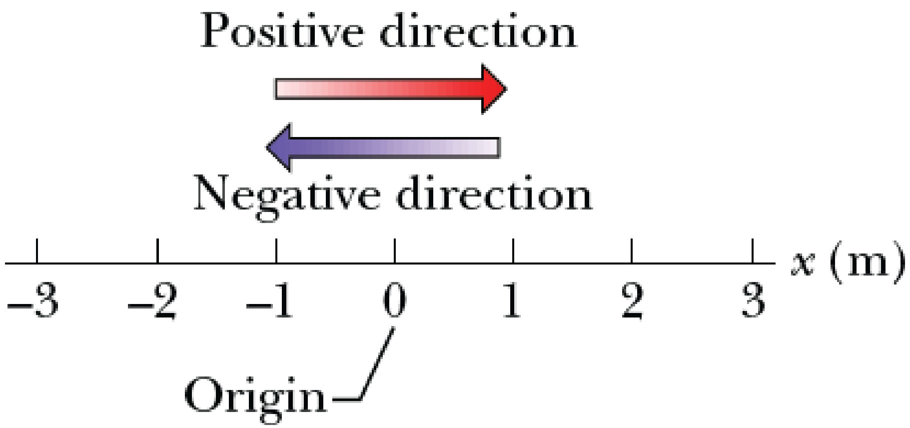
# Chapter 2

# Motion Along a Straight Line

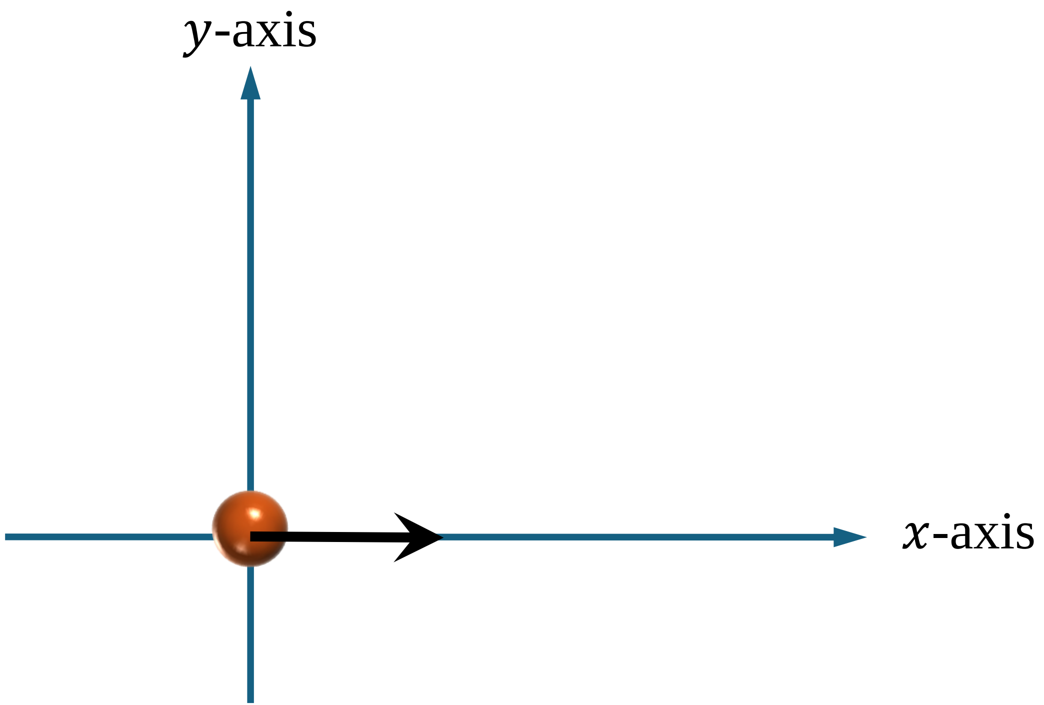


# Units of Chapter 2

- 2.1. Position, Displacement and Average Velocity
- 2.2. Instantaneous Velocity and Speed
- 2.3. Acceleration
- 2.4. Constant Acceleration
- 2.5. Free-Fall Acceleration

# 2.1. Position, Displacement, and Average Velocity

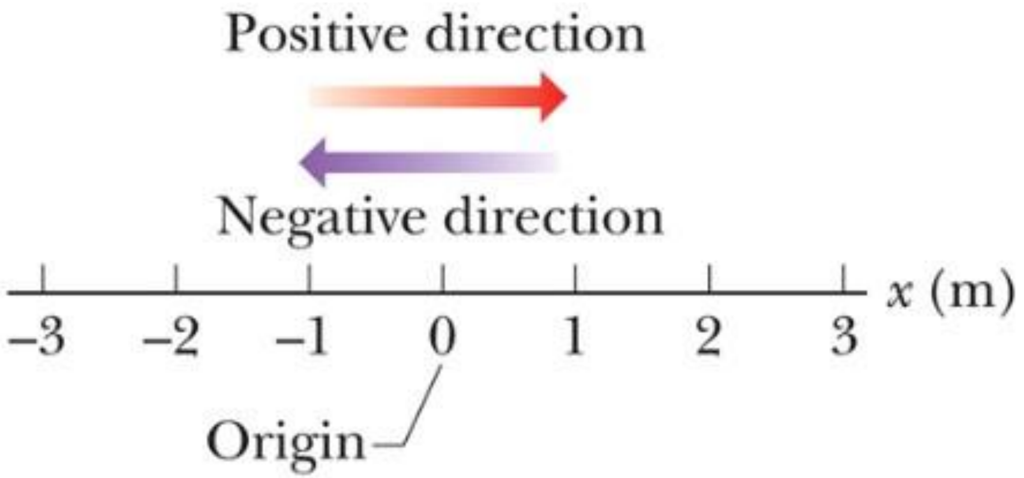
- The world, and everything in it, moves. Even seemingly stationary things, such as a roadway, move with Earth's rotation, Earth's orbit around the Sun.. The classification and comparison of motions (called **kinematics**) is often challenging.
- The motion is along a straight line only. The line may be vertical, horizontal, or slanted, but it must be straight.
- Any object moving along a straight line can be considered as moving along the $x$-axis (or the $y$-axis!).



# 2.1. Position, Displacement, and Average Velocity

## Position and Displacement

- Before describing motion, you must set up a coordinate system—define an origin (reference) and a positive direction. The **position ($x$)** of an object can then be determined with respect to the reference point.
- Position is a vector quantity. A vector quantity in 1D is represented using a number (magnitude) with a sign (direction). Position can be positive, zero or negative



# 2.1. Position, Displacement, and Average Velocity

## Position and Displacement

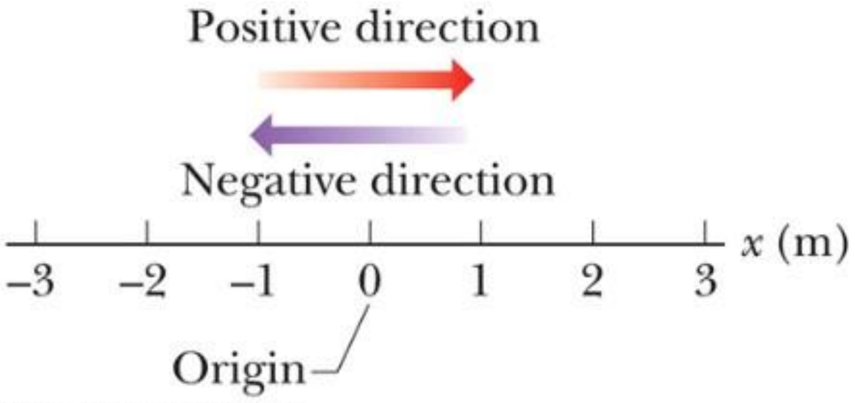
- **Distance ($d$)** is the total length of travel
  - Distance is always a positive scalar

- **Displacement ($\Delta x$)** is the change in position.

displacement = final position – initial position

$$\boxed{\Delta x = x_{\mathrm{f}} - x_{\mathrm{i}}}$$

  - Displacement is a vector quantity.



