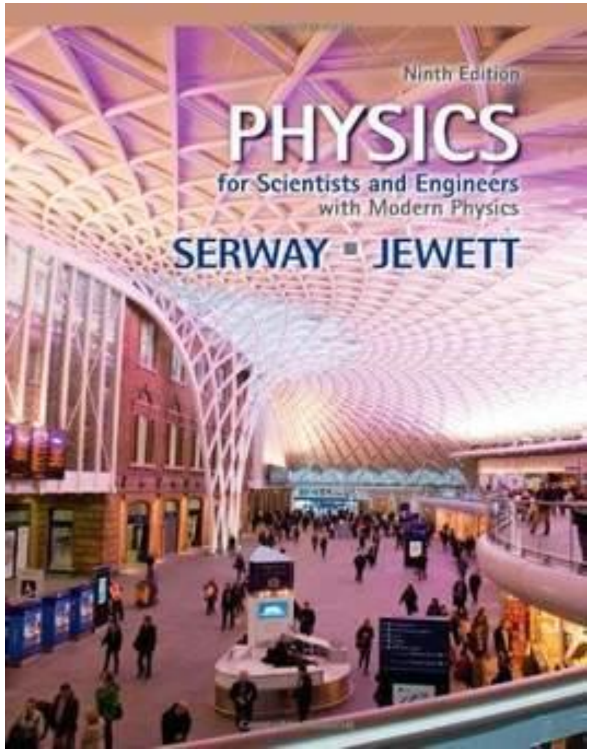
PHY 220
Electricity and Magnetism

Reference:

Serway R.A. and Jewett J.W., Physics for Scientists and Engineers with Modern Physics, 9th Edition,



Topics Outline

➢ Electric Fields
➢ Gauss's Law:
➢ Electric Potential
➢ Capacitance and dielectrics
➢ Sources of the Magnetic Field:
➢ Faraday's law
➢ Inductance

Chapter 1 :Electric Fields:

- **Outlines:**

✓Electric charges,

✓Coulomb's law,

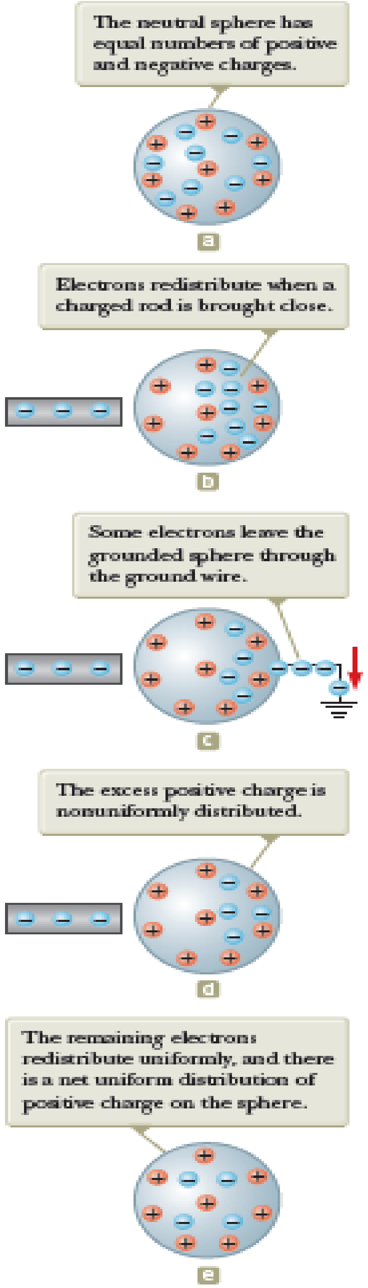
✓Electric field,

✓ Electric field of a continuous charge distribution,

✓ Motion of charged particles in a uniform electric field

Electric charges

- From simple experiments, it was found that there are two kinds of electric charges: positive and
- negative.
- Charges of the same sign repel one another and charges with opposite signs attract one another.
- Electric charge is always conserved.
- Electrical **conductors** are materials in which some of the electrons are free electrons that are not bound to atoms and can move relatively freely through the material; electrical **insulators** are materials in which all electrons are bound to atoms and cannot move freely through the material.



Coulomb's Law

- Charles Coulomb measured the magnitudes of the electric forces between charged objects.
- Coulomb's law is an equation giving the magnitude of the electric force (sometimes called the Coulomb force) between two point charges:

$$F_e = k_e \frac{|q_1||q_2|}{r^2}$$

where k_e is a constant called the **Coulomb constant.**

$$k_e = 8.987\,6 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$

This constant is also written in the form

$$k_e = \frac{1}{4\pi\epsilon_0}$$

where ε_o the constant is known as the **permittivity of free space** and has the value

$$\epsilon_0 = 8.854\,2 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$$

Table 23.1 **Charge and Mass of the Electron, Proton, and Neutron**

Particle	Charge (C)	Mass (kg)
Electron (e)	$-1.602\,176\,5 \times 10^{-19}$	$9.109\,4 \times 10^{-31}$
Proton (p)	$+1.602\,176\,5 \times 10^{-19}$	$1.672\,62 \times 10^{-27}$
Neutron (n)	0	$1.674\,93 \times 10^{-27}$

Example 23.1 The Hydrogen Atom

The electron and proton of a hydrogen atom are separated (on the average) by a distance of approximately 5.3×10^{-11} m. Find the magnitudes of the electric force and the gravitational force between the two particles.

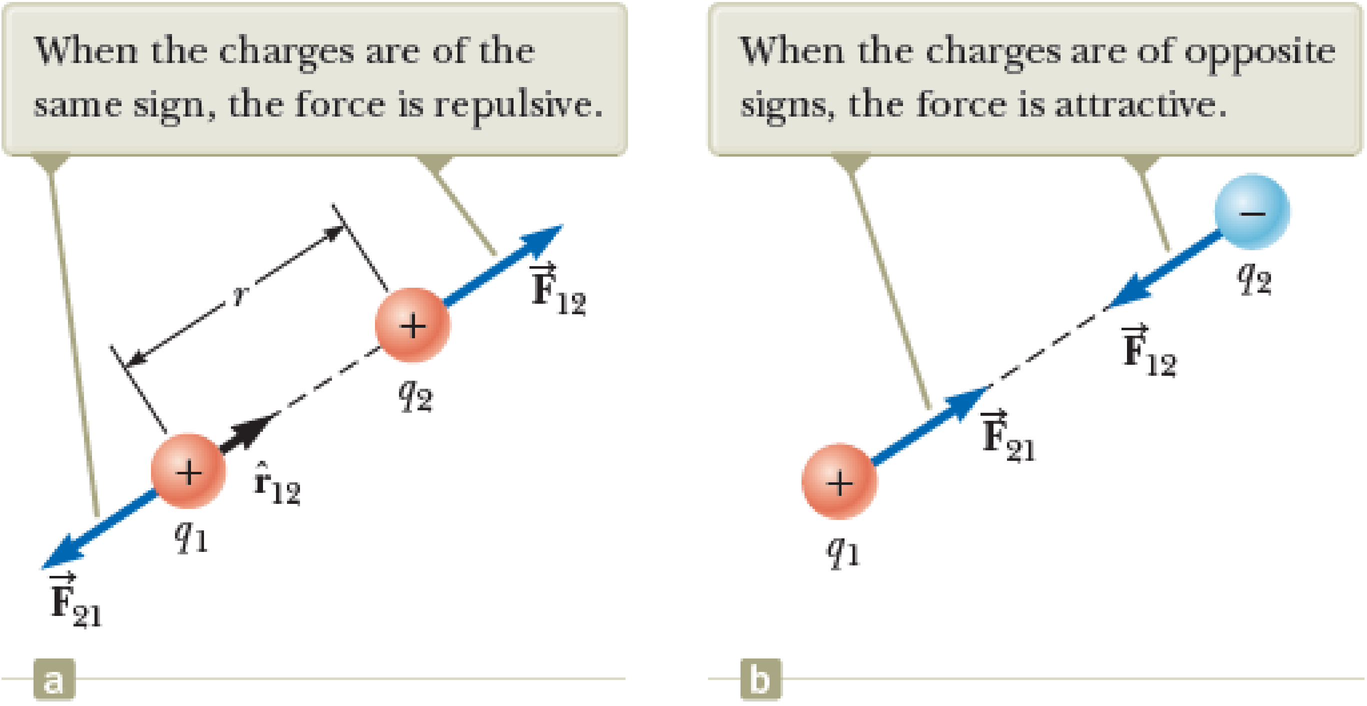
$$F_e = k_e \frac{|e||-e|}{r^2} = (8.988 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2) \frac{(1.60 \times 10^{-19}\ \text{C})^2}{(5.3 \times 10^{-11}\ \text{m})^2}$$

$$= \boxed{8.2 \times 10^{-8}\ \text{N}}$$

$$F_g = G \frac{m_e m_p}{r^2}$$

$$= (6.674 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2) \frac{(9.11 \times 10^{-31}\ \text{kg})(1.67 \times 10^{-27}\ \text{kg})}{(5.3 \times 10^{-11}\ \text{m})^2}$$

$$= \boxed{3.6 \times 10^{-47}\ \text{N}}$$



DO NOT FORGET !

Force is vector quantity so,

$$\vec{F}_e = k_e \frac{q_1 q_2}{r^2} \hat{r}$$

where $\hat{r}$ is a unit vector directed from q_1 toward q_2 or from q_2 toward q_1.

When more than two charges are present, the force between any pair of them is given by Equation 23.6. Therefore, the resultant force on any one of them equals the vector sum of the forces exerted by the other individual charges. For example, if four charges are present, the resultant force exerted by particles 2, 3, and 4 on particle 1 is

$$\vec{\mathbf{F}}_1 = \vec{\mathbf{F}}_{21} + \vec{\mathbf{F}}_{31} + \vec{\mathbf{F}}_{41}$$

Example 23.2 Find the Resultant Force

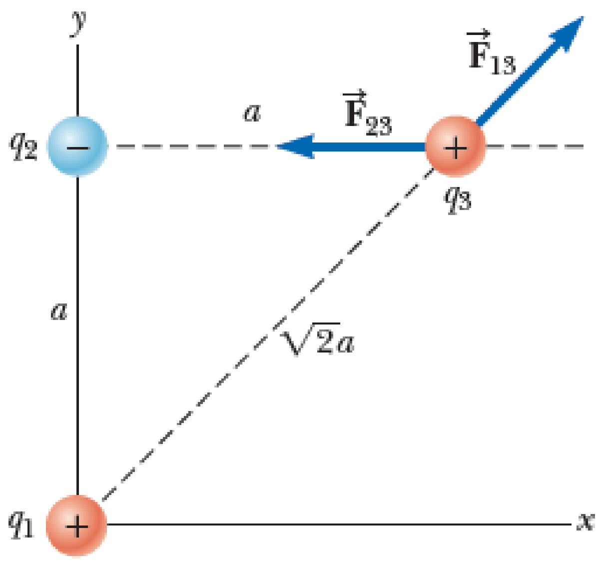
Consider three point charges located at the corners of a right triangle as shown in Figure 23.7, where $q_1 = q_3 = 5.00\ \mu C$, $q_2 = -2.00\ \mu C$, and $a = 0.100$ m. Find the resultant force exerted on q_3.

$$F_{23} = k_e \frac{|q_2||q_3|}{a^2}$$

$$= (8.988 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{(2.00 \times 10^{-6}\ \mathrm{C})(5.00 \times 10^{-6}\ \mathrm{C})}{(0.100\ \mathrm{m})^2} = 8.99\ \mathrm{N}$$

$$F_{13} = k_e \frac{|q_1||q_3|}{(\sqrt{2}\,a)^2}$$

$$= (8.988 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{(5.00 \times 10^{-6}\ \mathrm{C})(5.00 \times 10^{-6}\ \mathrm{C})}{2(0.100\ \mathrm{m})^2} = 11.2\ \mathrm{N}$$



$$F_{13x} = (11.2\ \mathrm{N}) \cos 45.0^\circ = 7.94\ \mathrm{N}$$

$$F_{13y} = (11.2\ \mathrm{N}) \sin 45.0^\circ = 7.94\ \mathrm{N}$$

$$F_{3x} = F_{13x} + F_{23x} = 7.94\ \mathrm{N} + (-8.99\ \mathrm{N}) = -1.04\ \mathrm{N}$$

$$F_{3y} = F_{13y} + F_{23y} = 7.94\ \mathrm{N} + 0 = 7.94\ \mathrm{N}$$

$$\vec{\mathbf{F}}_3 = \boxed{(-1.04\hat{\mathbf{i}} + 7.94\hat{\mathbf{j}})\ \mathrm{N}}$$

Electric Field

- If an arbitrary charge q is placed in an electric field **E**, it experiences an electric force given by

$$\vec{\mathbf{F}}_e = q\vec{\mathbf{E}} \qquad \text{(N/C)}$$

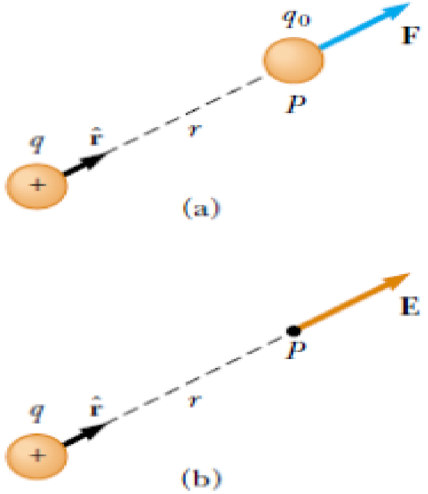
- The electric field is said to exist in the region of space around a charged object (the source charge). When another charged object (the test charge) enters this electric field an electric force acts on it.

- We can know the direction of the electric field from the charge q if q is positive, the force is in the same direction as the field. If q is negative, the force and the field are in opposite directions.

