Electric Potential

- When a test charge is placed in an electric field, it experiences a force.

$$\vec{\mathbf{F}}_e = q_o \vec{\mathbf{E}}$$

  - The force is conservative.

- If the test charge is moved in the field by some external agent, the work done by the field is equal to the negative of the work done by the external agent.
  - The path may be straight or curved and the integral performed along this path is called either a *path integral* or a *line integral.*

Electric Potential

- The work done within the charge-field system by the electric field on the charge is

$$W = \mathbf{F} \cdot d\mathbf{s} = q_0 \mathbf{E} \cdot d\mathbf{s}$$

- The potential energy of the charge-field system is changed by

$$W = \Delta U = U_B - U_A$$

is

$$\Delta U = -q_0 \int_A^B \mathbf{E} \cdot d\mathbf{s}$$

- Because the force is conservative, the line integral does not depend on the path taken by the charge.

Electric Potential

- The potential energy per unit charge, U/q_o, is called the **electric potential.**

$$V = \frac{U}{q_o}$$

  - The potential is characteristic of the field only.
    - The potential energy is characteristic of the charge-field system.
  - The potential is independent of the value of q_o.
  - The potential has a value at every point in an electric field.
  - Since energy is a scalar, the potential is a scalar quantity.

- As a charged particle moves in an electric field, it will experience a change in potential.

$$\Delta V = \frac{\Delta U}{q_o} = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

Electric Potential

- Assume a charge moves in an electric field without any change in its kinetic energy.
- The work performed on the charge is

$W = \Delta U = q_o \Delta V$ ⇒ $\Delta V = W/q_o = \Delta U/q_o$

- Units: $1\ V \equiv 1\ J/C$
  - V is a volt.

"*It takes one joule of work to move a 1-coulomb charge through a potential difference of 1 volt*".

Electric Potential

- In addition,

$$W = F.\,d\mathbf{s} = q_0\,\mathbf{E}.\,d\mathbf{s}$$

$$\Delta V = W/q_0 = \mathbf{E}.\,d\mathbf{s}$$

$$\mathbf{E} = \Delta V/\,ds$$

$$1\ N/C = 1\ V/m$$

This indicates we can interpret the electric field as ***a measure of the rate of change of the electric potential with respect to position***.

Potential Differences in a Uniform Electric Field

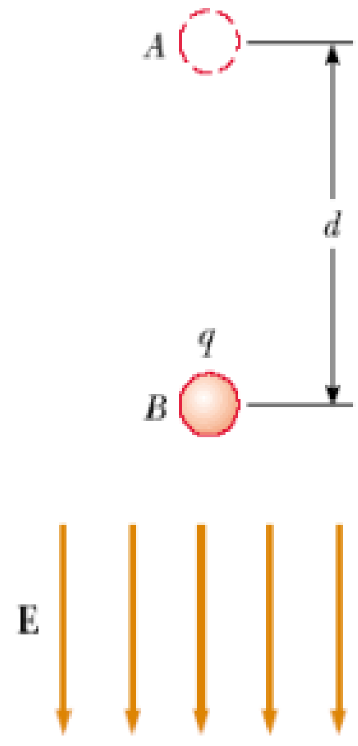
Consider a uniform electric field directed along the negative *y* axis, as shown in Figure 1-a. Let us calculate the potential difference between two points *A* and *B* separated by a distance **|s| = d**, where **s** is parallel to the field lines.

$$V_B - V_A = \Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -E\int_A^B d\mathbf{s} = -Ed$$

Because **E** is constant,

$$\Delta V = -E\int_A^B ds = -Ed$$

The negative sign indicates that the electric potential at point **B** is lower than at point **A**; that is, **VB** < **VA**.



(a)

Potential Differences in a Uniform Electric Field

$$\Delta U = q_0 \Delta V = -q_0 E d$$

From this result, we see that :-

(1) if q_0 is positive, then ΔU is negative.

(2) If q_0 is negative, then ΔU is positive.

equipotential surface

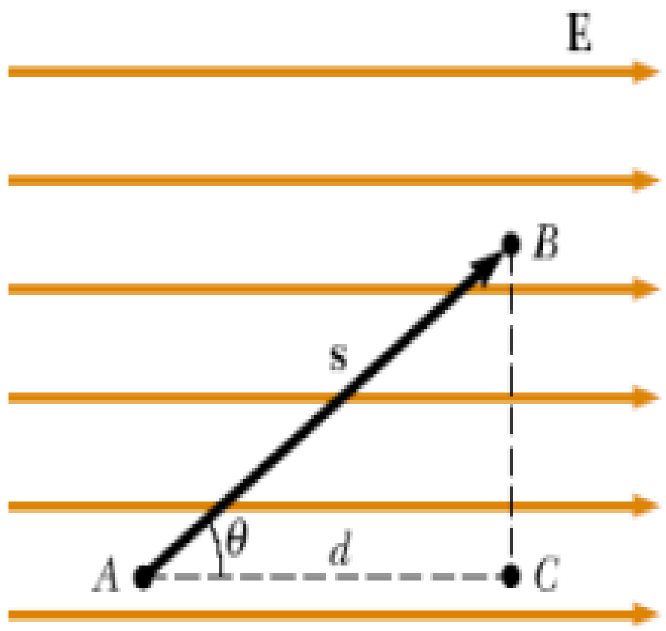
- a uniform electric field such that **the vector s is not parallel** to the field lines.
- In this case the potential difference is given by

$$\Delta V = -\int_A^B \mathbf{E}\cdot d\mathbf{s} = -\mathbf{E}\cdot\int_A^B d\mathbf{s} = -\mathbf{E}\cdot\mathbf{s}$$

$$\Delta U = q_0\,\Delta V = -q_0\,\mathbf{E}\cdot\mathbf{s}$$

The potential difference **VB - VA** is equal to the potential difference **VC - VA**.