# 2.1. Position, Displacement, and Average Velocity

## Position and Displacement

**Examples:** A particle moves . . .

- From $x = 5$ m to $x = 12$ m: $\Delta x = 7$ m (positive direction)
- From $x = 5$ m to $x = 1$ m: $\Delta x = -4$m (negative direction)
- From $x = 5$ m to $x = 200$ m to $x = 5$ m: $\Delta x = 0$ m

- The actual distance covered is irrelevant

**Example:** From $x = 5$ m to $x = 200$ m to $x = 5$ m:

Distance covered: 390 m. however, the displacement $\Delta x = 0$ m

# 2.1. Position, Displacement, and Average Velocity

## Position and Displacement

- Remark: Displacement is not distance. The **displacement** of an object **is not the same as the distance** it travels

**Example:** Throw a ball straight up and then catch it at the same point where you released it.

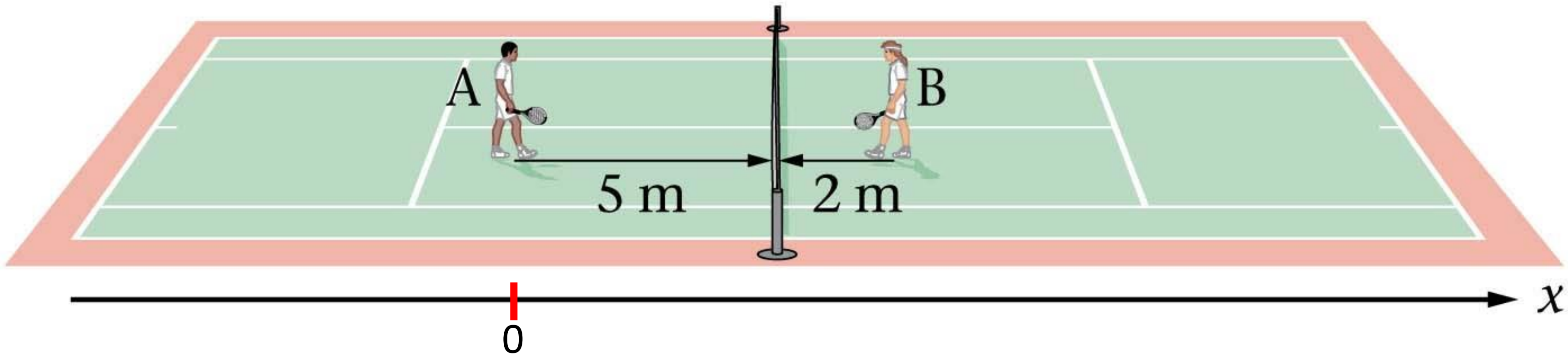
- The distance traveled is twice the maximum height (*distance is always a positive scalar!*)
- The displacement is zero (*displacement can be positive, negative, or zero!*)

# 2.1. Position, Displacement, and Average Velocity

**Example 1:** The two tennis players walk to the net to congratulate one another.

a) Find the distance traveled and the displacement of player A.

b) Repeat for player B.



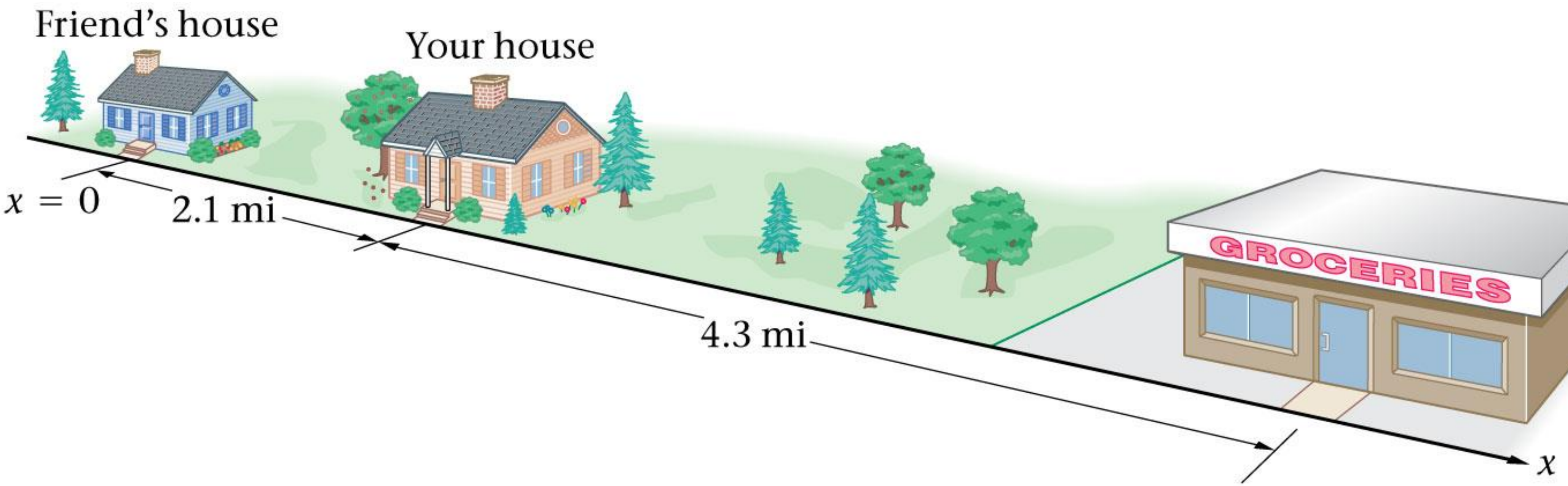
# 2.1. Position, Displacement, and Average Velocity

**Example:** Find the distance traveled and the displacement if you drive:

a) from your house to the grocery store and then to your friend's house.

b) from your house to the grocery store and then back to your house.

[Ans: (a) 10.7 mi, –2.1 mi (2.1 mi to the left) (b). 8.6 mi, 0 mi]



# 2.1. Position, Displacement, and Average Velocity

## Average Velocity and Average Speed

- **Average velocity** is the displacement traveled per unit time

$$\text{Average velocity} = \frac{\text{displacement}}{\text{elapsed time}}$$

$$\boxed{v_{\text{av}} = \frac{\Delta x}{\Delta t}}$$

  - The SI unit: m/s, vector quantity.

- **Average speed** is the ratio of the total distance covered to the time interval in which the distance was covered .

$$\text{Average speed} = \frac{\text{Total distance}}{\text{total time}}$$

# 2.1. Position, Displacement, and Average Velocity

- **Average speed** is the distance traveled per unit time:

  - The SI unit: m/s, scalar quantity.

**Speed vs. Velocity**

**Average speed** is a measurement of the total distance traveled over the time interval.

| Velocity | Speed |
|---|---|
| defined by both magnitude and direction | defined by magnitude, only |
| can be positive or negative | only positive |
| sign determines direction of the motion | sign is always positive; hence, direction is ignored |

# 2.1. Position, Displacement, and Average Velocity

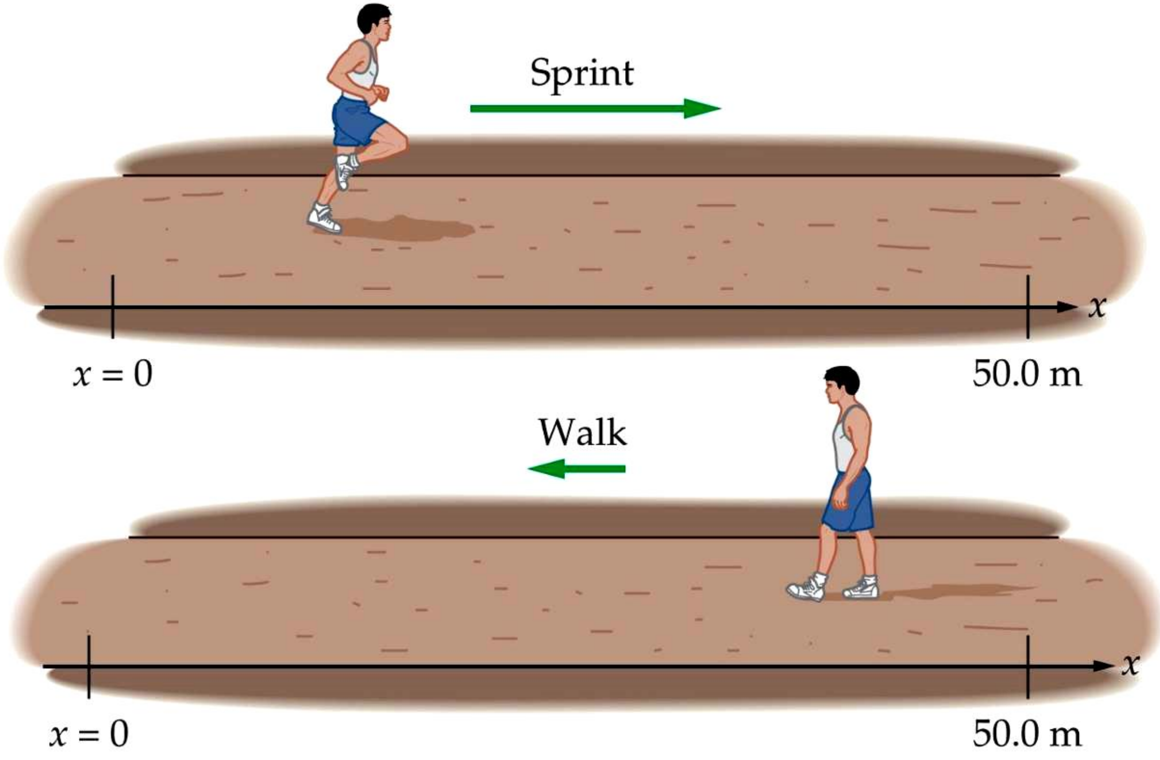
**Example 3:** An athlete sprints 50.0 m in 8.00 s, stops, and then walks slowly back to the starting line in 40.0 s. If the "sprint direction" is taken to be positive, what are

(a) the average sprint velocity

(b) the average walking velocity

(c) the average velocity for the complete round trip?