Consider a point charge q as a source charge. This charge creates an electric field at all points in space surrounding it. A test charge is placed at point P, a distance r from the source charge, as in Figure. According to Coulomb's law, the force exerted by q on the test charge is:

$$\vec{F}_e = k_e \frac{q\, q_\circ}{r^2}\, \hat{r}$$

the electric field created by q is: $\vec{E} = k_e \dfrac{q}{r^2}\, \hat{r}$

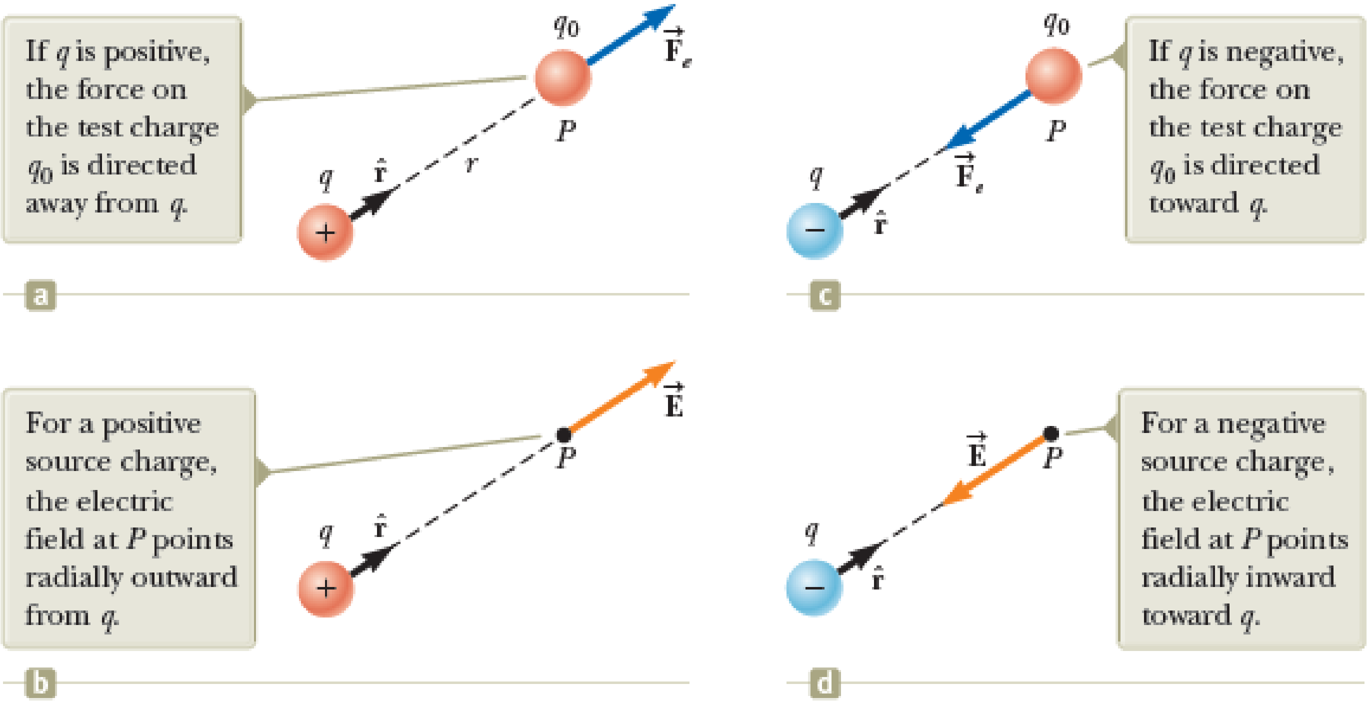


❑ **Electric field at point P due to a group of point charges:**

At any point *P*, the total electric field due to a group of source charges equals the vector sum of the electric fields of all the charges.

$$\vec{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{r}_i$$

Where r_i is the distance from the i^{th} source charge q_i to the point *P*.



Example 23.5 Electric Field Due to Two Charges

A charge $q_1 = 7.0\ \mu C$ is located at the origin, and a second charge $q_2 = -5.0\ \mu C$ is located on the x axis, 0.30 m from the origin (Fig. 23.14). Find the electric field at the point P, which has coordinates (0, 0.40) m.

$$E_1 = k_e \frac{|q_1|}{r_1^2} = (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{(7.0 \times 10^{-6}\ \mathrm{C})}{(0.40\ \mathrm{m})^2}$$
$$= 3.9 \times 10^5\ \mathrm{N/C}$$

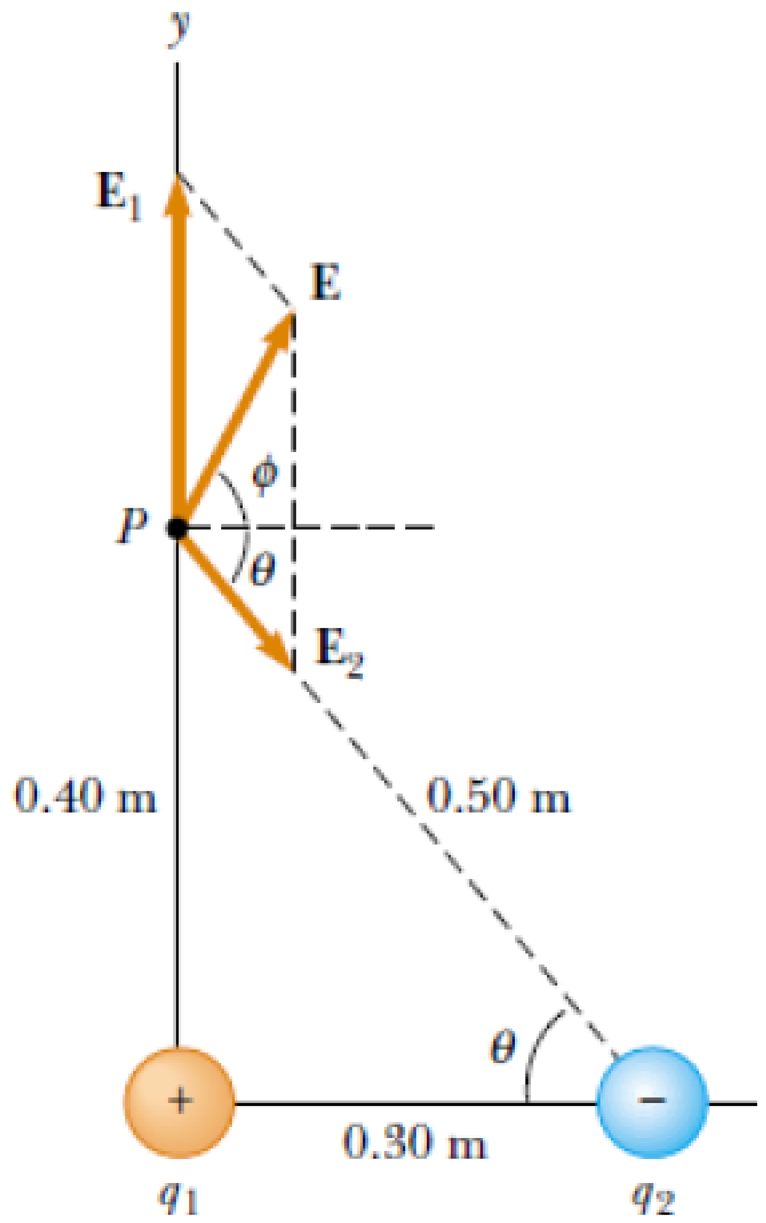
$$E_2 = k_e \frac{|q_2|}{r_2^2} = (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{(5.0 \times 10^{-6}\ \mathrm{C})}{(0.50\ \mathrm{m})^2}$$
$$= 1.8 \times 10^5\ \mathrm{N/C}$$

The vector $\mathbf{E}_1$ has only a y component. The vector $\mathbf{E}_2$ has an x component given by $E_2 \cos\theta = \frac{3}{5}E_2$ and a negative y component given by $-E_2 \sin\theta = -\frac{4}{5}E_2$. Hence, we can express the vectors as

$$\mathbf{E}_1 = 3.9 \times 10^5 \hat{\mathbf{j}}\ \mathrm{N/C}$$
$$\mathbf{E}_2 = (1.1 \times 10^5 \hat{\mathbf{i}} - 1.4 \times 10^5 \hat{\mathbf{j}})\ \mathrm{N/C}$$

The resultant field $\mathbf{E}$ at P is the superposition of $\mathbf{E}_1$ and $\mathbf{E}_2$:

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \boxed{(1.1 \times 10^5 \hat{\mathbf{i}} + 2.5 \times 10^5 \hat{\mathbf{j}})\ \mathrm{N/C}}$$



Example : Electric Field of a Dipole

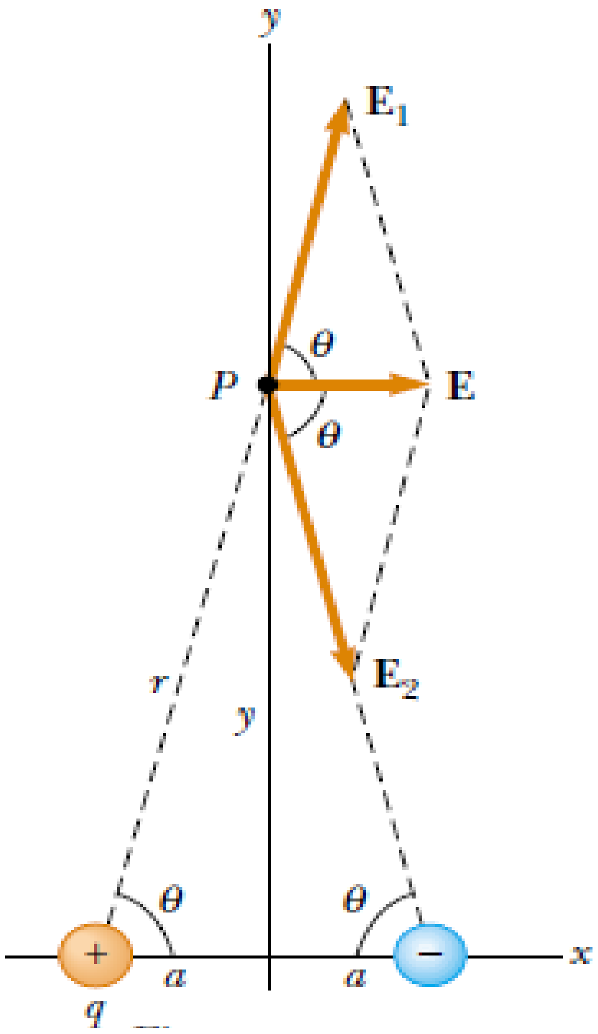
An **electric dipole** is defined as a positive charge q and a negative charge $-q$ separated by a distance $2a$. For the dipole shown in Figure 23.15, find the electric field $\mathbf{E}$ at P due to the dipole, where P is a distance $y >> a$ from the origin.

$$E_x = k_e \frac{q}{a^2 + y^2} \cos\theta + k_e \frac{q}{a^2 + y^2} \cos\theta = 2k_e \frac{q}{a^2 + y^2} \cos$$

$$E_y = k_e \frac{q}{a^2 + y^2} \sin\theta - k_e \frac{q}{a^2 + y^2} \sin\theta = 0$$

$$\cos\theta = \frac{a}{r} = \frac{a}{(a^2 + y^2)^{1/2}}$$

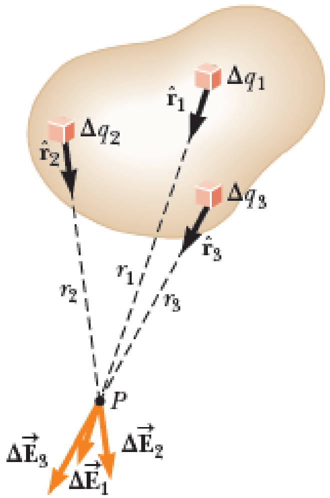
$$E_x = 2k_e \frac{q}{a^2 + y^2} \left[\frac{a}{(a^2 + y^2)^{1/2}}\right] = k_e \frac{2aq}{(a^2 + y^2)^{3/2}}$$



$y >> a$, neglect a^2 → $E \approx k_e \dfrac{2aq}{y^3}$

the magnitude of the electric field created by the dipole varies as $1/r^3$,

Electric Field of a Continuous Charge Distribution



- The electric field at *P* due to one charge element carrying charge Δq is

$$\Delta\vec{\mathbf{E}} = k_e \frac{\Delta q}{r^2}\hat{\mathbf{r}}$$

where *r* is the distance from the charge element to point *P* and $\hat{\mathbf{r}}$ is a unit vector directed from the element toward *P*. The total electric field at *P* due to all elements in the charge distribution is approximately

$$\vec{\mathbf{E}} \approx k_e \sum_i \frac{\Delta q_i}{r_i^{\,2}}\hat{\mathbf{r}}_i$$

where the index *i* refers to the *i*th element in the distribution. Because the number of elements is very large and the charge distribution is modeled as continuous, the total field at *P* in the limit $\Delta q \rightarrow 0$ is

$$\vec{\mathbf{E}} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^{\,2}}\hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2}\hat{\mathbf{r}}$$

- If a charge Q is uniformly distributed throughout a volume V, the **volume charge density** ρ is defined by

Volume charge density ▶

$$\rho \equiv \frac{Q}{V}$$

where ρ has units of coulombs per cubic meter (C/m^3).

- If a charge Q is uniformly distributed on a surface of area A, the **surface charge density** σ (Greek letter sigma) is defined by

Surface charge density ▶

$$\sigma \equiv \frac{Q}{A}$$

where σ has units of coulombs per square meter (C/m^2).

- If a charge Q is uniformly distributed along a line of length ℓ, the **linear charge density** λ is defined by

Linear charge density ▶

$$\lambda \equiv \frac{Q}{\ell}$$

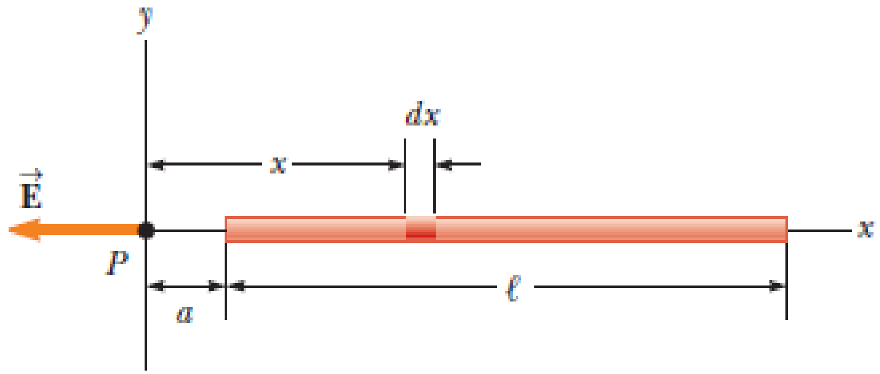
where λ has units of coulombs per meter (C/m).

