Current and Resistance

Electric Current

The amount of flow of electric charges through a piece of material depends on the material through which the charges are passing and the potential difference across the material.

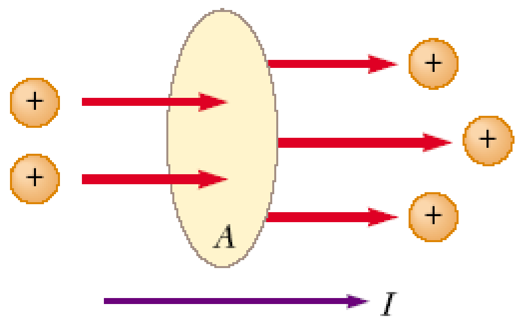


Figure (1)

To define current more precisely, suppose that charges are moving perpendicular to a surface of area *A*, as shown in Figure 1. (This area could be the cross-sectional area of a wire, for example.) ***The current is the rate at which charge flows through this surface***. If ΔQ is the amount of charge that passes through this area in a time interval Δt, the average current I_{av} is equal to the charge that passes through *A* per unit time:

$$I_{\text{av}} = \frac{\Delta Q}{\Delta t}$$

If the rate at which charge flows varies in time, then the current varies in time; we define **the instantaneous current *I*** as the differential limit of average current:

$$I \equiv \frac{dQ}{dt}$$

The SI unit of current is the ampere (A):

$$1\ \text{A} = \frac{1\ \text{C}}{1\ \text{s}}$$

That is, 1 A of current is equivalent to 1 C of charge passing through the surface area in 1 s.

The charges passing through the surface in Figure 1 can be positive or negative, or both. It is conventional to assign to ***the current the same direction as the flow of positive charge.***

In electrical conductors, such as copper or aluminum, the current is due to the motion of negatively charged electrons. Therefore, when we speak of current in an ordinary conductor, ***the direction of the current is opposite the direction of flow of electrons***.

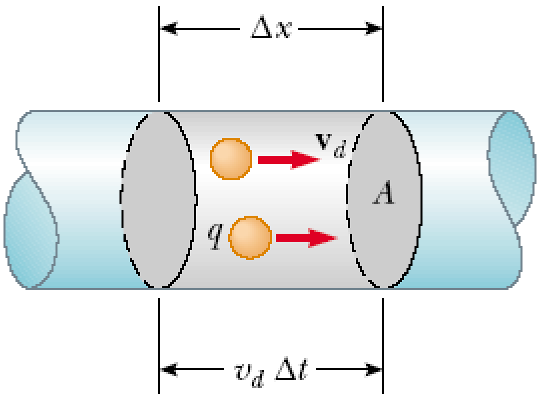


Figure (2)

Resistance

Consider a conductor of cross-sectional area A carrying a current I. **<u>The current density</u>** J in the conductor **<u>is defined as the current per unit area</u>**. Because the current $I = n\, q\, v_d\, A$, the current density is

$$J \equiv \frac{I}{A} = nqv_d$$

where J has SI units of A/m^2.

<u>In general, current density is a vector quantity</u>

<u>Note that</u> , ***<u>the current density is in the direction of charge motion for positive charge carriers and opposite the direction of motion for negative charge carriers</u>***.

A current density **J** and an electric field **E** are established in a conductor whenever a potential difference is maintained across the conductor. In some materials, the current density is proportional to the electric field:

$$\mathbf{J} = \sigma \mathbf{E}$$

where the constant of proportionality $\boldsymbol{\sigma}$ is called **<u>the conductivity of the conductor</u>**. Materials that obey above Equation are said to follow Ohm's law. More specifically, Ohm's law states that :-

<u>the ratio of the current density to the electric field is a constant & that is independent of the electric field producing the current.</u>

<u>Materials that obey Ohm's law are said to be ***ohmic*** *and* Materials that do not obey Ohm's law are said to be ***nonohmic***</u>.