[Ans: (a) 6.25 m/s, (b) −1.25 m/s (c) 0 ]



# 2.2. instantaneous Velocity

## Instantaneous Velocity

The *instantaneous velocity* is the velocity at a specific instant of time or specific point along the path.

The velocity at any instant is obtained from the average velocity by shrinking the time interval $\Delta t$ closer and closer to 0; as that time becomes infinitesimally small. As $\Delta t$ dwindles, the average velocity approaches a limiting value, which is the velocity at that time

The **instantaneous** ***x*-velocity** of a particle in **straight-line motion** ...

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

... equals the limit of the particle's average *x*-velocity as the time interval approaches zero ...

... and equals the instantaneous rate of change of the particle's *x*-coordinate.

In the language of calculus, the limit of $\frac{\Delta x}{\Delta t}$ as $t$ approaches zero is called the **derivative** of $x$ with respect to $t$ and is written $\frac{dx}{dt}$.

The derivative of a power function $x^n$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

# 2.2. Instantaneous Velocity

**Example :** (a) If a particle's position is given by $x = 4 - 12t + 3t^2$ (where $t$ is in seconds and $x$ is in meters), what is its velocity at $t = 1$ s?

(b) Is it moving in the positive or negative direction of $x$ just then?

(c) What is its speed just then?

# 2.2. Instantaneous Velocity

**Example 1:** A car is stopped at a traffic light. It then travels along a straight road such that its distance from the light is given by

$$x(t) = (2.40\ m/s^2)t^2 - (0.120\ m/s^3)t^3$$

(a) Calculate the average velocity of the car for the time interval $t$ = 0 to $t$ = 10.0 s

(b) Calculate the instantaneous velocity of the car at $t$ = 0, $t$ = 5.0 s, and $t$ = 10.0 s

# 2-3 Acceleration

**Average acceleration**

- Average acceleration is the rate of change of *velocity* with time

$$\boxed{a_{\text{av}} = \frac{\Delta v}{\Delta t} = \frac{v_{\text{f}} - v_{\text{i}}}{t_{\text{f}} - t_{\text{i}}}}$$

  - SI unit: $m/s^2$, vector quantity.

**Example 4:** An airplane has an average acceleration of 5.6 $m/s^2$. Starting from rest, how long does it take for the plane to reach a speed of 150 km/h?

[Ans: $t$ = 7.44 s]

$$a_{avr} = \frac{v_f - v_i}{time}$$

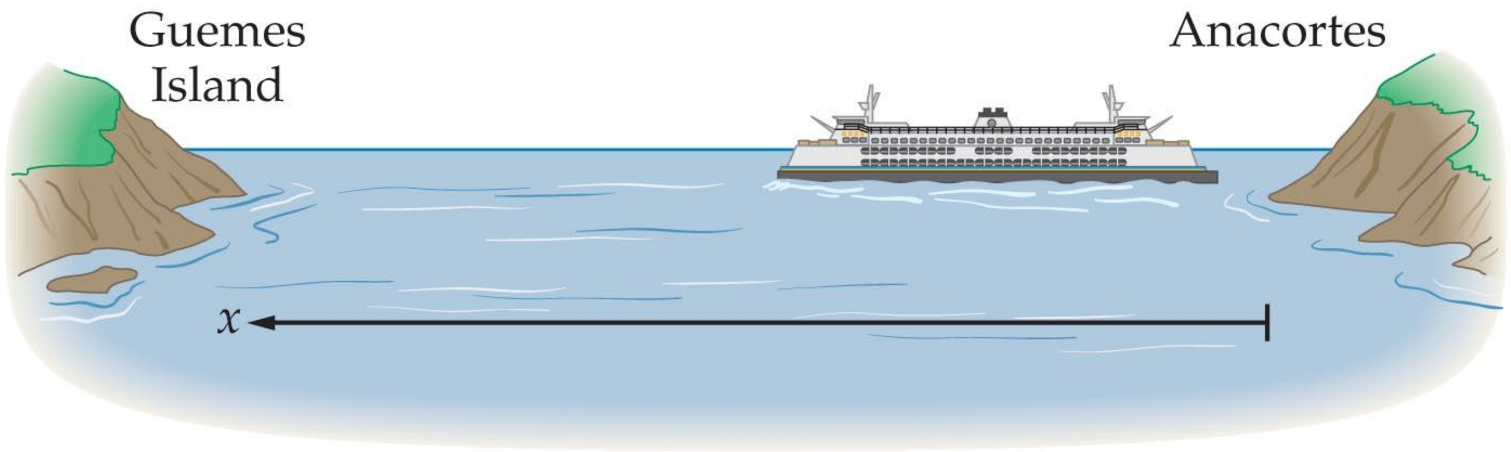
$$time = \frac{v_f - v_i}{a_{avr}} = \frac{150 \times (\quad 1000 \quad /3600)}{5.6}$$

$$time = \text{:} \quad 7.44 \qquad sec$$

# 2.3. Acceleration

**Example 5:** As a ferry approaches Guemes Island (traveling in the $+x$ direction), its speed is 7.4 m/s. (a) If the ferry slows to a stop in 12.3 s, what is its average acceleration? (b) As the ferry returns to the Anacortes dock, its speed is 7.3 m/s. If it comes to rest in 13.1 s, what is its average acceleration?

[Ans: (a) $-0.6\ \text{m/s}^2$, (b) $0.56\ \text{m/s}^2$]



# 2.3. Acceleration

**Example 6:** A train moving in a straight line with an initial velocity of 0.5 m/s accelerates at 2.0 m/s$^2$ for 2.0 s, coasts with zero acceleration for 3.0 s, and then accelerates at $-1.5$ m/s$^2$ for 1.0 s.

(a) What is the final velocity of the train?

(b) What is the average acceleration of the train? [Ans: (a) 3.0 m/s, (b) 0.42 m/s$^2$]

# 2.3. Acceleration

## Instantaneous acceleration

The instantaneous acceleration equals the derivative of the velocity with respect to time,

The instantaneous $x$-acceleration of a particle in straight-line motion ...

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \tag{2.5}$$

... equals the limit of the particle's average $x$-acceleration as the time interval approaches zero ...

... and equals the instantaneous rate of change of the particle's $x$-velocity.

- Combining Equations. 2.3.2 and 2.2.1:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

# 2.3. Acceleration

**Example 1:** Suppose the *x*-velocity $v_x$ of the car in Figure at any time *t* is given by the equation:

$$v_x = 60\frac{m}{s} + (0.50\ m/s^2)t^2$$

(a) Find the change in *x*-velocity of the car in the time interval $t_1$ = 1.0 s to $t_2$ = 3.0 s.

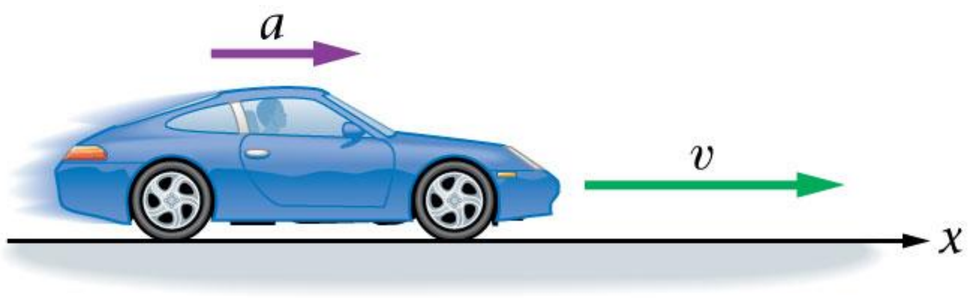
(b) Find the average *x*-acceleration in this time interval.