- If the charge is nonuniformly distributed over a volume, surface, or line, the amounts of charge dq in a small volume, surface, or length element are

$$dq = \rho\, dV \qquad dq = \sigma\, dA \qquad dq = \lambda\, d\ell$$

Example 23.7 The Electric Field Due to a Charged Rod

A rod of length ℓ has a uniform positive charge per unit length λ and a total charge Q. Calculate the electric field at a point P that is located along the long axis of the rod and a distance a from one end (Fig. 23.15).



$$dE = k_e \frac{dq}{x^2} = k_e \frac{\lambda\, dx}{x^2}$$

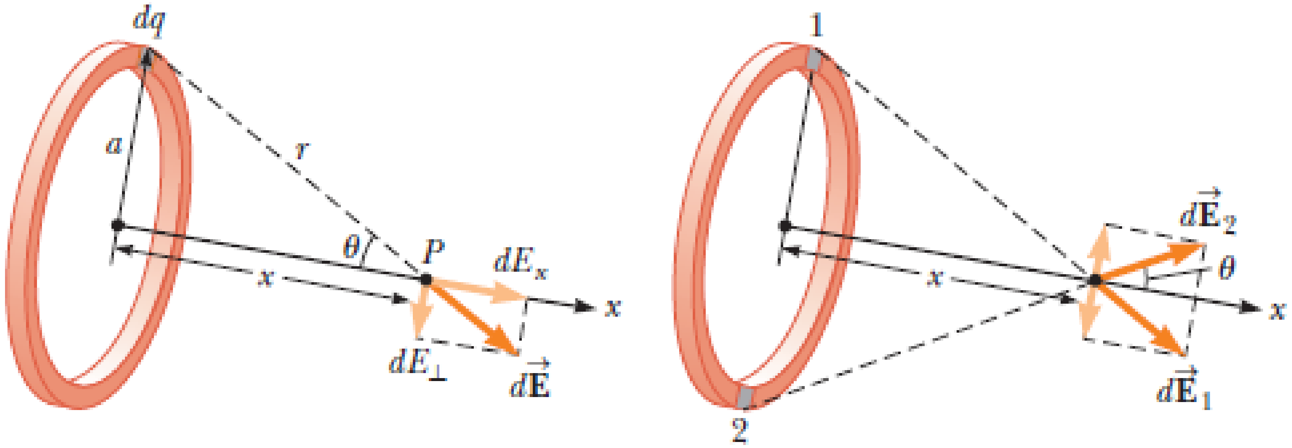
$$E = \int_a^{\ell + a} k_e \lambda \frac{dx}{x^2}$$

$$E = k_e \lambda \int_a^{\ell + a} \frac{dx}{x^2} = k_e \lambda \left[-\frac{1}{x} \right]_a^{\ell + a}$$

$$(1) \quad E = k_e \frac{Q}{\ell} \left(\frac{1}{a} - \frac{1}{\ell + a} \right) = \boxed{\frac{k_e Q}{a(\ell + a)}}$$

Example 23.8 The Electric Field of a Uniform Ring of Charge

A ring of radius a carries a uniformly distributed positive total charge Q. Calculate the electric field due to the ring at a point P lying a distance x from its center along the central axis perpendicular to the plane of the ring (Fig. 23.16a).



b

$$(1) \quad dE_x = k_e \frac{dq}{r^2}\cos\theta = k_e \frac{dq}{a^2 + x^2}\cos\theta$$

$$(2) \quad \cos\theta = \frac{x}{r} = \frac{x}{(a^2 + x^2)^{1/2}}$$

$$dE_x = k_e \frac{dq}{a^2 + x^2}\left[\frac{x}{(a^2 + x^2)^{1/2}}\right] = \frac{k_e x}{(a^2 + x^2)^{3/2}}\, dq$$

$$E_x = \int \frac{k_e x}{(a^2 + x^2)^{3/2}}\, dq = \frac{k_e x}{(a^2 + x^2)^{3/2}} \int dq$$

$$(3) \quad E = \boxed{\frac{k_e x}{(a^2 + x^2)^{3/2}}\, Q}$$

WHAT IF?

This result shows that the field is zero at $x = 0$.

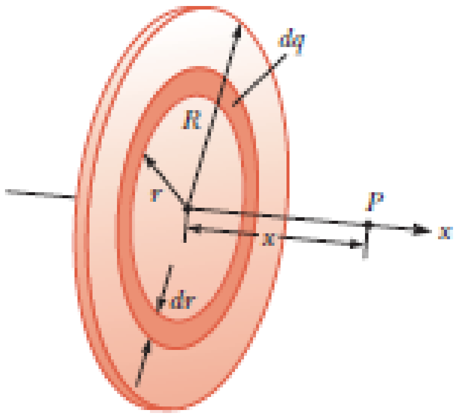
let $x << a$, which results in

$$E_x = \frac{k_e Q}{a^3}\, x$$

reduces to $k_e Q/x^2$ if $x >> a$, so the ring acts like a point charge

Example 23.9 The Electric Field of a Uniformly Charged Disk

A disk of radius R has a uniform surface charge density σ. Calculate the electric field at a point P that lies along the central perpendicular axis of the disk and a distance x from the center of the disk (Fig. 23.17).



$$dq = \sigma\ dA = \sigma(2\pi r\ dr) = 2\pi\sigma r\, dr$$

$$dE_x = \frac{k_e x}{(r^2 + x^2)^{3/2}}(2\pi\sigma r\ dr)$$

$$E_x = k_e x\pi\sigma\int_0^R \frac{2r\, dr}{(r^2 + x^2)^{3/2}}$$

$$= k_e x\pi\sigma\int_0^R (r^2 + x^2)^{-3/2} d(r^2)$$

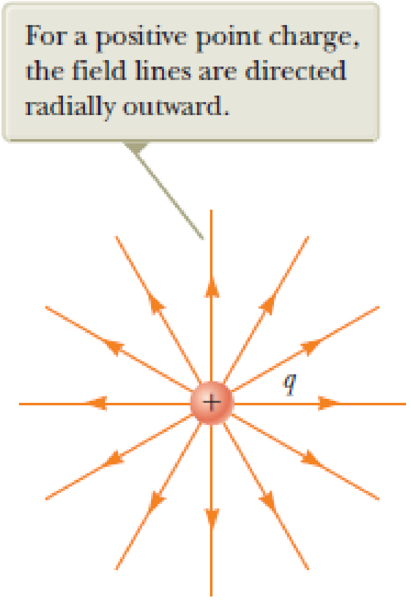
$$= k_e x\pi\sigma\left[\frac{(r^2 + x^2)^{-1/2}}{-1/2}\right]_0^R = \boxed{2\pi k_e\sigma\left[1 - \frac{x}{(R^2 + x^2)^{1/2}}\right]}$$

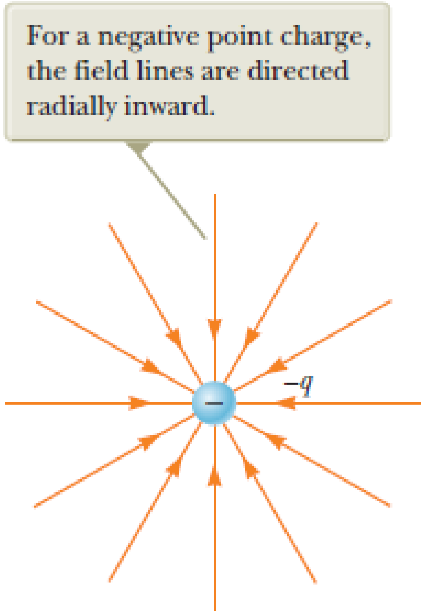
Electric Field Lines

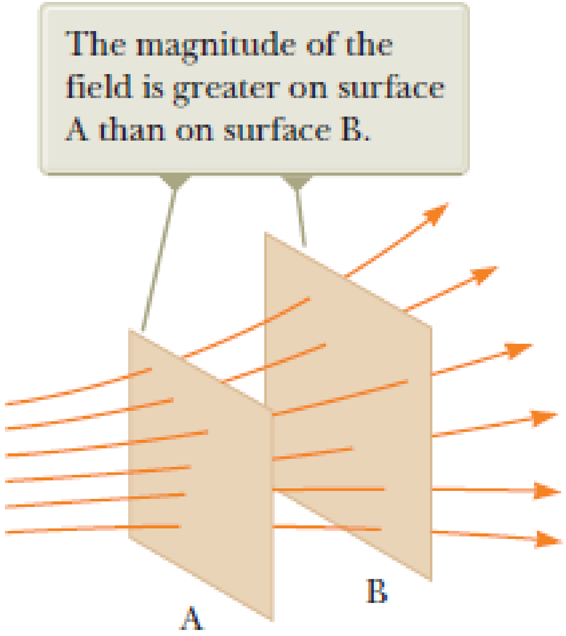
Convenient way of visualizing electric field patterns is to draw lines, called **electric field lines** and first introduced by Faraday, that are related to the electric field in a region of space in the following manner:

- The electric field vector **E** is tangent to the electric field line at each point.The line has a direction, indicated by an arrowhead, that is the same as that of the electric field vector.

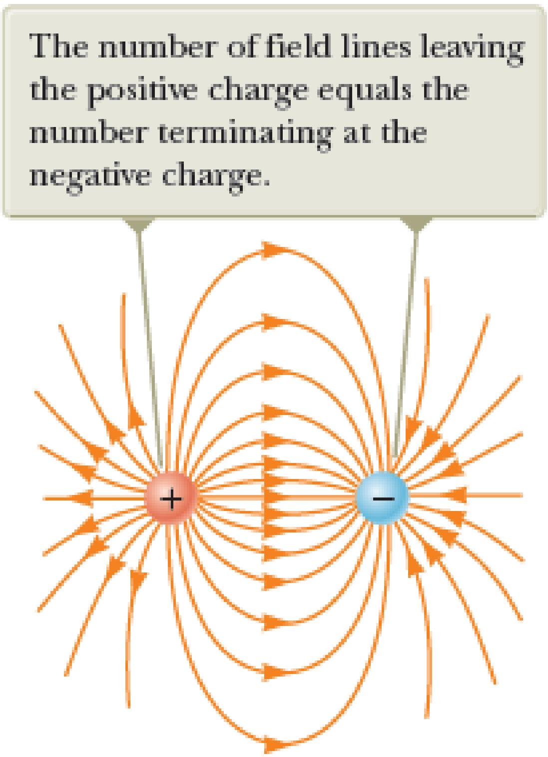
- The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region.

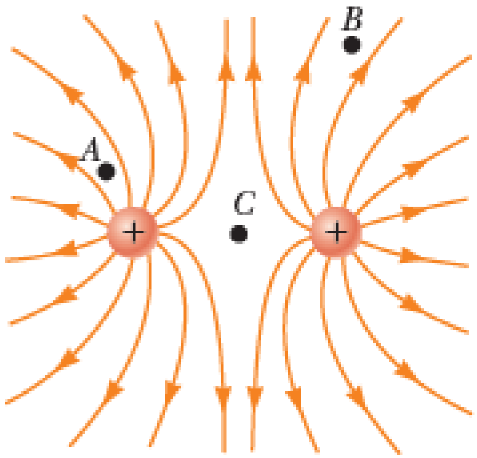


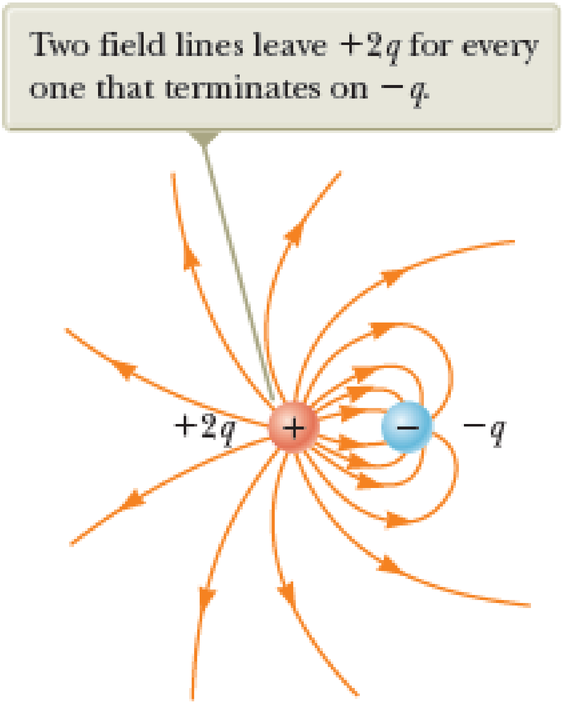




Electric Field Lines



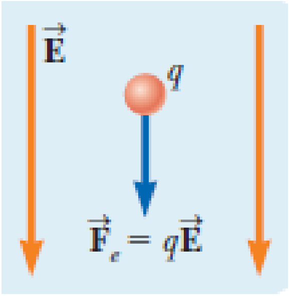




Motion of a Charged Particle in a Uniform Electric Field

When a particle of charge q and mass m is placed in an electric field $\vec{E}$, the electric force exerted on the charge is q$\vec{E}$according to Equation $\vec{F} = q\vec{E}$ in the particle in in the particle in a field model. If that is the only force exerted on the particle, it must be the net force, and it causes the particle to accelerate according to the particle under a net force model. Therefore:

$$\vec{F}_e = q\vec{E} = m\vec{a}$$



and the acceleration of the particle is

$$\vec{a} = \frac{q\vec{E}}{m}$$

- ❑ If **E** is uniform (that is, constant in magnitude and direction), and the particle is free to move, the electric force on the particle is constant and we can apply the particle under constant acceleration model to the motion of the particle. Therefore, the particle in this situation is described by *three* analysis models:
- ✓ Particle in a field,
- ✓ particle under a net force,
- ✓ and particle under constant acceleration.
- ❑ If the particle has a positive charge, its acceleration is in the direction of the electric field.
- ❑ If the particle has a negative charge, its acceleration is in the direction opposite the electric field.

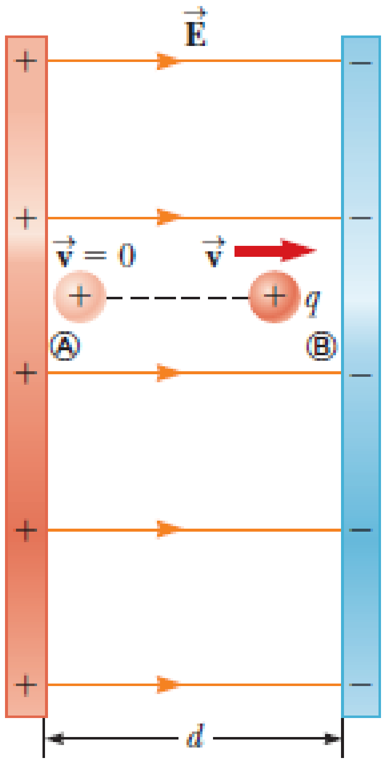
Example 23.10 An Accelerating Positive Charge: Two Models

A uniform electric field $\vec{\mathbf{E}}$ is directed along the x axis between parallel plates of charge separated by a distance d as shown in Figure 23.23. A positive point charge q of mass m is released from rest at a point Ⓐ next to the positive plate and accelerates to a point Ⓑ next to the negative plate.