We can obtain an equation useful in practical applications by considering a segment of straight wire of uniform cross-sectional area *A* and length L, as shown in Figure 3. A potential difference $\Delta V = V_b - V_a$ is maintained across the wire, creating in the wire an electric field and a current.

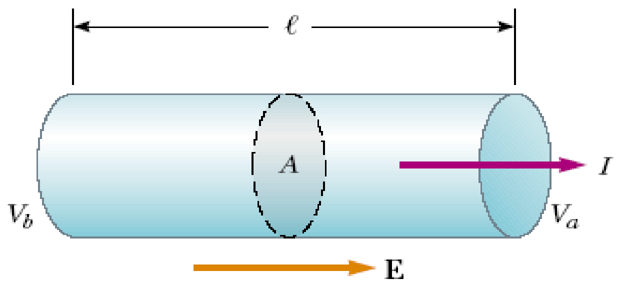


Figure (3)

$$\Delta V = E\ell$$

Therefore, we can express the magnitude of the current density in the wire as

$$J = \sigma E = \sigma \frac{\Delta V}{\ell}$$

$$\Delta V = \frac{\ell}{\sigma} J = \left(\frac{\ell}{\sigma A}\right) I = RI$$

The quantity R = L/ σA is called the resistance of the conductor.

We can define the resistance as the ratio of the potential difference across a conductor to the current in the conductor:

$$R \equiv \frac{\Delta V}{I}$$

The resistance has SI units of volts per ampere. One volt per ampere is defined to be one ohm (Ω):

$$1\ \Omega \equiv \frac{1\ \text{V}}{1\ \text{A}}$$

The inverse of conductivity is resistivity ρ

$$\rho = \frac{1}{\sigma}$$

where ρ has the units ohm-meters (Ω.m). Because ***R = L/ σ A***, we can express the resistance of a uniform block of material along the length L as

$$R = \rho \frac{\ell}{A}$$

Example2 The Resistance of a Conductor

Calculate the resistance of an aluminum cylinder that has a length of 10.0 cm and a cross-sectional area of 2.00 x 10^{-4} m^2. Repeat the calculation for a cylinder of the same dimensions and made of glass having a resistivity of 3.0 x 10^{10} Ω.m.

Solution

$$R = \rho \frac{\ell}{A} = (2.82 \times 10^{-8}\,\Omega\cdot\mathrm{m})\left(\frac{0.100\ \mathrm{m}}{2.00 \times 10^{-4}\ \mathrm{m}^2}\right)$$

$$= \boxed{1.41 \times 10^{-5}\,\Omega}$$

Similarly, for glass we find that

$$R = \rho \frac{\ell}{A} = (3.0 \times 10^{10}\ \Omega \cdot \mathrm{m}) \left(\frac{0.100\ \mathrm{m}}{2.00 \times 10^{-4}\ \mathrm{m}^2} \right)$$

$$= \boxed{1.5 \times 10^{13}\ \Omega}$$

Example 3 The Resistance of Nichrome Wire

(A) Calculate the resistance per unit length of a 22-gauge Nichrome wire, which has a radius of 0.321 mm.

(B) If a potential difference of 10 V is maintained across a 1.0-m length of the Nichrome wire, what is the current in the wire?

Note that "The resistivity of Nichrome is 1.5 x 10^{-6} Ω.m"

Solution

(A) The cross-sectional area of this wire is

$$A = \pi r^2 = \pi(0.321 \times 10^{-3}\ \mathrm{m})^2 = 3.24 \times 10^{-7}\ \mathrm{m}^2$$

$$\frac{R}{\ell} = \frac{\rho}{A} = \frac{1.5 \times 10^{-6}\ \Omega \cdot \mathrm{m}}{3.24 \times 10^{-7}\ \mathrm{m}^2} = \boxed{4.6\ \Omega/\mathrm{m}}$$