(c) Find the instantaneous *x*-acceleration at time $t_1$ = 1.0 s by taking *t* to be first 0.1 s, then 0.01 s, then 0.001 s.

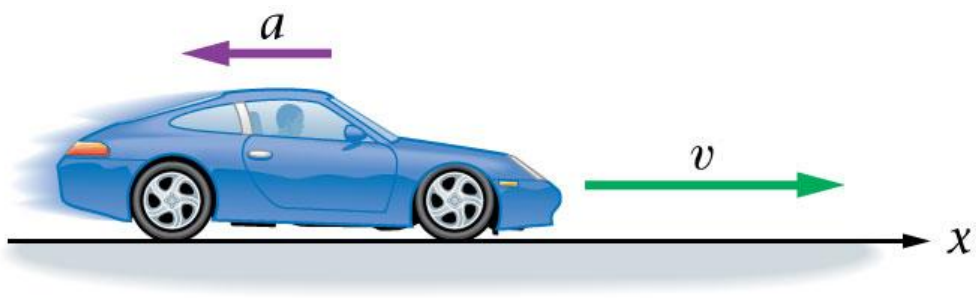
(d) Derive an expression for the instantaneous *x*-acceleration as a function of time, and use it to find *ax* at *t* = 1.0 s and *t* = 3.0 s.

# 2.3. Acceleration

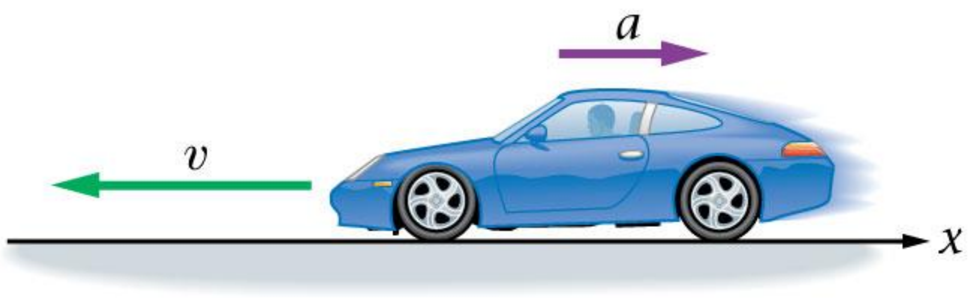
- Acceleration (increasing speed) and deceleration (decreasing speed) should not be confused with the directions of velocity and acceleration:



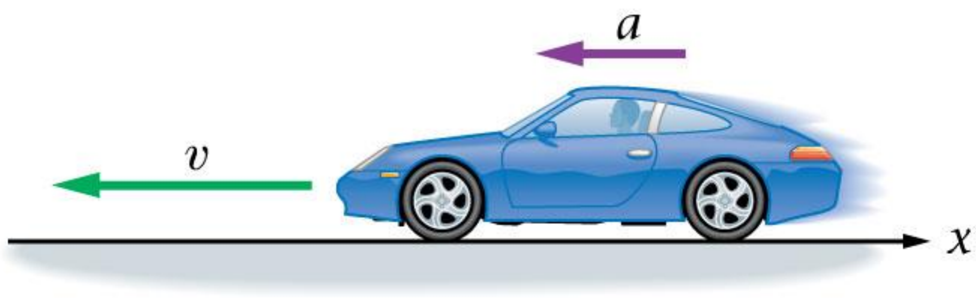

**(a)** Speed increases




**(b)** Speed decreases




**(c)** Speed decreases

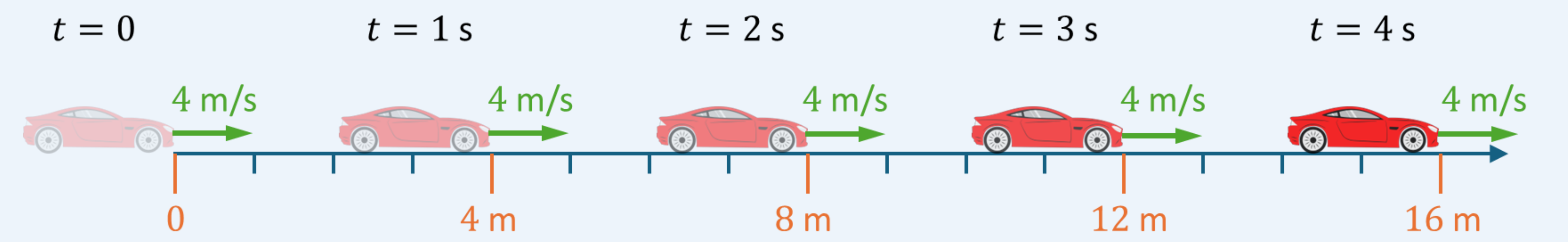



**(d)** Speed increases

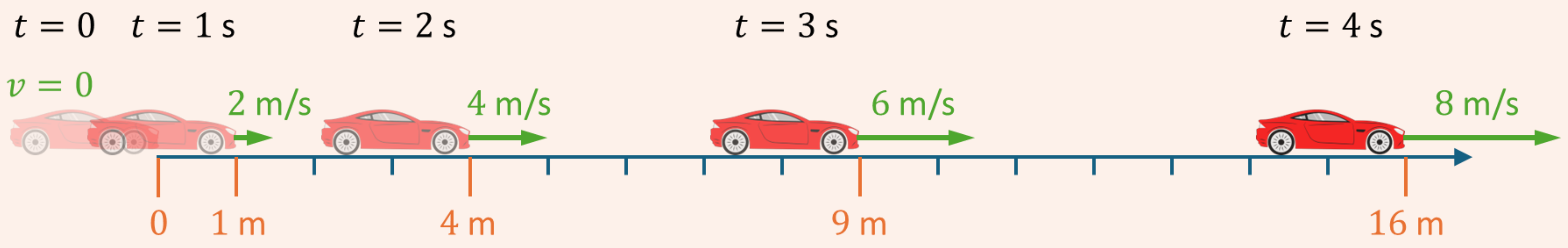
# 2.3. Acceleration

- When the velocity and acceleration of an object have the same sign (they point in the same direction), the *speed increases*.
- When the velocity and acceleration of an object have opposite signs (they point in opposite directions), the *speed decreases*.
- When a particle's speed decreases, it is often said to be decelerating.
- Deceleration can be caused by a positive or a negative acceleration, depending on the direction of the initial velocity.