(A) Find the speed of the particle at Ⓑ by modeling it as a particle under constant acceleration.

$$v_f^2 = v_i^2 + 2a(x_f - x_i) = 0 + 2a(d - 0) = 2ad$$

$$v_f = \sqrt{2ad} = \sqrt{2\left(\frac{qE}{m}\right)d} = \boxed{\sqrt{\frac{2qEd}{m}}}$$



(B) Find the speed of the particle at Ⓑ by modeling it as a nonisolated system in terms of energy.

$$W = \Delta K$$

$$F_e \Delta x = K_{Ⓑ} - K_{Ⓐ} = \tfrac{1}{2}mv_f^2 - 0 \rightarrow v_f = \sqrt{\frac{2F_e\Delta x}{m}}$$

$$v_f = \sqrt{\frac{2(qE)(d)}{m}} = \boxed{\sqrt{\frac{2qEd}{m}}}$$

Chapter 2 :Gauss's Law

Outlines:

- ✓Electric Flux
- ✓ Gauss's Law
- ✓ Application of Gauss's
- ✓ Law to Various Charge Distributions
- ✓ Conductors in Electrostatic Equilibrium

1-Electric Flux

Consider an electric field that is uniform in both magnitude and direction, as shown in Figure.
The field lines penetrate a rectangular surface of area *A*, whose plane is oriented perpendicular to the field. Note that the number of lines per unit area is proportional to the magnitude of the electric field. Therefore, the total number of lines penetrating the surface is proportional to the product *EA*. This product of the magnitude of the electric field *E* and surface area *A* perpendicular to the field is called the electric flux φ_E.

$$\Phi_E = EA \quad (\mathrm{N.m^2/C})$$

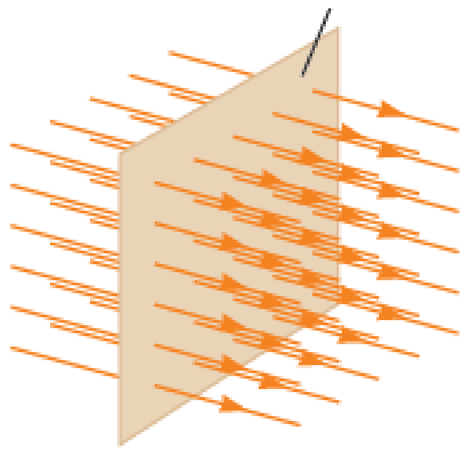


Figure 24.1 Field lines representing a uniform electric field penetrating a plane of area perpendicular to the field.

1-Electric Flux

- If the surface under consideration is not perpendicular to the field, the flux through it must be less than that given by $\Phi = EA$ where the normal to the surface of area A is at an angle u to the uniform electric field. Notice that the number of lines that cross this area A is equal to the number of lines that cross the area $A_\perp$,

$$\boxed{\Phi_E = EA_\perp = EA\cos\theta}$$

- In more general situations, the electric field may vary over a large surface

$$\boxed{\Phi_{E,i} = E_i\,\Delta A_i\,\cos\theta_i = \vec{\mathbf{E}}_i \cdot \Delta\vec{\mathbf{A}}_i}$$

Therefore, the general definition of electric flux is:

$$\boxed{\Phi_E \equiv \int_{\text{surface}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}}$$

(surface integral)

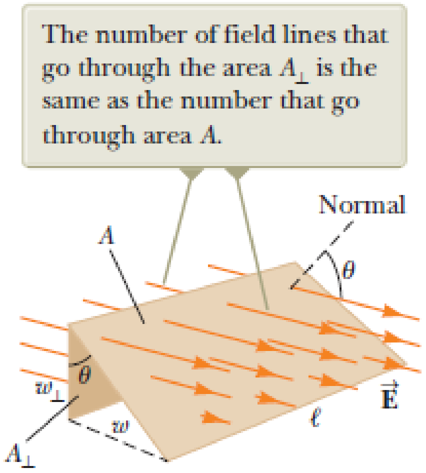


Figure 24.2 Field lines representing a uniform electric field penetrating an area A whose normal is at an angle θ to the field.

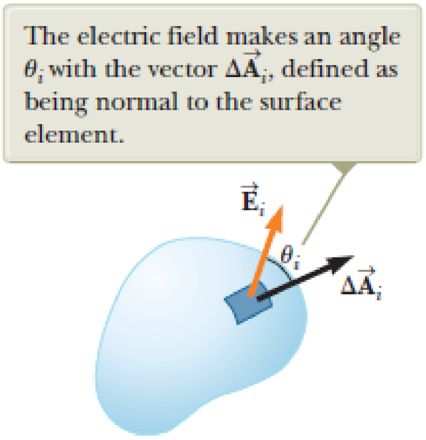


Figure 24.3 A small element of surface area ΔA_i in an electric field.

➢ When an area is constructed such that a closed surface is formed, we shall adopt the convention that the flux lines passing into the interior of the volume are negative and those passing out of the interior of the volume are positive.

Example 24.1 **Flux Through a Cube**

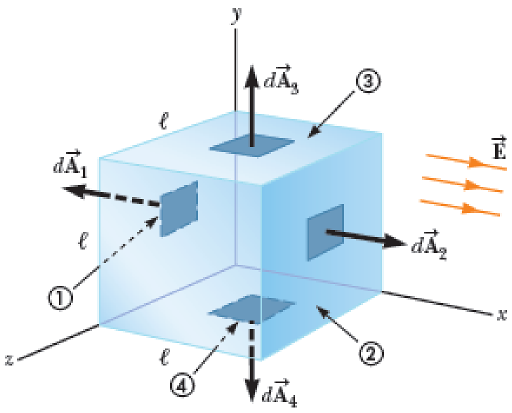
Consider a uniform electric field $\vec{\mathbf{E}}$ oriented in the x direction in empty space. A cube of edge length ℓ is placed in the field, oriented as shown in Figure 24.5. Find the net electric flux through the surface of the cube.

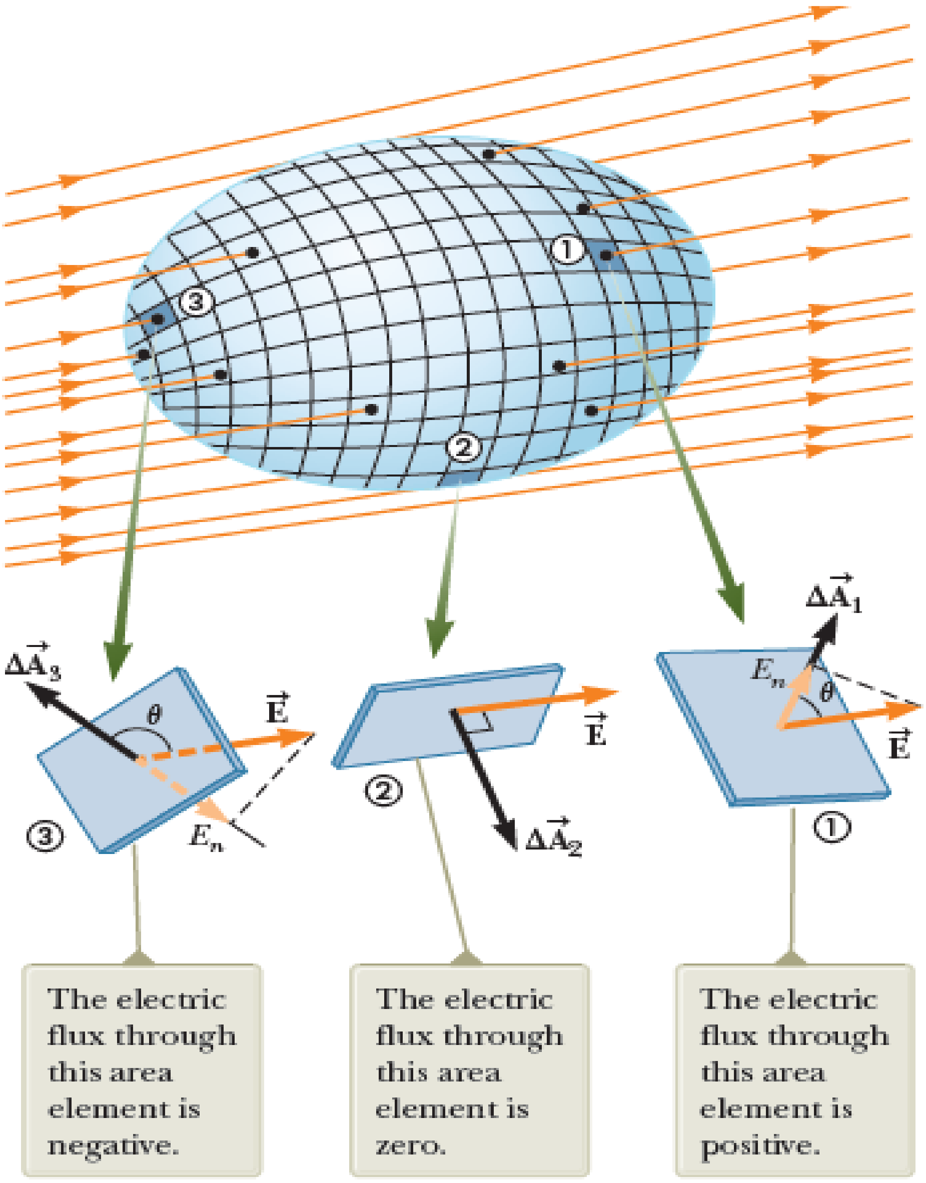
$$\Phi_E = \int_1 \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \int_2 \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

$$\int_1 \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_1 E(\cos 180^\circ)\, dA = -E\int_1 dA = -EA = -E\ell^2$$

$$\int_2 \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_2 E(\cos 0^\circ)\, dA = E\int_2 dA = +EA = E\ell^2$$

$$\Phi_E = -E\ell^2 + E\ell^2 + 0 + 0 + 0 + 0 = \boxed{0}$$

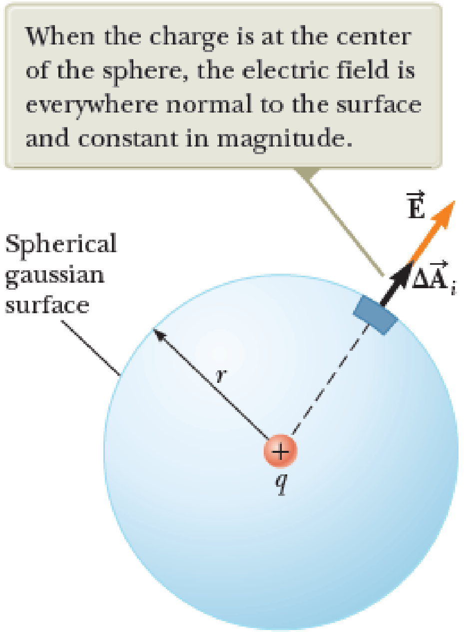




2-Gauss's Law

- we describe a general relationship between the net electric flux through a closed surface (often called a ***gaussian surface***) and the charge enclosed by the surface. This relationship, known as *Gauss's law,* is of fundamental importance in the study of electric fields.

- we know that the magnitude of the electric field everywhere on the surface of the sphere is $\mathrm{E} = Kq/r^2$.
- The field lines are directed radially outward and hence are perpendicular to the surface at every point on the surface. That is, at each surface point, **E** is parallel to the vector $\Delta\mathbf{A}_i$ representing a local element of area $\Delta\mathbf{A}_i$ surrounding the surface point.



$$\vec{\mathbf{E}} \cdot \Delta\vec{\mathbf{A}}_i = E\,\Delta A_i$$

the net flux through the Gaussian surface is

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E\,dA = E \oint dA$$

The surface is spherical $\longrightarrow$ $\oint dA = A = 4\pi r^2$.

$$\Phi_E = k_e \frac{q}{r^2}(4\pi r^2) = 4\pi k_e q$$

that $k_e = 1/4\pi\epsilon_0$,

$$\boxed{\Phi_E = \frac{q}{\epsilon_0}}$$

the net flux through the spherical surface is proportional to the charge inside the surface where $\vec{\mathbf{E}}$ is the total electric field at any point on the surface produced by the vector addition of the electric fields at that point due to the individual charges. Consider the system of charges shown in Figure 24.9. The surface S surrounds only one charge, q_1; hence, the net flux through S is q_1/ϵ_0. The flux through S due to charges q_2, q_3, and q_4 outside it is zero because each electric field line from these charges that enters S at one point leaves it at another. The surface S' surrounds charges q_2 and q_3; hence, the net flux through it is $(q_2 + q_3)/\epsilon_0$. Finally, the net flux through surface S'' is zero because there is no charge inside this surface. That is, *all* the electric field lines that enter S'' at one point leave at another. Charge q_4 does not contribute to the net flux through any of the surfaces.

The mathematical form of **Gauss's law** is a generalization of what we have just described and states that the net flux through *any* closed surface is

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0}$$

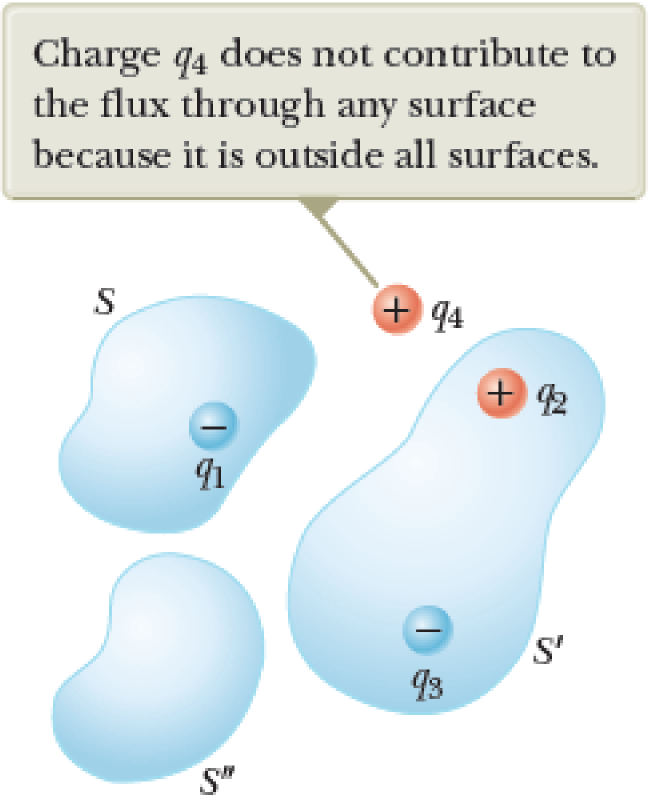


Figure 24.9 The net electric flux through any closed surface depends only on the charge *inside* that surface. The net flux through surface S is q_1/ϵ_0, the net flux through surface S' is $(q_2 + q_3)/\epsilon_0$, and the net flux through surface S'' is zero.

