b}$?

30. Many of the operations are done efficiently on most modern graphical calculators using their built-in vector manipulation and rectangular ↔ polar "shortcuts." In this solution, we employ the "traditional" methods (such as Eq. 3.1.6).

(a) The magnitude of $\vec{a}$ is $a = \sqrt{(4.0\ \text{m})^2 + (-3.0\ \text{m})^2} = 5.0\ \text{m}$.

(b) The angle between $\vec{a}$ and the $+x$ direction is $\tan^{-1}[(-3.0\ \text{m})/(4.0\ \text{m})] = -37°$. The vector is 37° *clockwise* from the axis defined by $\hat{i}$.

(c) The magnitude of $\vec{b}$ is $b = \sqrt{(6.0\ \text{m})^2 + (8.0\ \text{m})^2} = 10\ \text{m}$.

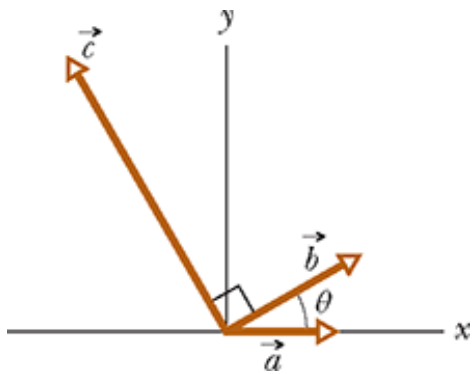
(d) The angle between $\vec{b}$ and the $+x$ direction is $\tan^{-1}[(8.0\ \text{m})/(6.0\ \text{m})] = 53°$.

(e) $\vec{a} + \vec{b} = (4.0\ \text{m} + 6.0\ \text{m})\hat{i} + [(-3.0\ \text{m}) + 8.0\ \text{m}]\hat{j} = (10\ \text{m})\hat{i} + (5.0\ \text{m})\hat{j}$. The magnitude of this vector is $|\vec{a} + \vec{b}| = \sqrt{(10\ \text{m})^2 + (5.0\ \text{m})^2} = 11\ \text{m}$; we round to two significant figures in our results.

(f) The angle between the vector described in part (e) and the $+x$ direction is $\tan^{-1}[(5.0\ \text{m})/(10\ \text{m})] = 27°$.

(g) $\vec{b} - \vec{a} = (6.0\ \text{m} - 4.0\ \text{m})\hat{i} + [8.0\ \text{m} - (-3.0\ \text{m})]\hat{j} = (2.0\ \text{m})\hat{i} + (11\ \text{m})\hat{j}$. The magnitude of this vector is $|\vec{b} - \vec{a}| = \sqrt{(2.0\ \text{m})^2 + (11\ \text{m})^2} = 11\ \text{m}$, which is, interestingly, the same result as in part (e) (exactly, not just to 2 significant figures). This curious coincidence is made possible by the fact that $\vec{a} \perp \vec{b}$.

43 M SSM The three vectors in Fig. 3.13 have magnitudes a = 3.00 m, b = 4.00 m, and c = 10.0 m and angle θ= 30.0°. What are (a) the x component and (b) the y component of $\vec{a}$; (c) the x component and (d) the y component of $\vec{b}$; and (e) the x component and (f) the y component of $\vec{c}$?



ANALYZE (a) The x component of $\vec{a}$ is $a_x = a\cos 0° = a = 3.00$ m.

(b) Similarly, the y component of $\vec{a}$ is $a_y = a\sin 0° = 0$.

(c) The x component of $\vec{b}$ is $b_x = b\cos 30° = (4.00\text{ m})\cos 30° = 3.46$ m.

(d) The y component is $b_y = b\sin 30° = (4.00\text{ m})\sin 30° = 2.00$ m.

(e) The x component of $\vec{c}$ is $c_x = c\cos\ 120° = (10.0\text{ m})\cos 120° = -5.00$ m.

(f) The y component is $c_y = c\sin 30° = (10.0\text{ m})\sin 120° = 8.66$ m.