# 2.4. Constant Acceleration




Constant velocity, $a = 0$




Constant acceleration, $a = +2\ \mathrm{m/s^2}$

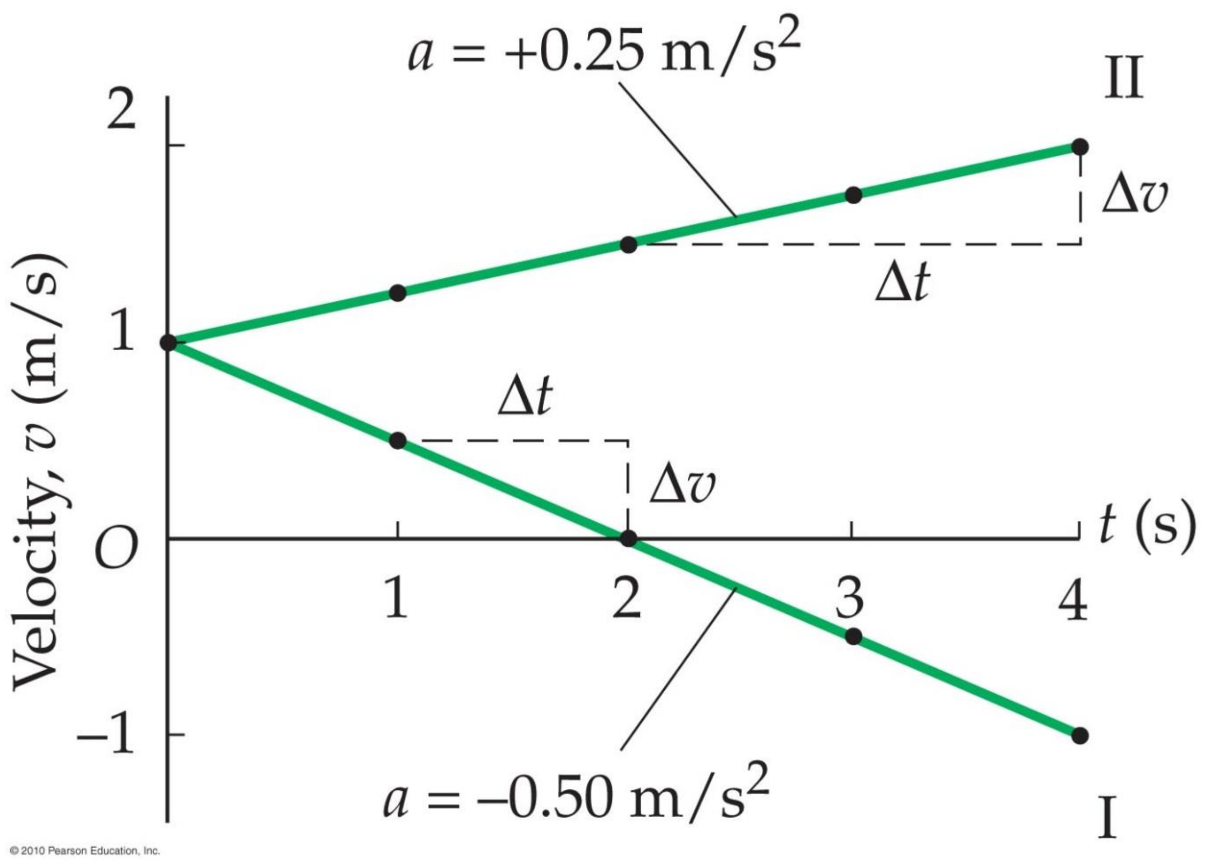
# 2.4. Constant Acceleration

- In case of constant acceleration, the plot of $v$ vs. $t$ will show a straight line. The slope of the line is determined by the acceleration.

**Exercise:**

(a) Describe how the velocity changes in magnitude and direction for curves I and II.

(b) Sketch a curve that represents a constant velocity of 1 m/s. What's the value of acceleration in this case?



# 2.4. Constant Acceleration

- If the acceleration is constant, the velocity changes linearly.
- One can relate the different variables together:

| Equation Number | Equation | Missing Quantity |
|---|---|---|
| 2.4.1 | $v = v_0 + at$ | $x - x_0$ |
| 2.4.5 | $x - x_0 = v_0 t + \frac{1}{2} a t^2$ | $v$ |
| 2.4.6 | $v^2 = v_0^2 + 2a\left(x - x_0\right)$ | $t$ |
| 2.4.7 | $x - x_0 = \frac{1}{2}\left(v_0 + v\right)t$ | $a$ |

***a* Make sure that the acceleration is indeed constant before using the equations in this table.**

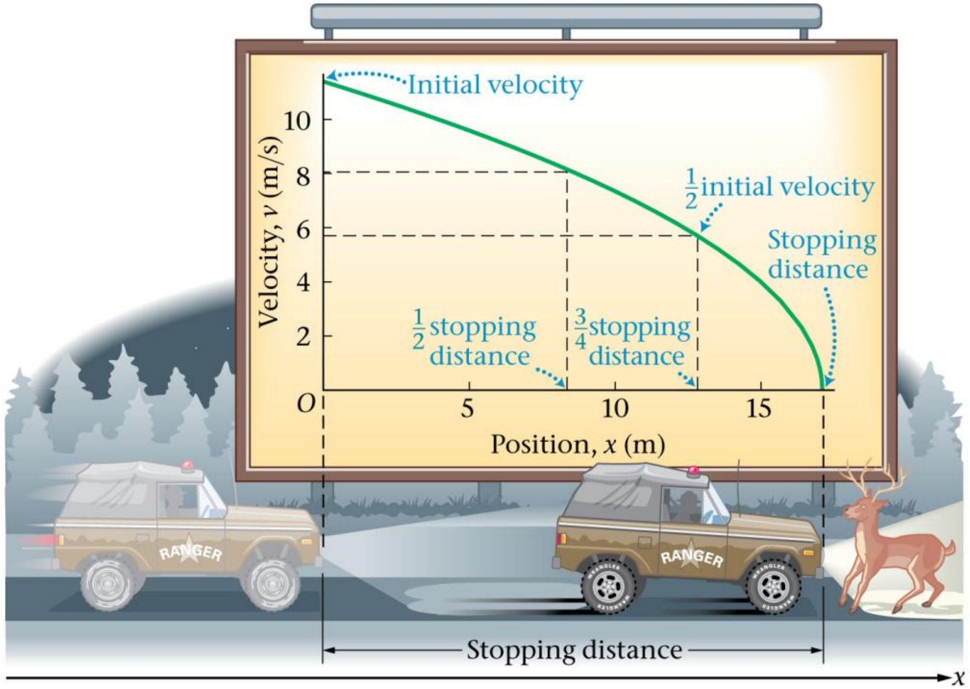
- Note that position does not change linearly with time when the object accelerates.

# 2.4. Constant Acceleration

**Example 7:** A park ranger driving at 11.4 m/s suddenly sees a deer “frozen” in the headlights. He applies the brakes and slows with an acceleration of 3.80 m/s$^2$.

(a) If the deer is 20.0 m from the ranger’s vehicle when the brakes are applied, how close does the ranger come to hitting the deer?

(b) How much time is needed for the ranger’s vehicle to stop? [Ans: (a) 2.9 m, (b) 3.0 s]



**Problem add**: An electric car accelerates from rest at a traffic light to a speed of 15 m/s with acceleration of 2.5 $m/s^2$. it then slows to a stop at the next light at a constant deceleration of 1.5 $m/s^2$. how far the car travel between the traffic lights?
[Ans:  120 m]

# 2.5. Free-Fall Acceleration

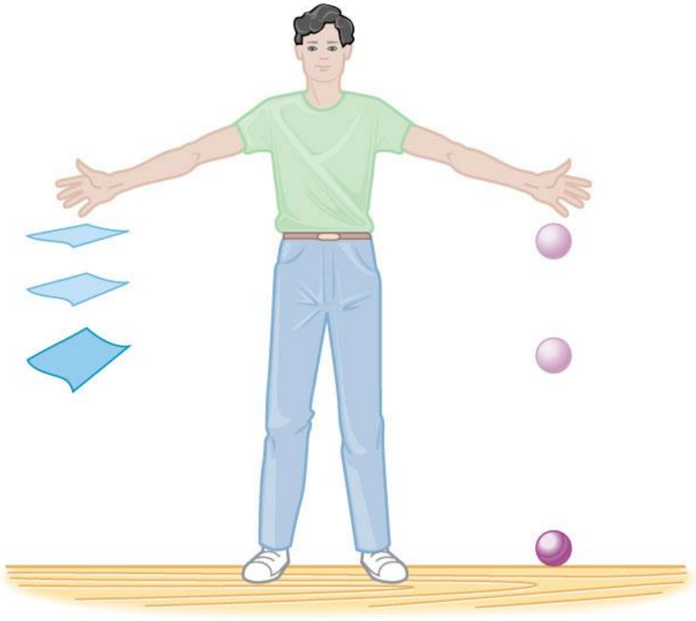
- Free fall is the motion of an object subject only to the influence of gravity. The acceleration due to gravity is a constant, $g$.

**TABLE 2-6 Values of $g$ at Different Locations on Earth $(\mathrm{m/s^2})$**

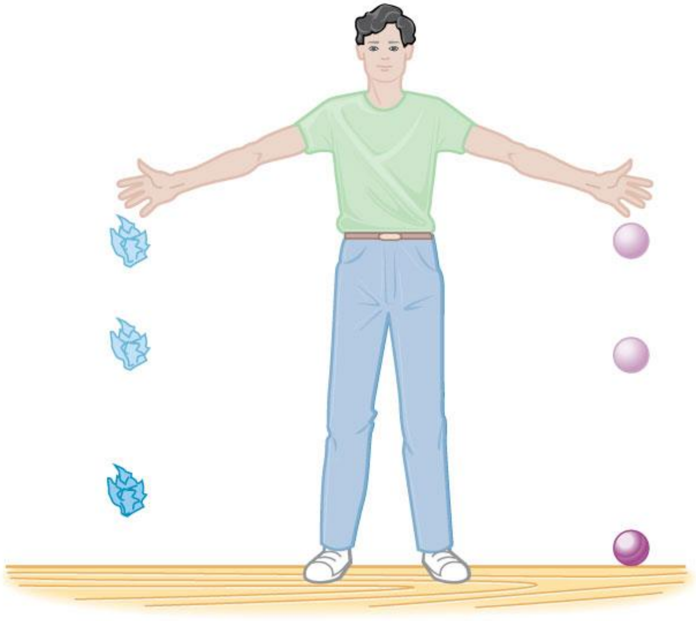
| Location | Latitude | $g$ |
|---|---|---|
| North Pole | 90° N | 9.832 |
| Oslo, Norway | 60° N | 9.819 |
| Hong Kong | 30° N | 9.793 |
| Quito, Ecuador | 0° | 9.780 |