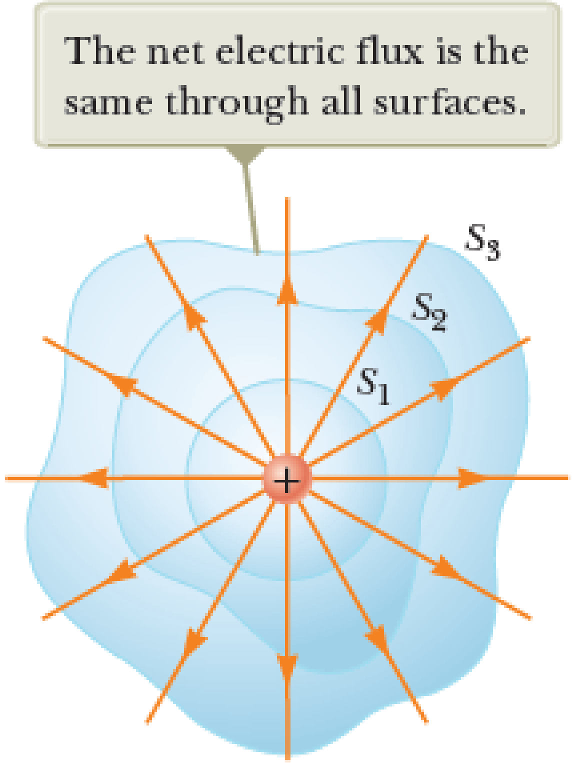


Figure 24.7 Closed surfaces of various shapes surrounding a positive charge.

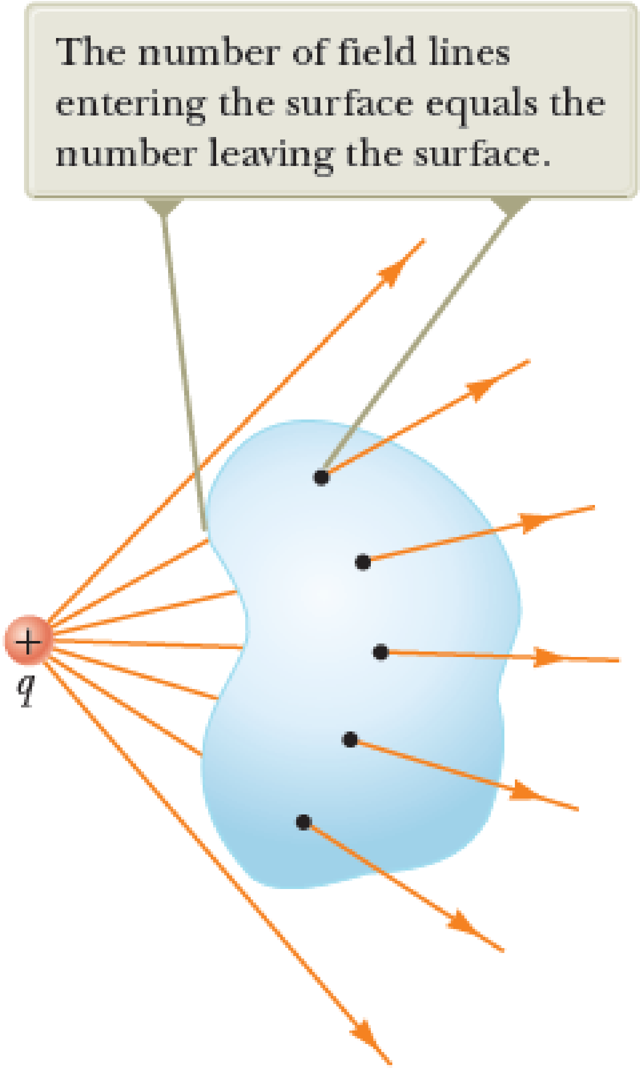


Figure 24.8 A point charge located *outside* a closed surface.

EXAMPLE: Flux Due to a Point Charge

A spherical Gaussian surface surrounds a point charge q. Describe what happens to the total flux through the surface if:

(A) the charge is tripled,

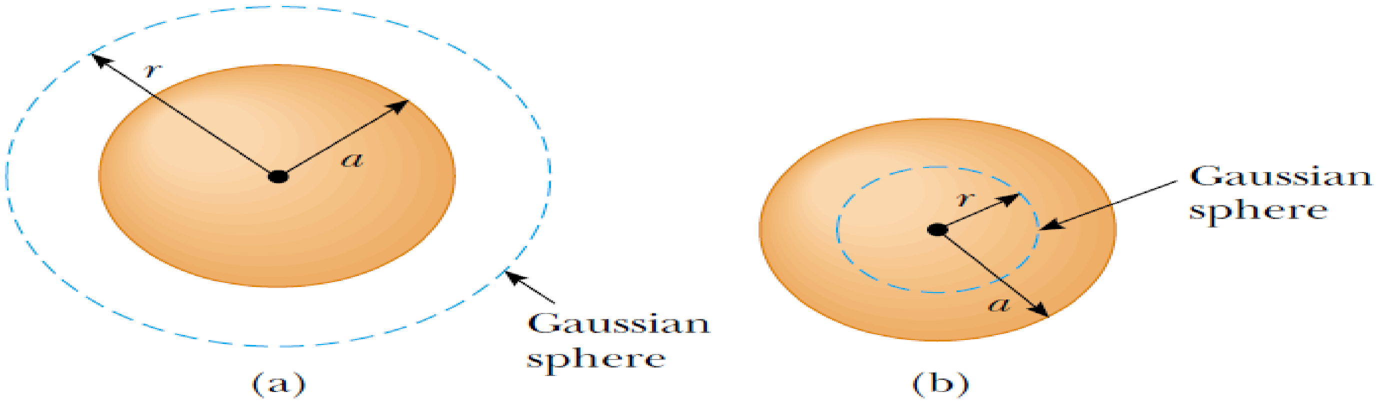
(B) the radius of the sphere is doubled,

(C) the surface is changed to a cube,

(D) the charge is moved to another location inside the surface.

A Spherically Symmetric Charge Distribution

- An insulating solid sphere of radius *a has a uniform volume* charge density ρ and carries a total positive charge *Q as in figure below*



A. Calculate the magnitude of the electric field at a point outside the sphere.

B. Find the magnitude of the electric field at a point inside the sphere.

Solution

(A)

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E\, dA = \frac{Q}{\epsilon_0}$$

$$\oint E\, dA = E \oint dA = E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2} = \boxed{k_e \frac{Q}{r^2} \quad (\text{for } r > a)}$$

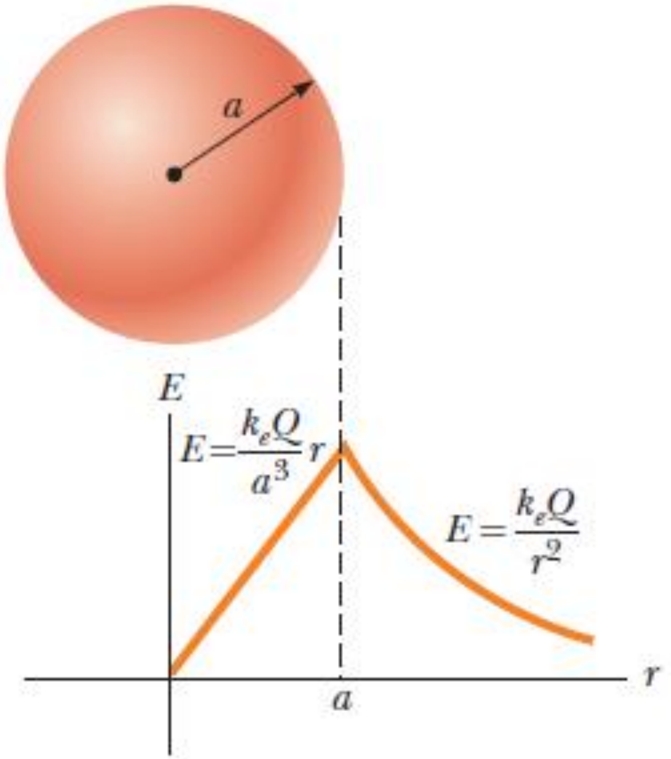
(B) In this case the Gaussian surface of volume $V`$ is less than Q. $\Rightarrow$ $q_{\text{in}} = \rho V' = \rho(\frac{4}{3}\pi r^3)$

$$E = \frac{q_{\text{in}}}{4\pi\epsilon_0 r^2} = \frac{\rho(\frac{4}{3}\pi r^3)}{4\pi\epsilon_0 r^2} = \frac{\rho}{3\epsilon_0} r$$

$$\rho = Q/\tfrac{4}{3}\pi a^3 \text{ and } \epsilon_0 = 1/4\pi k$$

$$E = \frac{Q/\frac{4}{3}\pi a^3}{3(1/4\pi k_e)} r = \boxed{k_e \frac{Q}{a^3} r \quad (\text{for } r < a)}$$

WHAT IF? Suppose the radial position $r = a$ is approached from inside the sphere and from outside. Do we obtain the same value of the electric field from both directions?



the electric field approaches a value from the outside given by:

$$E = \lim_{r \to a} \left(k_e \frac{Q}{r^2} \right) = k_e \frac{Q}{a^2}$$

the electric field approaches a value from the inside given by:

$$E = \lim_{r \to a} \left(k_e \frac{Q}{a^3} r \right) = k_e \frac{Q}{a^3} a = k_e \frac{Q}{a^2}$$

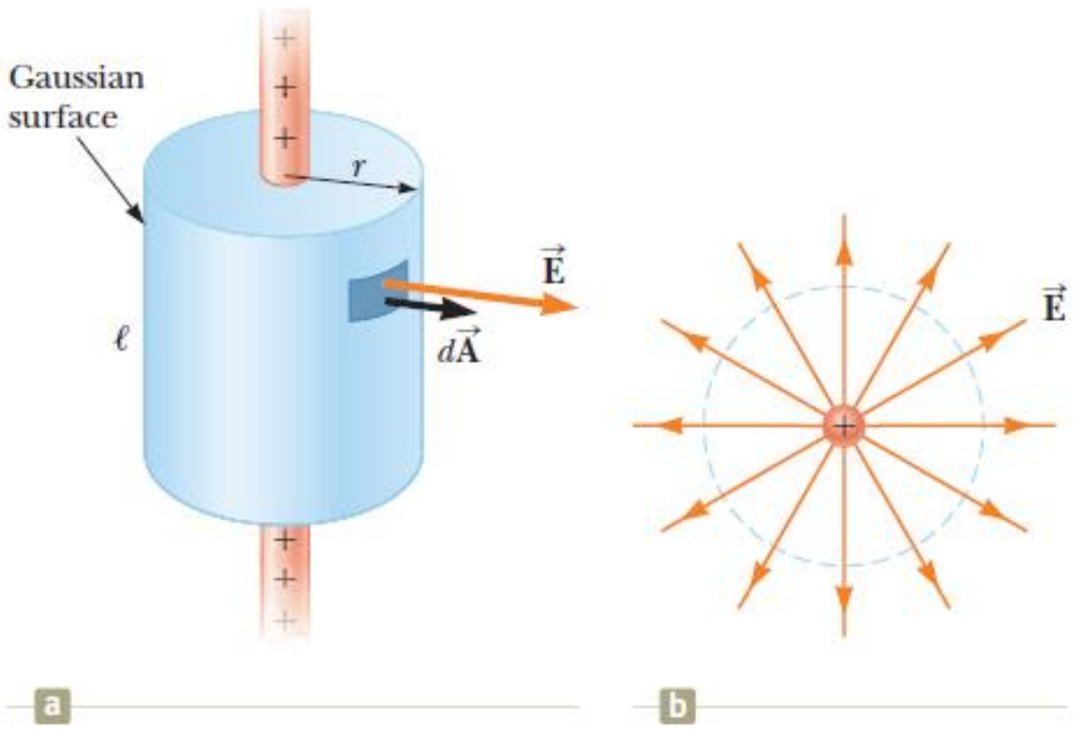
Example 24.4 A Cylindrically Symmetric Charge Distribution

Find the electric field a distance *r* from a line of positive charge of infinite length and constant charge per unit length λ

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E \oint dA = EA = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E(2\pi r\ell) = \frac{\lambda\ell}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \boxed{2k_e \frac{\lambda}{r}}$$



Example 24.5 A Plane of Charge

Find the electric field due to an infinite plane of positive charge with uniform surface charge density σ.

A gaussian surface that reflects the symmetry is a small cylinder whose axis is perpendicular to the plane and whose ends each have an area *A* and are equidistant from the plane. Because **E**s is parallel to the curved surface of the cylinder and therefore perpendicular to ***dA***
at all points on this surface

$$\Phi_E = 2EA = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

$$E = \boxed{\frac{\sigma}{2\epsilon_0}}$$

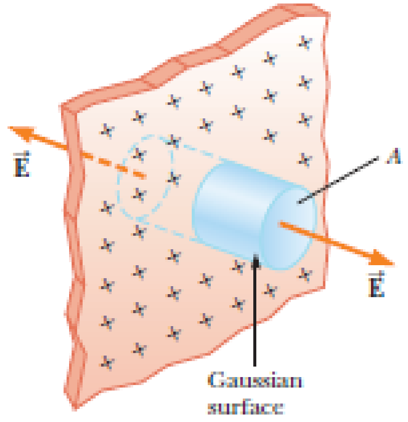


Figure 24.13 (Example 24.5) A cylindrical gaussian surface penetrating an infinite plane of charge. The flux is EA through each end of the gaussian surface and zero through its curved surface.

Conductors in Electrostatic Equilibrium

- A good electrical conductor contains charges (electrons)

  are not bound to any atom and therefore are free to move about within the material.
- When there is no net motion of charge within a conductor, the conductor is in electrostatic equilibrium.
- A conductor in electrostatic equilibrium has the following properties:

1. The electric field is zero everywhere inside the conductor.
2. If an isolated conductor carries a charge, the charge resides on its surface.
3. 3. The electric field just outside a charged conductor is perpendicular to the surface of the conductor and has a magnitude σ/ε_0, where σ is the surface charge density at that point.

Chapter 3 :Electric Potential

Outlines:

- ✓ Potential energy and electric potential,
- ✓ Electric potential difference in a uniform electric field,
- ✓ Electric potential due to point charges,
- ✓ Obtaining the value of the electric field from the electric potential,
- ✓ Electric potential due to continuous charge distributions,
- ✓ Electric potential due to charged conductor,
- ✓ Application of electrostatics.