- Therefore, **VB = VC**.



All points in a plane perpendicular to a uniform electric field are at the same electric potential.

Electric Potential and Potential Energy Due to Point Charges

- To find the electric potential at a point located a distance ***r*** from the charge,

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

- The electric field due to the point charge is

$$\mathbf{E} = k_e \frac{q}{r^2} \hat{\mathbf{r}}$$

$$\mathbf{E} \cdot d\mathbf{s} = k_e \frac{q}{r^2} \hat{\mathbf{r}} \cdot d\mathbf{s}$$

$$V_B - V_A = -k_e q \int_{r_A}^{r_B} \frac{dr}{r^2} = \frac{k_e q}{r} \Big]_{r_A}^{r_B}$$

$$V_B - V_A = k_e q \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$

Electric Potential and Potential Energy Due to Point Charges

$$V_B - V_A = k_e q \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$

This equation shows us that the integral of **E. ds** is ***independent*** of the path between points ***A*** and ***B***.

Multiplying by a charge ***qo*** that moves between points ***A*** and ***B***, we see that the integral of ***qo*** **E. ds** is also ***independent*** of path.

The electric potential created by a point charge at any distance ***r*** from the charge is

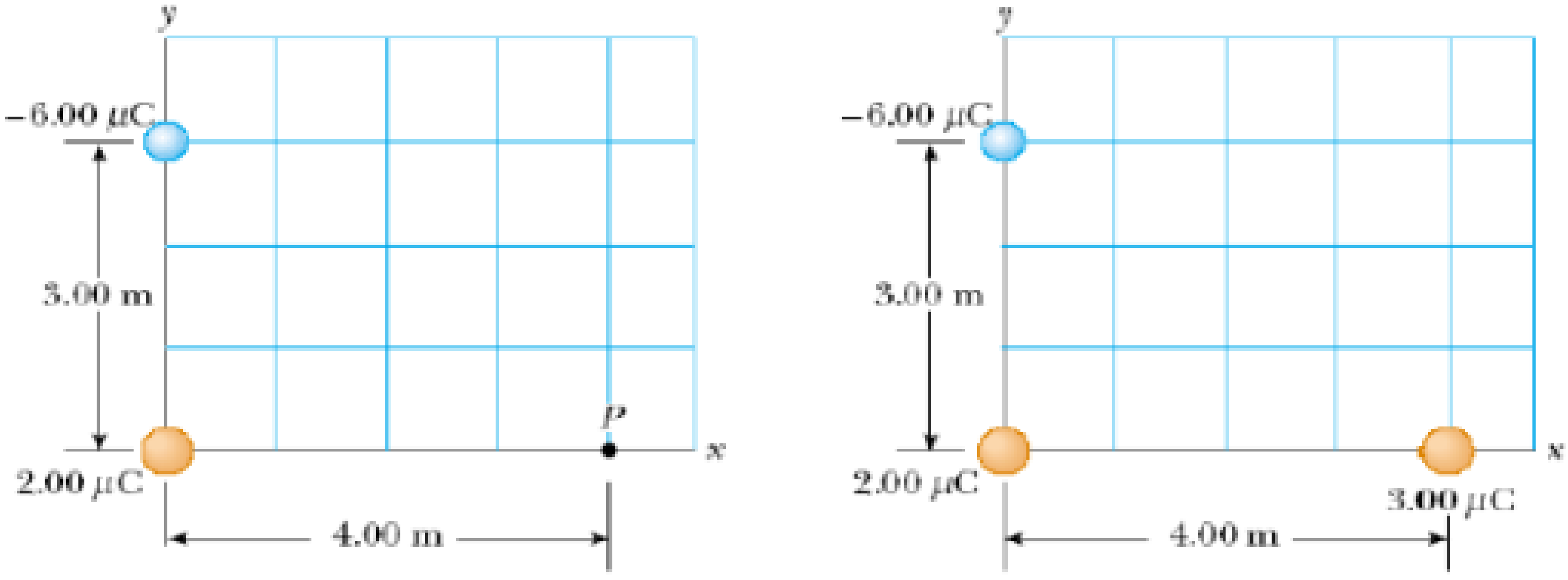
$$V = k_e \frac{q}{r}$$

Example 3 (The Electric Potential Due to Two Point Charges)

A charge $q_1 = 2.00\ \mu C$ is located at the origin, and a charge $q_2 = -6.00\ \mu C$ is located at (0, 3.00) m, as shown in Figure 6-a.

(A) Find the total electric potential due to these charges at the point P, whose coordinates are (4.00, 0) m.

(B) Find the change in potential energy of the system of two charges plus a charge $q_3 = 3.00\ \mu C$ as the latter charge moves from infinity to point P (Figure6-b).



(a) (b)