(B)

$$I = \frac{\Delta V}{R} = \frac{10\ \mathrm{V}}{4.6\ \Omega} = \boxed{2.2\ \mathrm{A}}$$

Resistance and Temperature

Over a limited temperature range, the resistivity of a conductor varies approximately linearly with temperature according to the expression

$$\rho = \rho_0[1 + \alpha(T - T_0)]$$

where ρ is the resistivity at some temperature T (in degrees Celsius), ρ_0 is the resistivity at some reference temperature T_0 (usually taken to be 20°C), and ***α is the temperature coefficient of resistivity***.

$$\alpha = \frac{1}{\rho_0}\frac{\Delta\rho}{\Delta T}$$

where $\Delta\rho = \rho - \rho_o$ is the change in resistivity in the temperature interval $\Delta T = T - T_0$.

we can write the variation of resistance as

$$R = R_0[1 + \alpha(T - T_0)]$$

Electrical Power

Let us consider now the rate at which the system loses electric potential energy as the charge Q passes through the resistor:

$$\frac{dU}{dt} = \frac{d}{dt}(Q\Delta V) = \frac{dQ}{dt}\Delta V = I\Delta V$$

where I is the current in the circuit.

The power P, representing the rate at which energy is delivered to the resistor, is

$$\boxed{\mathcal{P} = I\Delta V}$$

$$\boxed{\mathcal{P} = I^2R = \frac{(\Delta V)^2}{R}}$$

When I is expressed in amperes, ΔV in volts, and R in ohms, **<u>the SI unit of power is the watt</u>**

<u>Example 4 Power in an Electric Heater</u>

An electric heater is constructed by applying a potential difference of 120 V to a Nichrome wire that has a total resistance of 8.00 **Ω**. Find the current carried by the wire and the power rating of the heater.

Solution

$$I = \frac{\Delta V}{R} = \frac{120\ \text{V}}{8.00\ \Omega} = \boxed{15.0\ \text{A}}$$

$$\mathcal{P} = I^2R = (15.0\ \text{A})^2(8.00\ \Omega) = 1.80 \times 10^3\ \text{W}$$

$$\mathcal{P} = \boxed{1.80\ \text{kW}}$$

Direct Current Circuits

Electromotive Force

Because the potential difference at the battery terminals is constant in a particular circuit, the current in the circuit is constant in magnitude and direction and is called direct current. A battery is called either a *source of electromotive force* or, more commonly, a *source of emf.* (The phrase *electromotive force* is an unfortunate historical term, describing not a force but rather a potential difference in volts.) **The emf $\mathcal{E}$ of a battery is the maximum possible voltage that the battery can provide between its terminals**.

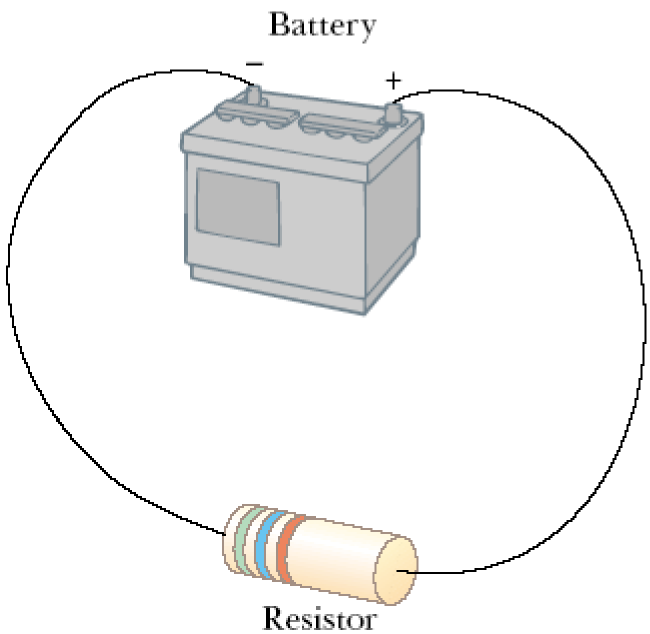


Figure (1)

Consider the circuit shown in Figure 28.1, consisting of a battery connected to a resistor. We shall generally assume that the connecting wires have no resistance. The positive terminal of the battery is at a higher potential than the negative terminal. Because a real battery is made of matter, there is resistance to the flow of charge within the battery. This resistance is called internal resistance *r*.