Potential energy and electric potential

- When a test charge q_0 is placed in an electric field created by some source charge distribution, the electric force acting on the test charge is $\mathbf{F} = q_0\mathbf{E}$. The force is *conservative* because the force between charges described by *Coulomb's law* is conservative.
- When the test charge is moved in the field by some external agent, the work done by the field on the charge is equal to the negative of the work done by the external agent causing the displacement.
- For infinitesimal displacement $\overrightarrow{ds}$ of a charge, the work done by the electric field on the charge is:

$$\boxed{W_{\text{int}} = \vec{\mathbf{F}}_e \cdot d\vec{\mathbf{s}} = q\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}}$$

- The potential energy of the charge–field system is changed by an amount :

$$\boxed{dU = -W_{\text{int}} = -q\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}}$$

- For a finite displacement of the charge from point A to point B, the change in *potential energy* of the system $\Delta U = U_B - U_A$ is:

$$\boxed{\Delta U = -q\int_{Ⓐ}^{Ⓑ} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}}$$

- The potential energy per unit charge U/q_0 is independent of the value of q_0 and has a value at every point in an electric field, is called the ***electric potential*** (or simply the *potential)* **V**.

$$\boxed{V = \frac{U}{q}}$$

the unit of electric potential is volt $\Longrightarrow$ **(J/c = v)**

Potential energy and electric potential

- The **potential difference** between two points A and B in an electric field is defined as the change in electric potential energy of the system when a charge *q* is moved between the points :

$$\Delta V \equiv \frac{\Delta U}{q} = -\int_{Ⓐ}^{Ⓑ} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

- The potential energy per unit charge U/q_0 is independent of the value of q_0 and has a value at every point in an electric field, is called the electric potential (or simply the potential) V. Thus, the electric potential at any point in an electric field is

- Potential energy is a scalar quantity, and The electric potential also is a scalar quantity.

- The electric field is a measure of the rate of change of the electric potential with respect to position.

- ***Electric field lines always point in the direction of decreasing electric potential.***

Potential energy and electric potential

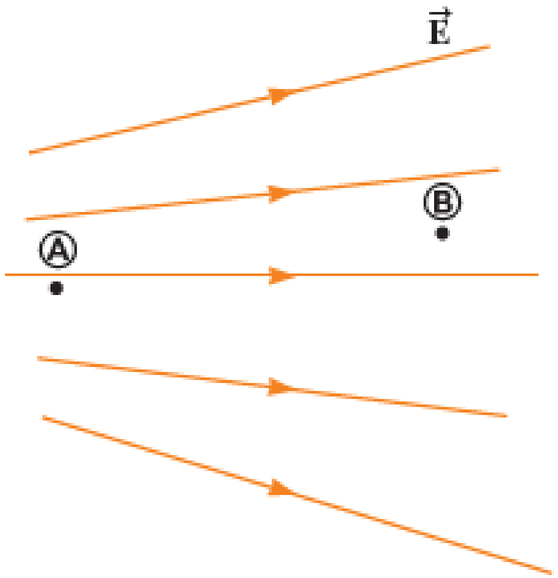
A unit of energy commonly used in atomic and nuclear physics is the **electron volt** (eV), which is defined as the energy a charge–field system gains or loses when a charge of magnitude ***e*** (that is, an electron or a proton) is moved through a potential difference of 1 V.

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ C} \cdot \text{V} = 1.60 \times 10^{-19} \text{ J}$$

Example :How would you describe the potential difference $\Delta V = V_{\text{B}} - V_{\text{A}}$?

a) It is positive.
b) It is negative.
c) It is zero

How would you describe the change in potential energy of the charge–field system for this process?



Potential Difference in a Uniform Electric Field

- The electrostatic force is conservative. As in mechanics, work is

$$W = Fd\cos\vartheta$$

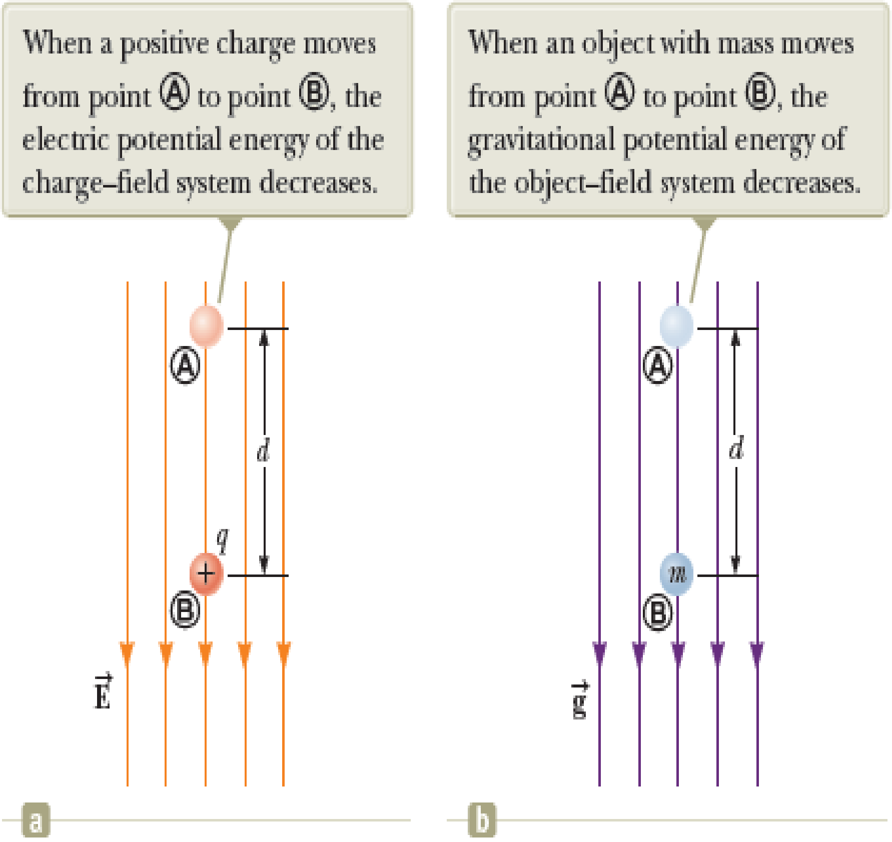
- Work done on the positive charge by moving it from A to B

$$W = Fd\cos\vartheta = qEd$$

The work done by a conservative force equals the **negative** of the change in potential energy ΔPE

$$\Delta PE = -W = -qEd$$

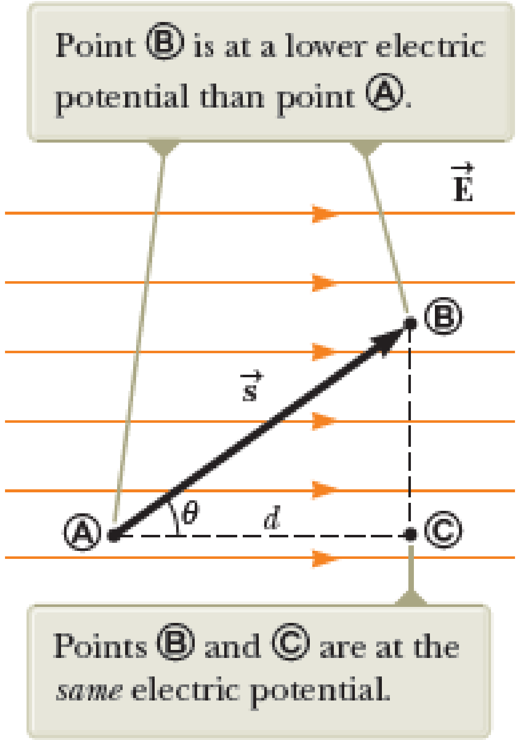
- This equation is valid only for the case of a *uniform electric field*.



Potential Difference in a Uniform Electric Field

- Now consider the more general case of a charged particle that moves between A and B in a uniform electric field such that the vector s is not parallel to the field lines, as shown in the Figure below.

- A uniform electric field directed along the positive x axis. Point B is at a lower electric potential than point A. Points B and C are at the same electric potential.Then:



$$\Delta V = -\int_{Ⓐ}^{Ⓑ} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\vec{\mathbf{E}} \cdot \int_{Ⓐ}^{Ⓑ} d\vec{\mathbf{s}} = -\vec{\mathbf{E}} \cdot \vec{\mathbf{s}}$$

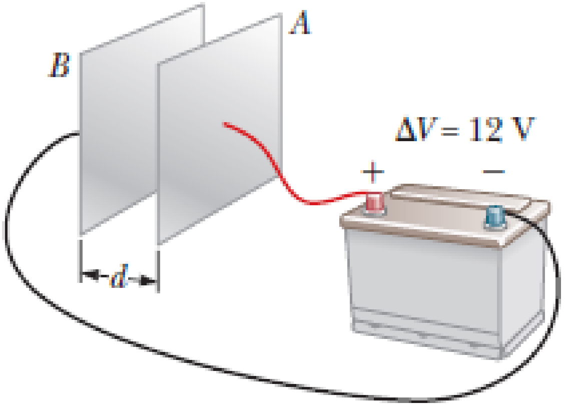
The change in potential energy of the charge–field system is

$$\Delta U = q\Delta V = -q\vec{\mathbf{E}} \cdot \vec{\mathbf{s}}$$

Example 25.1 The Electric Field Between Two Parallel Plates of Opposite Charge

A battery has a specified potential difference ΔV between its terminals and establishes that potential difference between conductors attached to the terminals. A 12-V battery is connected between two parallel plates as shown in Figure 25.5. The separation between the plates is $d = 0.30$ cm, and we assume the electric field between the plates to be uniform. (This assumption is reasonable if the plate separation is small relative to the plate dimensions and we do not consider locations near the plate edges.) Find the magnitude of the electric field between the plates.

$$E = \frac{|V_B - V_A|}{d} = \frac{12\text{ V}}{0.30 \times 10^{-2}\text{ m}} = \boxed{4.0 \times 10^{3}\text{ V/m}}$$



Example 25.2 Motion of a Proton in a Uniform Electric Field AM

A proton is released from rest at point Ⓐ in a uniform electric field that has a magnitude of 8.0×10^4 V/m (Fig. 25.6). The proton undergoes a displacement of magnitude $d = 0.50$ m to point Ⓑ in the direction of $\vec{\mathbf{E}}$. Find the speed of the proton after completing the displacement.

$$\Delta K + \Delta U = 0$$

$$\left(\tfrac{1}{2}mv^2 - 0\right) + e\,\Delta V = 0$$

$$v = \sqrt{\frac{-2e\,\Delta V}{m}} = \sqrt{\frac{-2e(-Ed)}{m}} = \sqrt{\frac{2eEd}{m}}$$

$$v = \sqrt{\frac{2(1.6 \times 10^{-19}\ \text{C})(8.0 \times 10^4\ \text{V})(0.50\ \text{m})}{1.67 \times 10^{-27}\ \text{kg}}}$$

$$= \boxed{2.8 \times 10^6\ \text{m/s}}$$

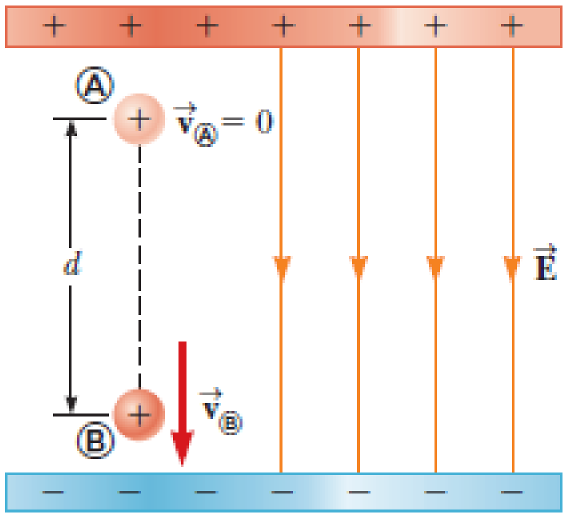


Figure 25.6 (Example 25.2) A proton accelerates from Ⓐ to Ⓑ in the direction of the electric field.

Electric Potential and Potential Energy Due to Point Charges

- If we have an isolated positive point charge q_0 produces an electric field directed radially outward from the charge. To find the electric potential at a point located a distance *r* from the charge, let's begin with the general expression for potential difference:

$$V_{B} - V_{A} = -\int_{A}^{B} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

At any point in space, the electric field due to the point charge is

$$\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = k_e \frac{q}{r^2}\hat{\mathbf{r}} \cdot d\vec{\mathbf{s}}$$

The expression for the potential difference becomes:

$$V_{B} - V_{A} = -k_e q \int_{r_A}^{r_B} \frac{dr}{r^2} = k_e \frac{q}{r}\bigg|_{r_A}^{r_B}$$

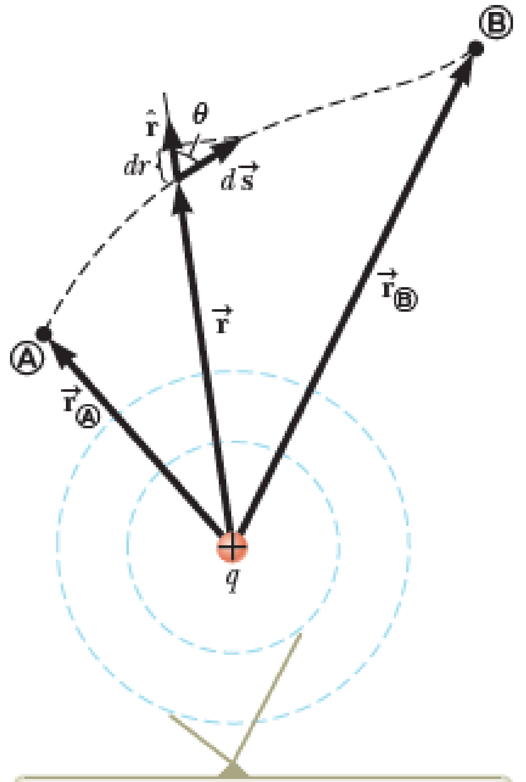
$$V_{B} - V_{A} = k_e q \left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

- the electric potential due to a point charge at any distance *r* from the charge is:

$$V = k_e \frac{q}{r}$$

- For a group of point charges, the total electric potential at P in the form:

$$V = k_e \sum_i \frac{q_i}{r_i}$$

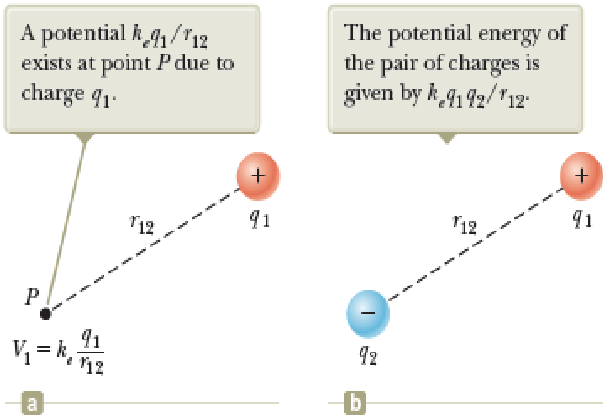


The two dashed circles represent intersections of spherical equipotential surfaces with the page.