# 2.5. Free-Fall Acceleration

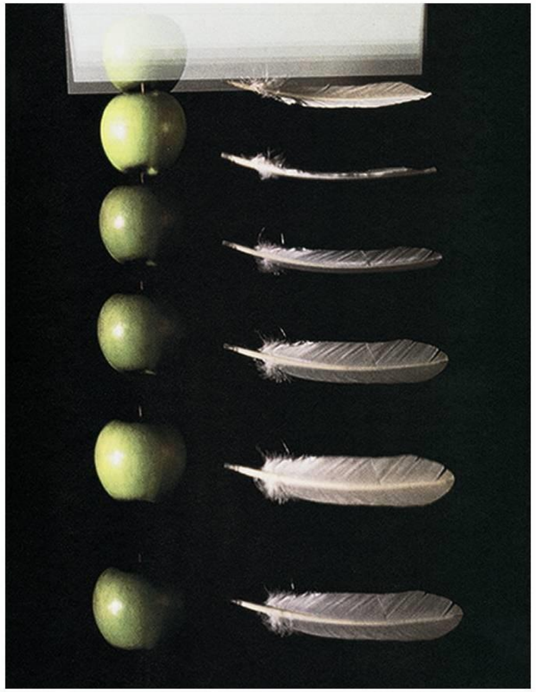
- An object falling in air is subject to air resistance (and therefore is not freely falling). However, air resistance can be negligible in many cases!



**(a)** Dropping a sheet of paper and a rubber ball



**(b)** Dropping a wadded-up sheet of paper and a rubber ball



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# 2.5. Free-Fall Acceleration

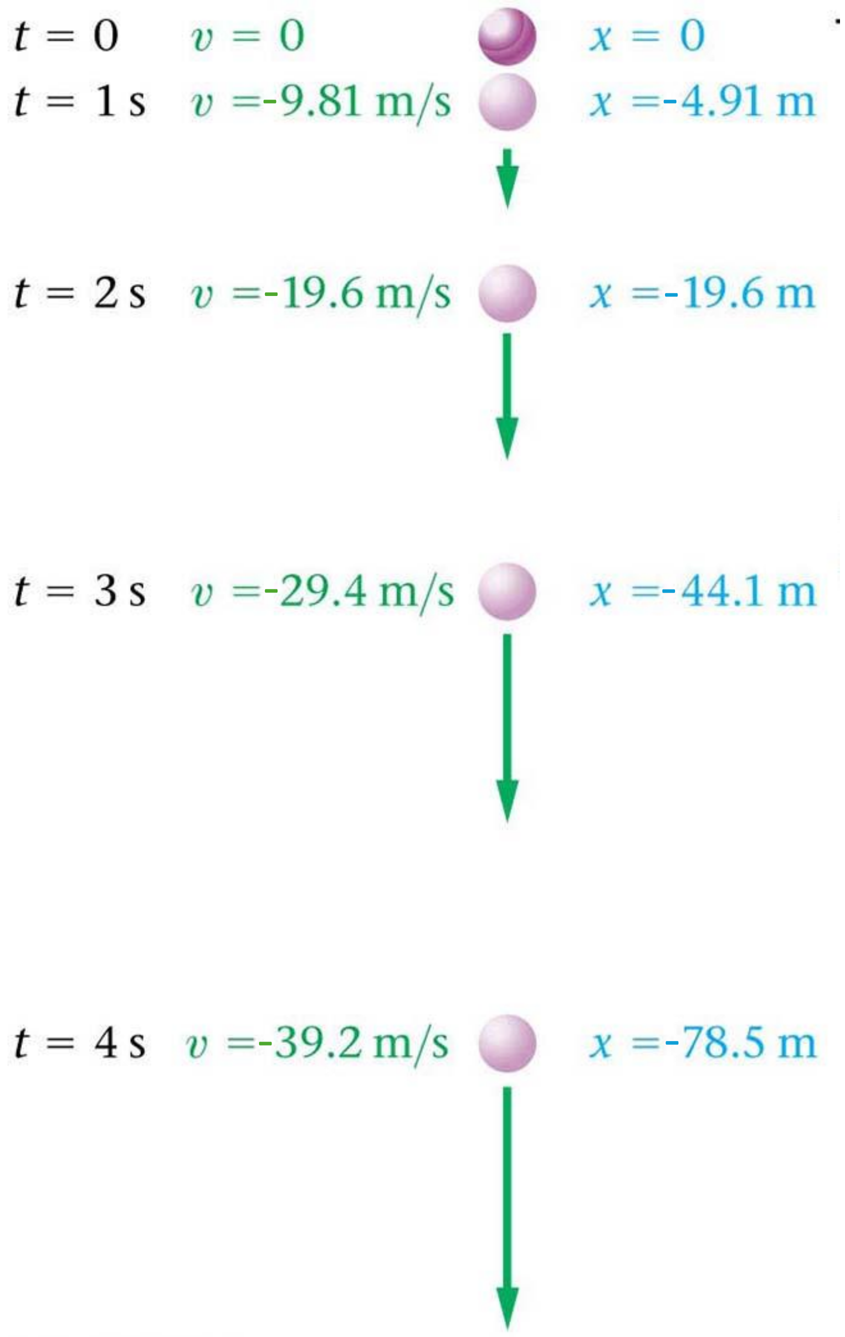
- All objects in free fall accelerate at the same rate, regardless of their weight.
- The object can be moving up or down—it is still in free fall!
- An object is in free fall as soon as it is released, whether it is dropped from rest, thrown downward, or thrown upward.
- Free fall acceleration is $\boldsymbol{g} \approx \mathbf{9.81\ m/s^2}$. The direction of $g$ is always downward, towards earth's center.
- The equations of motion for constant acceleration apply.
- We usually choose upward as the positive direction. Hence:

$$\boxed{a = -g = -9.81\ \mathrm{m/s^2}}$$

(b)

# 2.5 Free-Fall Acceleration

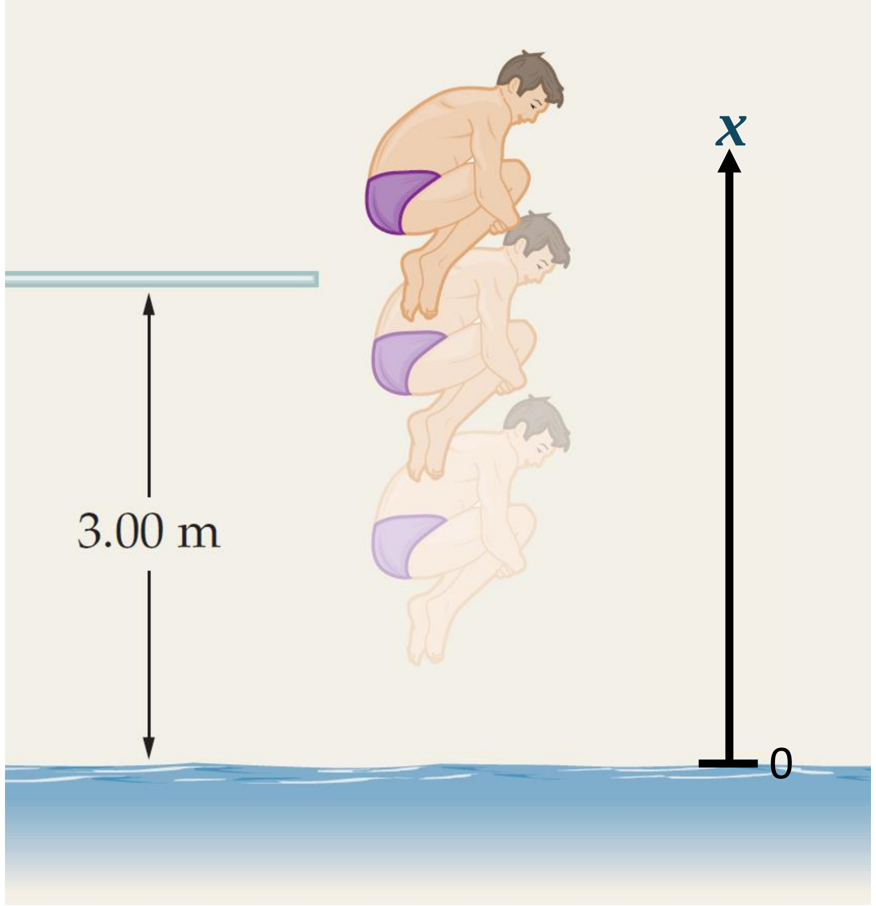
- In the figure, position and velocity are shown as functions of time for the special case of an object dropped from rest ($v_0 = 0$).
- It is apparent that velocity depends linearly on $t$, whereas position depends on $t^2$.