For an idealized battery with zero internal resistance, the potential difference across the battery (called its *terminal voltage*) equals its emf. However, for a real battery, the terminal voltage is *not* equal to the emf for a battery in a circuit in which there is a current.

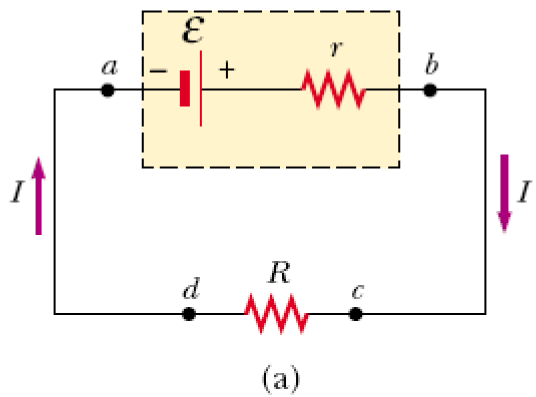


Figure (2)

To understand why this is so, consider the circuit diagram in Figure 2, where the battery of Figure 1 is represented by the dashed rectangle containing an ideal, resistance-free emf ε in series with an internal resistance *r*. Now imagine moving through the battery from *a* to *b* and measuring the electric potential at various locations. As we pass from the negative terminal to the positive terminal, the potential *increases* by an amount ε. However, as we move through the resistance *r*, the potential *decreases* by an amount I_r, where *I* is the current in the circuit. Thus, the terminal voltage of the battery

$$\Delta V = \varepsilon - Ir$$

$$\mathcal{E} = IR + Ir \qquad (1)$$

$$I = \frac{\mathcal{E}}{R + r}$$

This equation shows that the current in this simple circuit depends on both the load resistance R external to the battery and the internal resistance r. If R is much greater than r, as it is in many real-world circuits, we can neglect r.

If we multiply Equation 2 by the current I, we obtain

$$I\mathcal{E} = I^2R + I^2r$$

This equation indicates that, because power P = *I V*, the total power output *I* $\mathcal{E}$ of the battery is delivered to the external load resistance in the amount I^2R and to the internal resistance in the amount I^2r.

Example1 Terminal Voltage of a Battery

A battery has an emf of 12.0 V and an internal resistance of 0.05 Ω Its terminals are connected to a load resistance of 3.00 Ω.

(A) Find the current in the circuit and the terminal voltage of the battery.

$$I = \frac{\mathcal{E}}{R + r} = \frac{12.0\ \text{V}}{3.05\ \Omega} = \boxed{3.93\ \text{A}}$$

$$\Delta V = \mathcal{E} - Ir = 12.0\ \text{V} - (3.93\ \text{A})(0.05\ \Omega) = \boxed{11.8\ \text{V}}$$