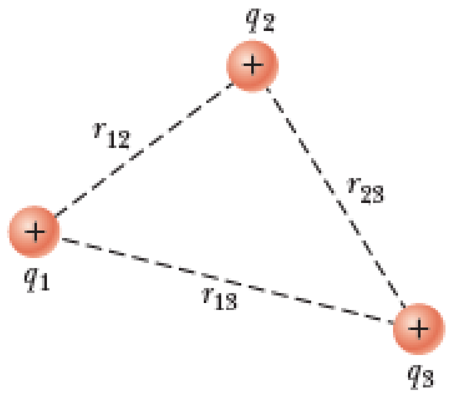
Electric Potential and Potential Energy Due to Point Charges

- The potential energy of two particles, separated by a distance r_{12}

$$\boxed{U = k_e \frac{q_1 q_2}{r_{12}}}$$



- Note that if the charges are of the same sign, U is positive. This is consistent with the fact that positive work must be done by an external agent on the system to bring the two charges near one another (because charges of the same sign repel).
- If the charges are of opposite sign, U is negative; this means that negative work is done by an external agent against the attractive force between the charges of opposite sign as they are brought near each other;a force must be applied opposite to the displacement to prevent q_1 from accelerating toward q_2 .



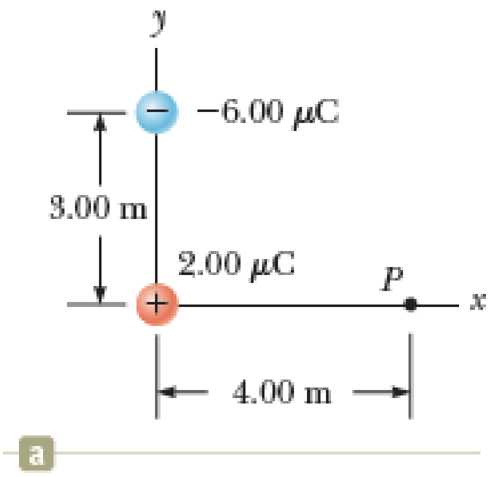
- the total potential energy of the system of three charges shown in Figure below:

$$\boxed{U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)}$$

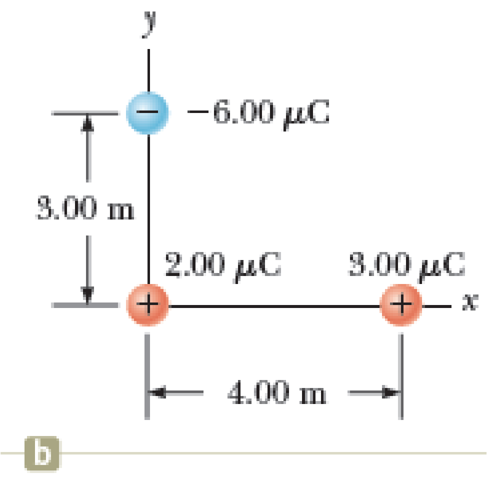
Example 25.3 The Electric Potential Due to Two Point Charges

As shown in Figure 25.10a, a charge $q_1 = 2.00\ \mu C$ is located at the origin and a charge $q_2 = -6.00\ \mu C$ is located at $(0, 3.00)$ m.

(A) Find the total electric potential due to these charges at the point P, whose coordinates are $(4.00, 0)$ m.



(B) Find the change in potential energy of the system of two charges plus a third charge $q_3 = 3.00\ \mu C$ as the latter charge moves from infinity to point P (Fig. 25.10b).



SOLUTION

$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$V_P = (8.988 \times 10^9\ \mathrm{N \cdot m^2/C^2})\left(\frac{2.00 \times 10^{-6}\ \mathrm{C}}{4.00\ \mathrm{m}} + \frac{-6.00 \times 10^{-6}\ \mathrm{C}}{5.00\ \mathrm{m}}\right)$$

$$= \boxed{-6.29 \times 10^3\ \mathrm{V}}$$

$$U_f = q_3 V_P$$

$$\Delta U = U_f - U_i = q_3 V_P - 0 = (3.00 \times 10^{-6}\ \mathrm{C})(-6.29 \times 10^3\ \mathrm{V})$$

$$= \boxed{-1.89 \times 10^{-2}\ \mathrm{J}}$$

Obtaining the Value of the Electric Field from the Electric Potential

- In this section, we'll show how to calculate the value of the electric field if the electric potential is known in a certain region. As we seen ,The electric field and the electric potential are related as:

$$\Delta V = -\int_A^B \vec{E}.d\vec{s}$$
$$\rightarrow dV = -\vec{E}.d\vec{s}$$

- If the electric field has only one component E_x

$$E_x = -\frac{dV}{dx}$$

That is, the *x* component of the electric field is equal to the negative of the derivative of the electric potential with respect to *x*. Similar statements can be made about the *y* and *z* components.

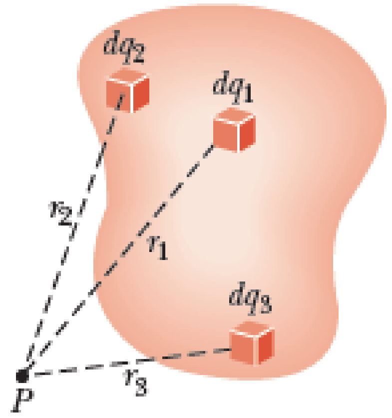
- The electric field components E_x, E_y and E_z can readily be found from $V(x, y, z)$ as the partial derivative

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

Electric Potential Due to Continuous Charge Distributions

- We consider the potential due to a small charge element *dq*, treating this element as a point charge then the electric potential *dV* at some point P due to the charge element *dq* is

$$\boxed{dV = k_e \frac{dq}{r}}$$

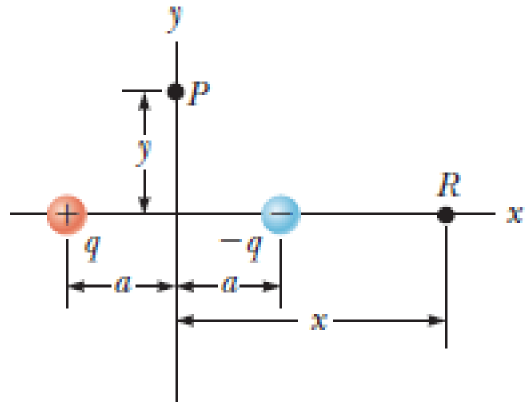


we can express V as

$$\boxed{V = k_e \int \frac{dq}{r}}$$

Example 25.4 The Electric Potential Due to a Dipole

An electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance $2a$ as shown in Figure 25.13. The dipole is along the x axis and is centered at the origin.



(A) Calculate the electric potential at point P on the y axis.

$$V_P = k_e \sum_i \frac{q_i}{r_i} = k_e\left(\frac{q}{\sqrt{a^2+y^2}} + \frac{-q}{\sqrt{a^2+y^2}}\right) = \boxed{0}$$

(B) Calculate the electric potential at point R on the positive x axis.

$$V_R = k_e \sum_i \frac{q_i}{r_i} = k_e\left(\frac{-q}{x-a} + \frac{q}{x+a}\right) = \boxed{-\frac{2k_e qa}{x^2-a^2}}$$

(C) Calculate V and E_x at a point on the x axis far from the dipole.

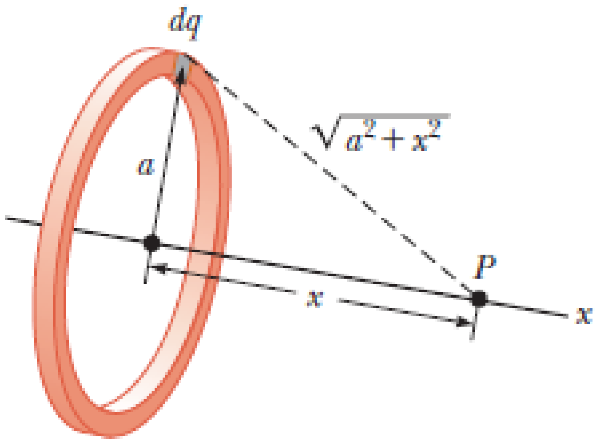
$$V_R = \lim_{x>>a}\left(-\frac{2k_e qa}{x^2-a^2}\right) \approx \boxed{-\frac{2k_e qa}{x^2} \quad (x >> a)}$$

$$E_x = -\frac{dV}{dx} = -\frac{d}{dx}\left(-\frac{2k_e qa}{x^2}\right)$$

$$= 2k_e qa\frac{d}{dx}\left(\frac{1}{x^2}\right) = \boxed{-\frac{4k_e qa}{x^3} \quad (x >> a)}$$

Example 25.5 Electric Potential Due to a Uniformly Charged Ring

(A) Find an expression for the electric potential at a point P located on the perpendicular central axis of a uniformly charged ring of radius a and total charge Q.



$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{a^2 + x^2}}$$

$$V = \frac{k_e}{\sqrt{a^2 + x^2}} \int dq = \boxed{\frac{k_e Q}{\sqrt{a^2 + x^2}}}$$

(B) Find an expression for the magnitude of the electric field at point P.

$$E_x = -\frac{dV}{dx} = -k_e Q \frac{d}{dx}(a^2 + x^2)^{-1/2}$$

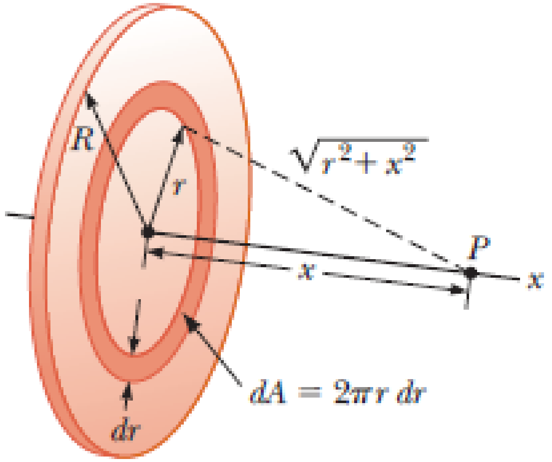
$$= -k_e Q(-\tfrac{1}{2})(a^2 + x^2)^{-3/2}(2x)$$

$$E_x = \boxed{\frac{k_e x}{(a^2 + x^2)^{3/2}} Q}$$

Example 25.6 Electric Potential Due to a Uniformly Charged Disk

A uniformly charged disk has radius R and surface charge density σ.

(A) Find the electric potential at a point P along the perpendicular central axis of the disk.



$$dq = \sigma\,dA = \sigma(2\pi r\,dr) = 2\pi\sigma r\,dr$$

$$dV = \frac{k_e\,dq}{\sqrt{r^2+x^2}} = \frac{k_e 2\pi\sigma r\,dr}{\sqrt{r^2+x^2}}$$

$$V = \pi k_e\sigma \int_0^R \frac{2r\,dr}{\sqrt{r^2+x^2}} = \pi k_e\sigma \int_0^R (r^2+x^2)^{-1/2}\,2r\,dr$$

This integral is of the common form $\int u^n\,du$, where $n = -\frac{1}{2}$ and $u = r^2 + x^2$, and has the value $u^{n+1}/(n+1)$.

$$V = \boxed{2\pi k_e\sigma[(R^2+x^2)^{1/2} - x]}$$

(B) Find the x component of the electric field at a point P along the perpendicular central axis of the disk.

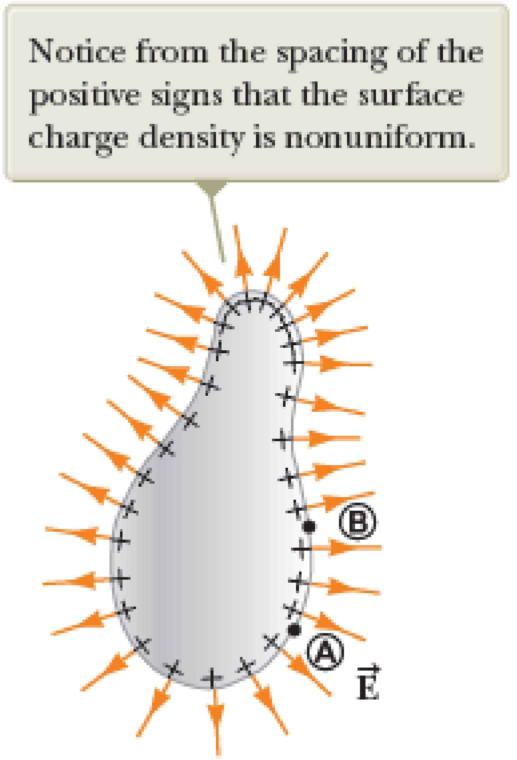
$$E_x = -\frac{dV}{dx} = \boxed{2\pi k_e\sigma\left[1 - \frac{x}{(R^2+x^2)^{1/2}}\right]}$$

➢ Electric Potential Due to a Charged Conductor

- From Gausses law, we found that when a solid conductor in equilibrium it carries a net charge, the charge resides on the outer surface of the conductor, and that the field inside is zero.

- Consider two points *A and B on the* surface of a charged conductor, as shown in Figure below.

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = 0$$



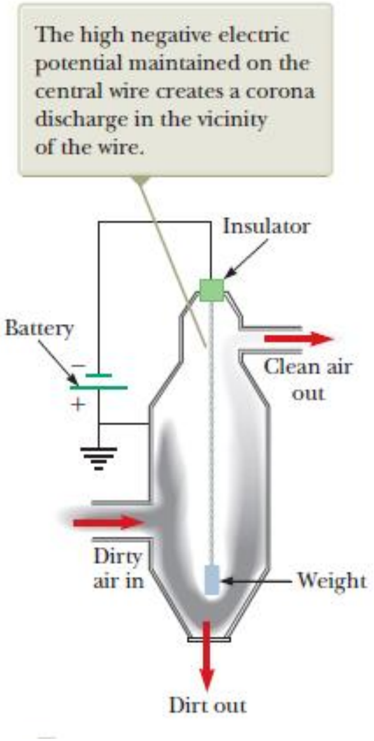
- E is always perpendicular to the displacement *ds; therefore* E• *ds = 0.*

- Therefore, *V is constant everywhere* on the surface of a charged conductor in equilibrium.