# 2.5. Free-Fall Acceleration

**Example 8:** A person steps off the end of a 3-m-high diving board and drops to the water below. (a) How long does it take for the person to reach the water? (b) What is the person’s speed on entering the water?

[Ans: (a) 0.782 s, (b) 7.67 m/s]



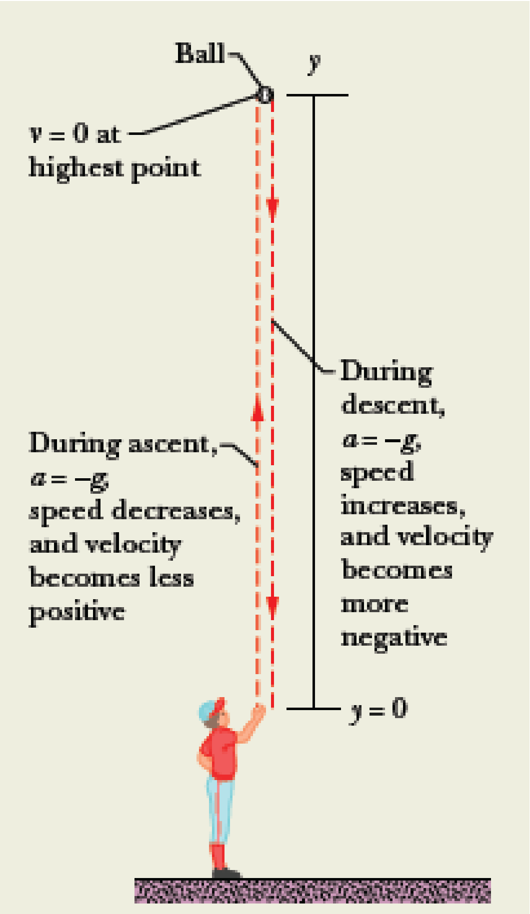
# 2.5. Free-Fall Acceleration

**Problem 2.5.1 Time for full up-down flight, baseball toss**

In Fig. 2.5.2, a pitcher tosses a baseball up along a *y* axis, with an initial speed of 12 m/s.

(a) How long does the ball take to reach its maximum height?

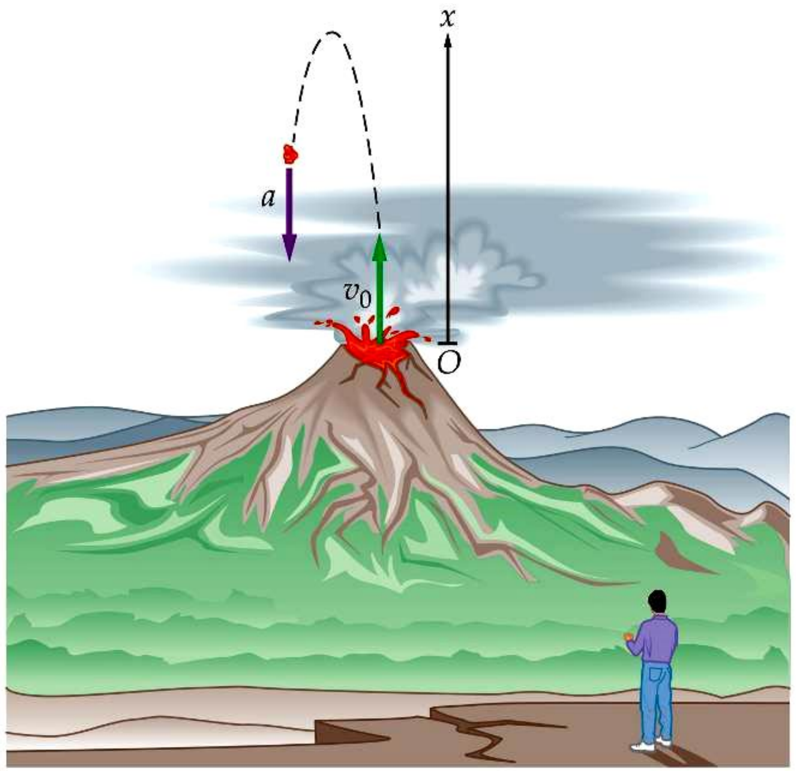
(b) What is the ball's maximum height above its release point?



# 2.5. Free-Fall Acceleration

**Example 9:** A volcano shoots out blobs of molten lava, called lava bombs, from its summit. A geologist observing the eruption uses a stopwatch to time the flight of a particular lava bomb that is projected straight upward. If the time for it to rise and fall back to its launch height is $4.75$ s, what is its initial speed? and how high did it go?

[Ans: 23.3 m/s, 27.7 m]

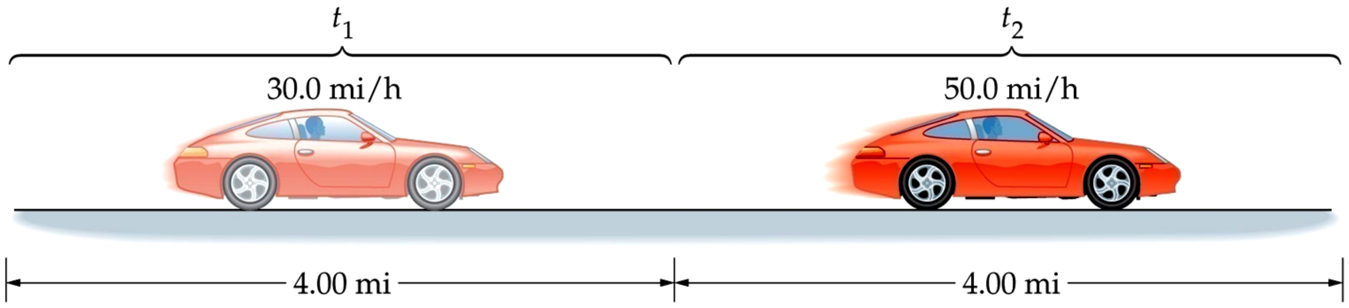


# Summary of Chapter 2

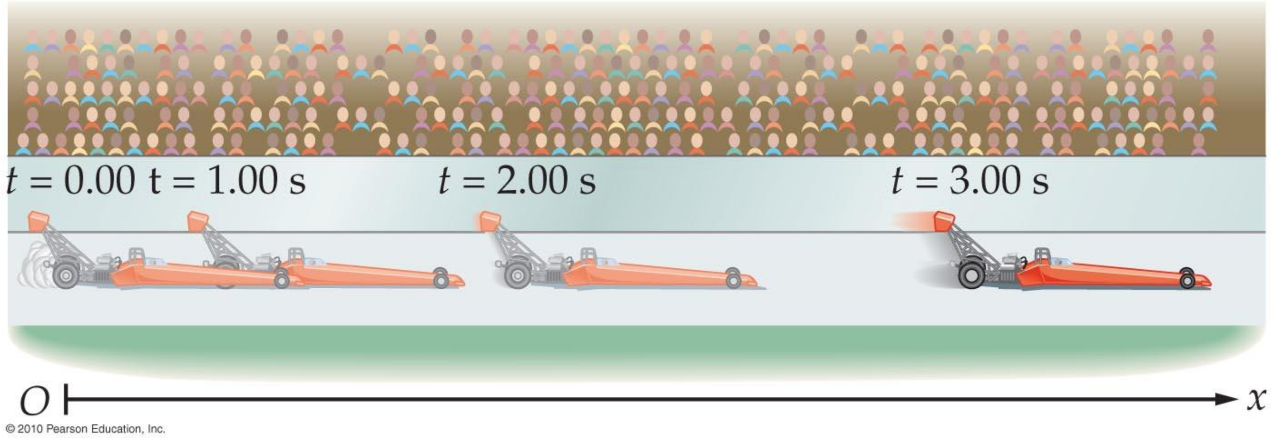
- Distance: total length of travel
- Displacement: change in position
- Average speed: distance/time
- Average velocity: displacement/time
- Average acceleration: change in velocity divided by change in time
- Deceleration: velocity and acceleration have opposite signs
- Constant acceleration: equations of motion relate position, velocity, acceleration, and time.
- Freely falling objects: constant downward acceleration of $a = -g = 9.81$ m/s$^2$

# Additional Exercises

- You drive 4 mi at 30 mi/h and then another 4 mi at 50 mi/h. What is your average speed for the 8-mi trip? [Ans: 37.5 mi/h]