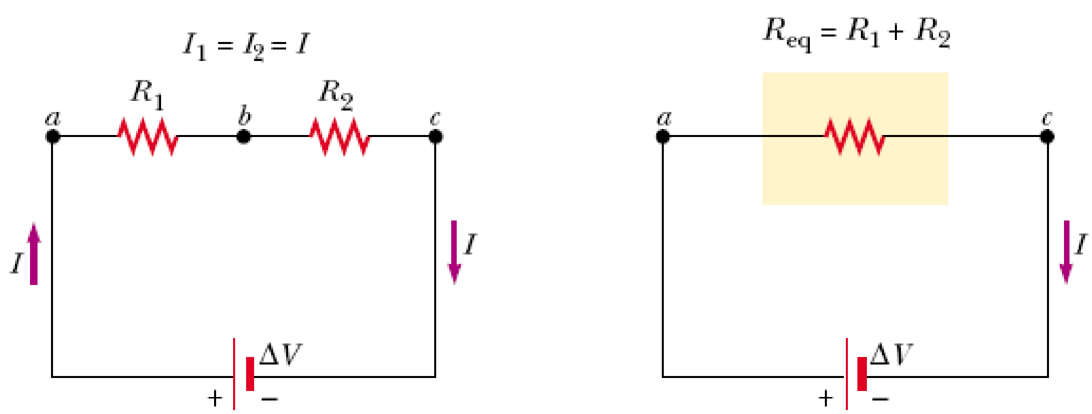
(B) Calculate the power delivered to the load resistor, the power delivered to the internal resistance of the battery, and the power delivered by the battery.

The power delivered to the load resistor is

$$\mathcal{P}_R = I^2R = (3.93\ \text{A})^2\ (3.00\ \Omega) = \boxed{46.3\ \text{W}}$$

The power delivered to the internal resistance is

$$\mathcal{P}_r = I^2r = (3.93\ \text{A})^2\ (0.05\ \Omega) = \boxed{0.772\ \text{W}}$$



(a) **(b)**

Figure (3)

Resistors in Series and Parallel

When two or more resistors are connected together as in Figure 3-a, they are said to be in *series*. The potential difference applied across the series combination of resistors will divide between the resistors. In Figure 3-a, because the voltage drop from *a* to *b* equals ***IR_1*** and the voltage drop from *b* to *c* equals ***IR_2***, the voltage drop from *a* to *c* is

$$\Delta V = IR_1 + IR_2 = I(R_1 + R_2)$$

The potential difference across the battery is also applied to the **equivalent resistance** R_{eq} in Figure 3-b:

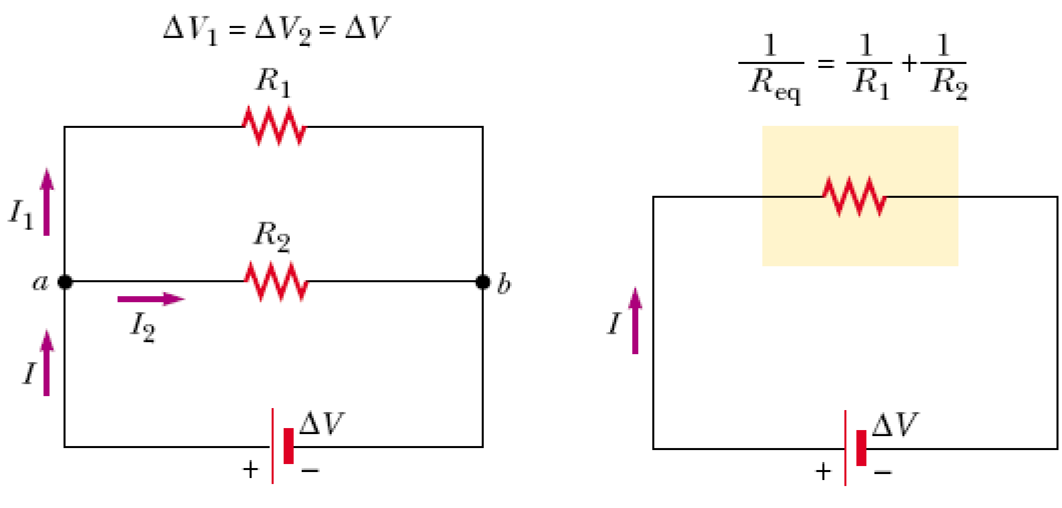
$$\Delta V = IR_{eq}$$

$$\Delta V = IR_{eq} = I(R_1 + R_2) \longrightarrow R_{eq} = R_1 + R_2$$

The equivalent resistance of three or more resistors connected in series is

$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$

<u>This relationship indicates that the equivalent resistance of a series connection of resistors is the numerical sum of the individual resistances and is always greater than any individual resistance.</u>



(a) **(b)**

Figure (4)

Now consider two resistors connected in *parallel,* as shown in Figure 4. The current I that enters point a must equal the total current leaving that point:

$$I = I_1 + I_2$$

Where I_1 is the current in R_1 and I_2 is the current in R_2.

$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\Delta V}{R_{eq}}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

An extension of this analysis to three or more resistors in parallel gives

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

We can see from this expression that the inverse of the equivalent resistance of two or more resistors connected in parallel is equal to the sum of the inverses of the individual resistances. Furthermore, the equivalent resistance is always less than the smallest resistance in the group.

Example 2 Find the Equivalent Resistance

Four resistors are connected as shown in Figure 5-a .

(A) Find the equivalent resistance between points *a* and *c*.

The combination of resistors can be reduced in steps, as shown in Figure 5. The 8.0-Ω and 4.0-Ω resistors are in series; thus, the equivalent resistance between *a* and *b* is 12.0 Ω . The 6.0-Ω and 3.0-Ω resistors are in parallel, so we find that the equivalent resistance from *b* to *c* is 2.0 Ω. Hence, the equivalent resistance from *a* to *c* is 14.0 Ω.

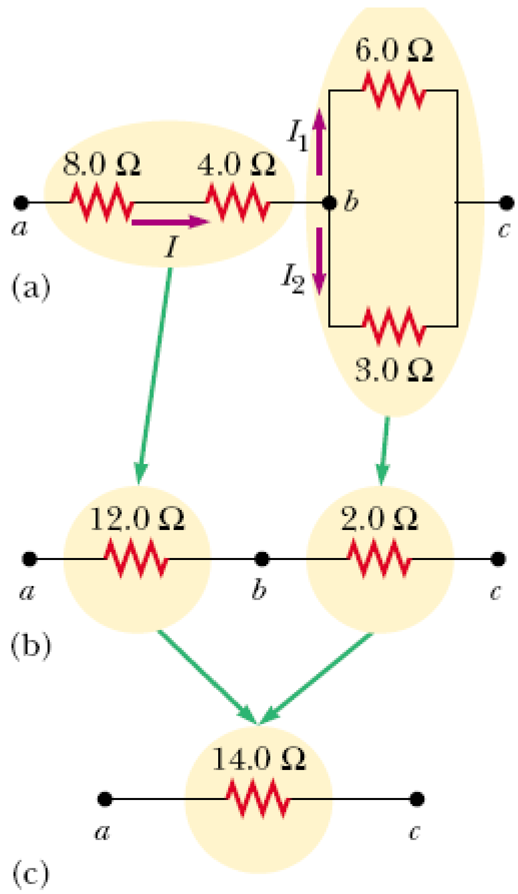


Figure (5)

(B) What is the current in each resistor if a potential difference of 42 V is maintained between *a* and *c*?

The currents in the 8.0-Ω and 4.0-Ω resistors are the same because they are in series. In addition, this is the same as the current that would exist in the 14.0-Ω equivalent resistor subject to the 42-V potential difference. Therefore,

$$I = \frac{\Delta V_{ac}}{R_{eq}} = \frac{42\text{ V}}{14.0\ \Omega} = 3.0\text{ A}$$

This is the current in the 8.0-Ω and 4.0-Ω resistors. When this 3.0-A current enters the junction at *b*, however, it splits, with part passing through the 6.0-Ω resistor (I_1) and part through the 3.0-Ω resistor (I_2). Because the potential difference is V_{bc} across each of these parallel resistors, we see that (6.0 Ω) I_1 = (3.0 Ω)I_2, or $I_2 = 2I_1$. Using this result and the fact that $I_1 + I2 = 3.0$ A, we find that $I_1 = 1.0$ A and $I_2 = 2.0$ A.