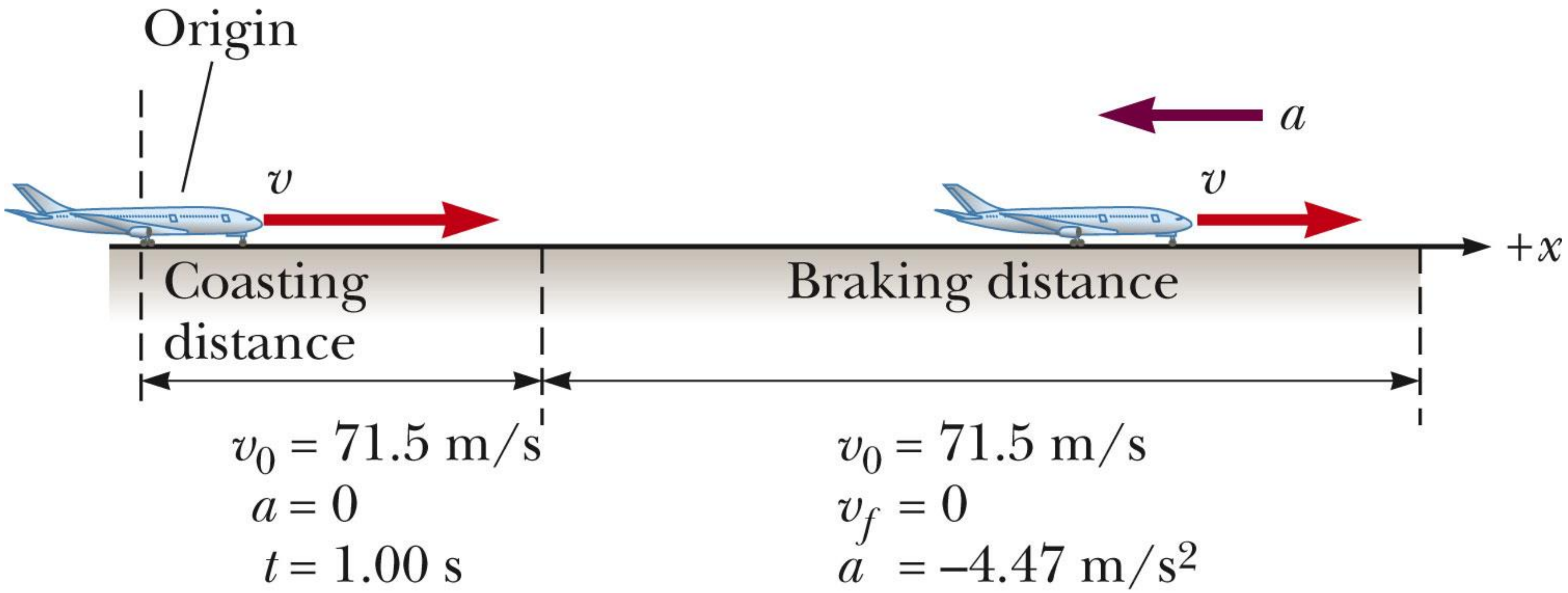



- A drag racer starts from rest and accelerates at 7.4 m/s2. How far has it traveled in (a) 1 s, (b) 2 s, (3) 3 s?



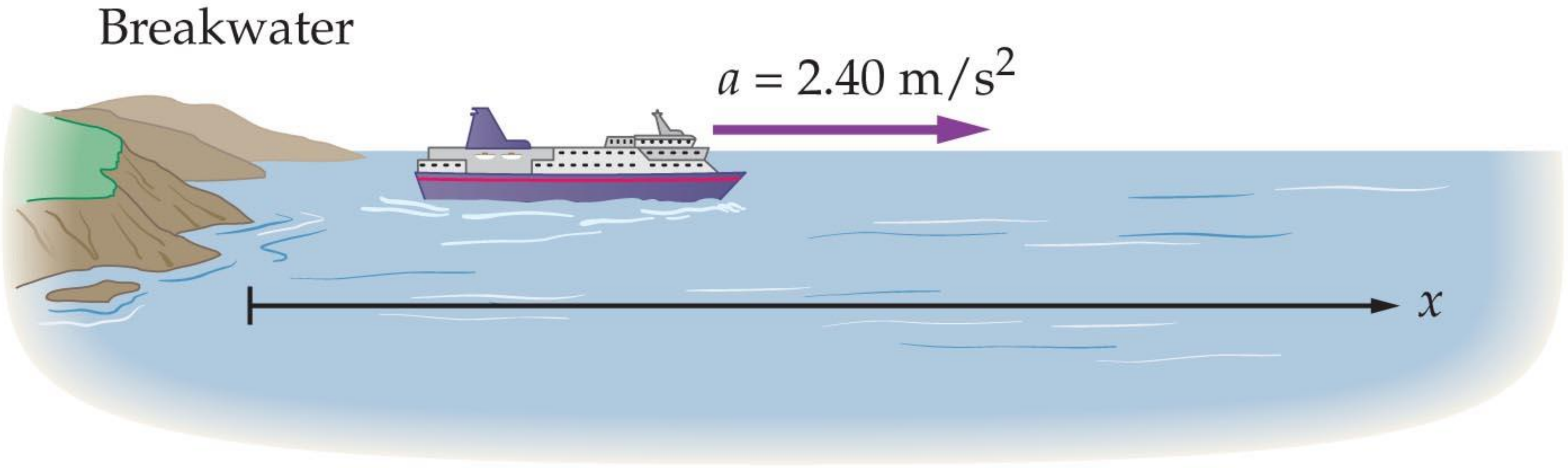
# Additional Exercises

- A typical jetliner lands at a speed of 71.5 m/s and decelerates at the rate of 4.47 $m/s^2$. If the plane travels at a constant speed of 71.5 m/s for 1.00 s after landing before applying the brakes, what is the total displacement of the aircraft between touchdown on the runway and coming to rest?



# Additional Exercises

- A boat moving with a speed of 1.50 m/s starts to accelerate at 2.40 m/s$^2$. (a) How fast is the boat moving after accelerating for 5.00 s? (b) How far has the boat travelled in this time?



# Additional Exercises

- **Problem 33**: At the starting gun, a runner accelerates at 1.9 $m/s^2$ for 5.2 s. The runner's acceleration is zero for the rest of the race. What is the speed of the runner (a) at 2.0 s, and (b) at the end of the race. [Ans: (a) 3.8 m/s, (b) 9.9 m/s]

- **Problem 43**: Landing with a speed of 81.9 m/s, travelling due south, a jet comes to rest in 949 m. Assuming the jet slows with constant acceleration, find the magnitude and direction of its acceleration. [Ans: 3.53 $m/s^2$ to the north]

- **Problem 41**. As a train accelerates away from a station, it reaches a speed of 4.7 m/s in 5.0 s. If the train's acceleration remains constant, what is its speed after an additional 6.0 s has elapsed

- **Problem 57**. You are driving through town at 12 m/s when suddenly a ball rolls out in front of you. You apply breaks and begin decelerating at 3.5 $m/s^2$.
  a) How far do you travel before stopping?
  b) When you have traveled half the distance in part (a), is your speed 6.0 m/s, greater than 6.0 m/s, or less than 6 m/s?

# Additional Exercises

- **Problem 77**. *An experiment to measure your reaction time*. Have a friend hold a ruler by one end, letting the other end hang down vertically. At the lower end, hold your thumb and index finger on either side of the ruler, ready to grip it. Have your friend release the ruler without warning. Catch it as quickly as you can. If you catch the ruler 5.2 cm from the lower end, what is your reaction time?

- **Problem 88**. An Arrow shot vertically upwards reaches 30 m above its launch point in 2 seconds.
  - (a) What was the arrow's initial speed?
  - (b) How long did it take for the arrow to first reach the height of 15 m ?
  - (c) What is the maximum height gained by the arrow?

**Problem:** The position of a particle moving along the $x$ axis is given in centimeters by

$x = 9.75 + 1.50t^3$, where $t$ is in seconds.

(a) Calculate the average velocity during the time interval $t = 2.00$ s to $t = 3.00$ s;

(b) Calculate the instantaneous velocity at $t = 2.00$ s;

(c) Calculate the instantaneous velocity at $t = 3.00$ s;

(d) Calculate the instantaneous velocity at $t = 2.50$ s;

(e) Calculate the instantaneous velocity when the particle is midway between its positions at $t = 2.00$ s and $t = 3.00$ s.

**44** E When startled, an armadillo will leap upward. Suppose it rises 0.544 m in the first 0.200 s. (a) What is its initial speed as it leaves the ground? (b) What is its speed at the height of 0.544 m? (c) How much higher does it go?