Example 3 Three Resistors in Parallel

Three resistors are connected in parallel as shown in Figure 6-a. A potential difference of 18.0 V is maintained between points *a* and *b*.

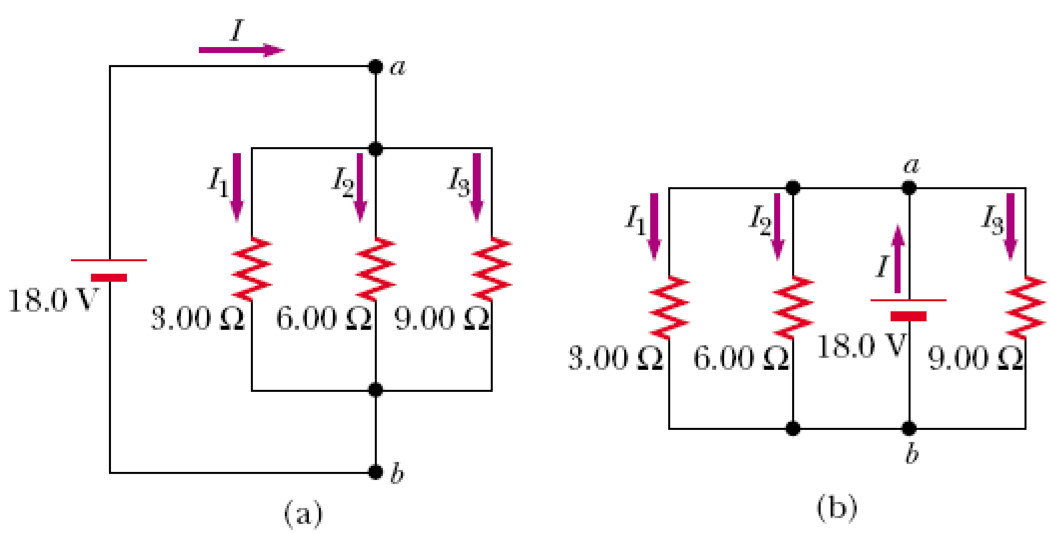


Figure 6

(A) Find the current in each resistor.

$$I_1 = \frac{\Delta V}{R_1} = \frac{18.0\ \text{V}}{3.00\ \Omega} = \boxed{6.00\ \text{A}}$$

$$I_2 = \frac{\Delta V}{R_2} = \frac{18.0\ \text{V}}{6.00\ \Omega} = \boxed{3.00\ \text{A}}$$

$$I_3 = \frac{\Delta V}{R_3} = \frac{18.0\ \text{V}}{9.00\ \Omega} = \boxed{2.00\ \text{A}}$$

(B) Calculate the power delivered to each resistor and the total power delivered to the combination of resistors.

We apply the relationship P = I_2R to each resistor and obtain

$$3.00\text{-}\Omega: \quad \mathcal{P}_1 = I_1{}^2R_1 = (6.00\ \text{A})^2(3.00\ \Omega) = \boxed{108\ \text{W}}$$

$$6.00\text{-}\Omega: \quad \mathcal{P}_2 = I_2{}^2R_2 = (3.00\ \text{A})^2(6.00\ \Omega) = \boxed{54.0\ \text{W}}$$

$$9.00\text{-}\Omega: \quad \mathcal{P}_3 = I_3{}^2R_3 = (2.00\ \text{A})^2(9.00\ \Omega) = \boxed{36.0\ \text{W}}$$

(C) Calculate the equivalent resistance of the circuit.

$$\frac{1}{R_{\text{eq}}} = \frac{1}{3.00\ \Omega} + \frac{1}{6.00\ \Omega} + \frac{1}{9.00\ \Omega}$$

$$R_{\text{eq}} = \frac{18.0\ \Omega}{11.0} = \boxed{1.64\ \Omega}$$