Applications of Electrostatics

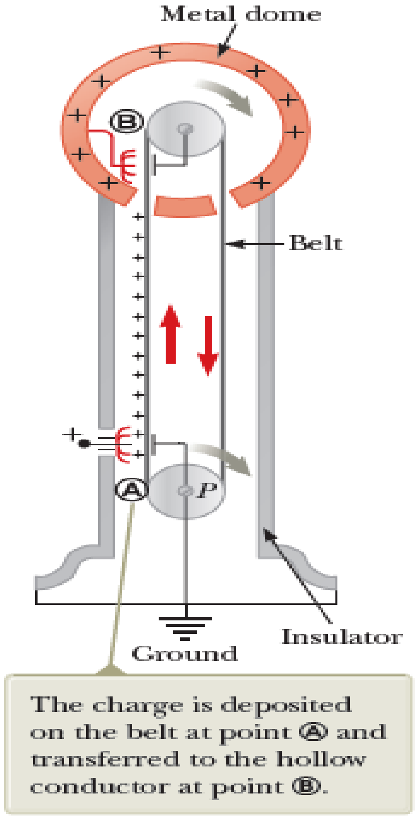
1-The Electrostatic Precipitator

One important application of electrical discharge in gases is the electrostatic precipitator. This device removes particulate matter from combustion gases, thereby reducing air pollution. Precipitators are especially useful in coal-burning power plants and industrial operations that generate large quantities of smoke. Current systems are able to eliminate more than 99% of the ash from smoke.



2-The Van de Graaff Generator

Experimental results show that when a charged conductor is placed in contact with the inside of a hollow conductor, all the charge on the charged conductor is transferred to the hollow conductor. In principle, the charge on the hollow conductor and its electric potential can be increased without limit by repetition of the process.

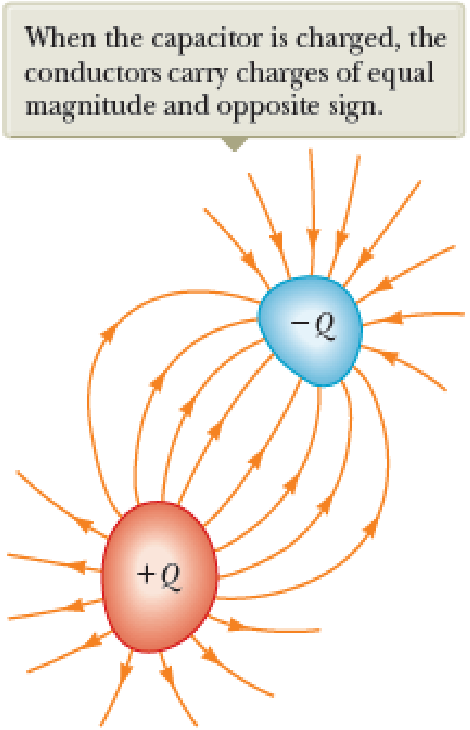


Chapter 4 :Capacitance and Dielectrics

Outlines:

- ✓ Definition of capacitance,
- ✓ Calculating capacitance for parallel plate capacitors,
- ✓ Combination of capacitors,
- ✓ Energy stored in a charged capacitor,
- ✓ Capacitors with dielectrics,
- ✓ RC circuits.

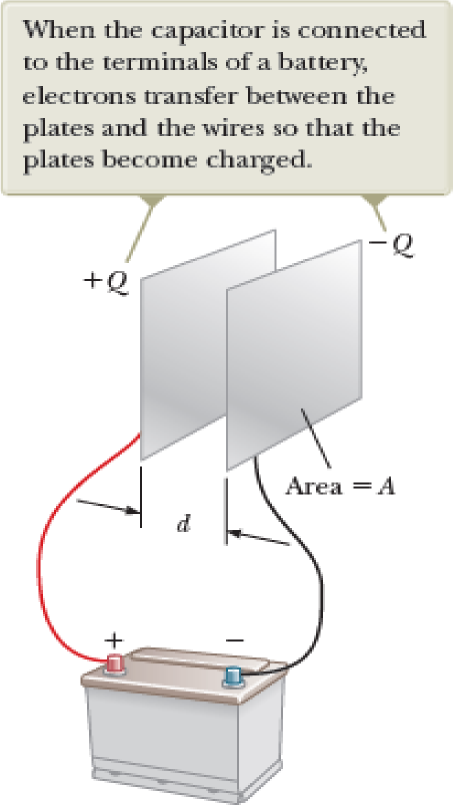
Definition of capacitance



- The capacitance C of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between the conductors:

$$C \equiv \frac{Q}{\Delta V}$$

- The SI unit of capacitance is the **farad** (F): $1\ \text{F} = 1\ \text{C/V}$



Quick Quiz A capacitor stores charge Q at a potential difference ΔV. What happens if the voltage applied to the capacitor by a battery is doubled to $2\,\Delta V$?

a) The capacitance falls to half its initial value, and the charge remains the same.
b) The capacitance and the charge both fall to half their initial values.
c) The capacitance and the charge both double.
d) The capacitance remains the same, and the charge doubles.

Calculating Capacitance

There are some shapes of capacitor:

- Parallel plate capacitor ,
- Cylindrical capacitor ,
- Spherical capacitor.

The capacitance depends on shape of conductor which related to electrical potential.

Spherical capacitor

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q / a} = \frac{a}{k_e} = 4\pi\epsilon_0 a$$

Parallel-Plate Capacitors

$$C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\epsilon_0 A}$$

$$C = \frac{\epsilon_0 A}{d}$$

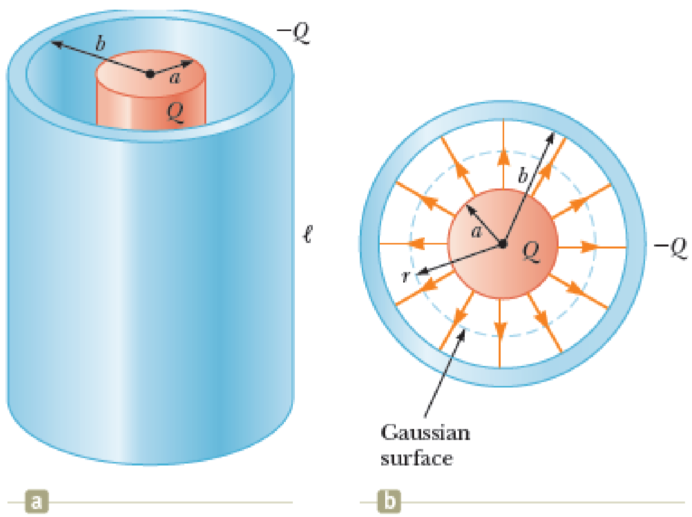
Cylindrical capacitor

$$C = \frac{Q}{\Delta V} = \frac{Q}{(2k_e Q/\ell)\ln(b/a)} = \frac{\ell}{2k_e \ln(b/a)}$$

Example(26.1)

Example 26.1 The Cylindrical Capacitor

A solid cylindrical conductor of radius a and charge Q is coaxial with a cylindrical shell of negligible thickness, radius $b > a$, and charge $-Q$ (Fig. 26.4a). Find the capacitance of this cylindrical capacitor if its length is ℓ.



$$V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

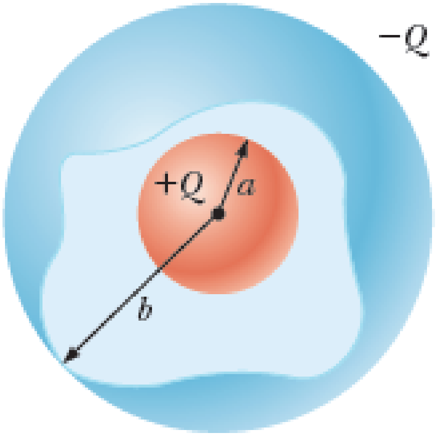
$$V_b - V_a = -\int_a^b E_r dr = -2k_e\lambda \int_a^b \frac{dr}{r} = -2k_e\lambda \ln\left(\frac{b}{a}\right)$$

and use $\lambda = Q/\ell$:

$$C = \frac{Q}{\Delta V} = \frac{Q}{(2k_e Q/\ell)\ln(b/a)} = \boxed{\frac{\ell}{2k_e \ln(b/a)}}$$

Example 26.2 The Spherical Capacitor

A spherical capacitor consists of a spherical conducting shell of radius b and charge $-Q$ concentric with a smaller conducting sphere of radius a and charge Q (Fig. 26.5, page 782). Find the capacitance of this device.



$$V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$V_b - V_a = -\int_a^b E_r\, dr = -k_e Q \int_a^b \frac{dr}{r^2} = k_e Q \left[\frac{1}{r}\right]_a^b$$

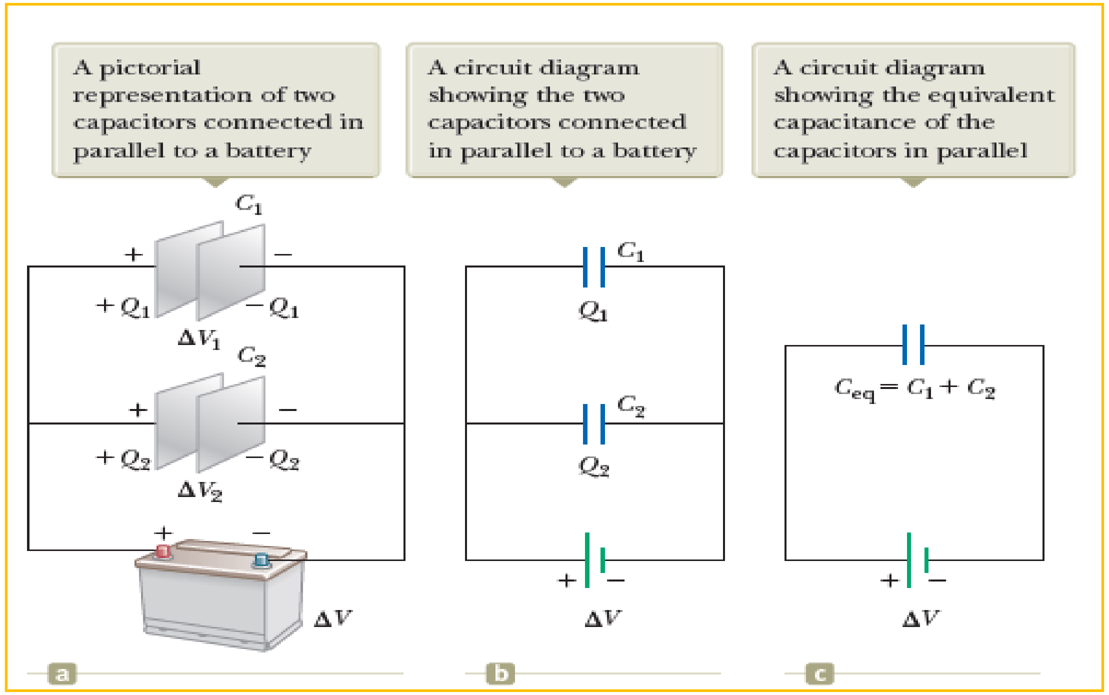
$$(1) \quad V_b - V_a = k_e Q\left(\frac{1}{b} - \frac{1}{a}\right) = k_e Q \frac{a - b}{ab}$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{|V_b - V_a|} = \boxed{\frac{ab}{k_e(b - a)}}$$

Combinations of Capacitors

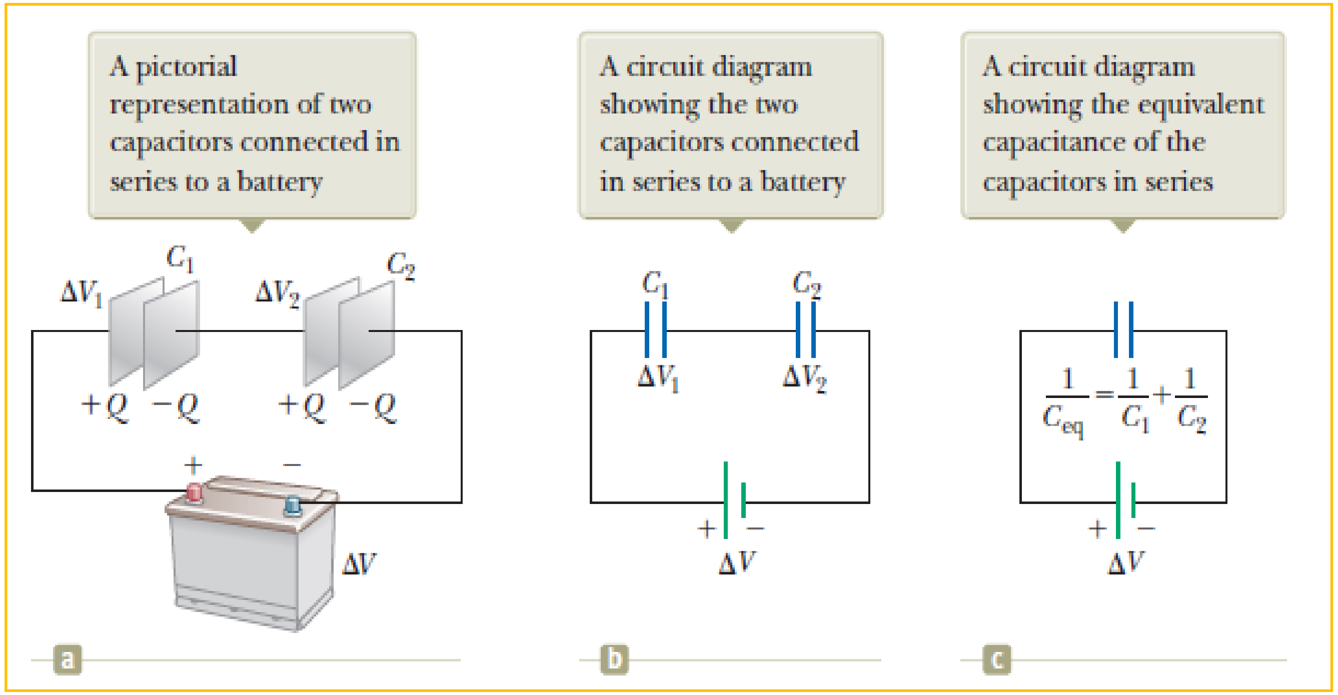
- Parallel Combination

$$C_{eq} = C_1 + C_2 + C_3 + \cdots$$



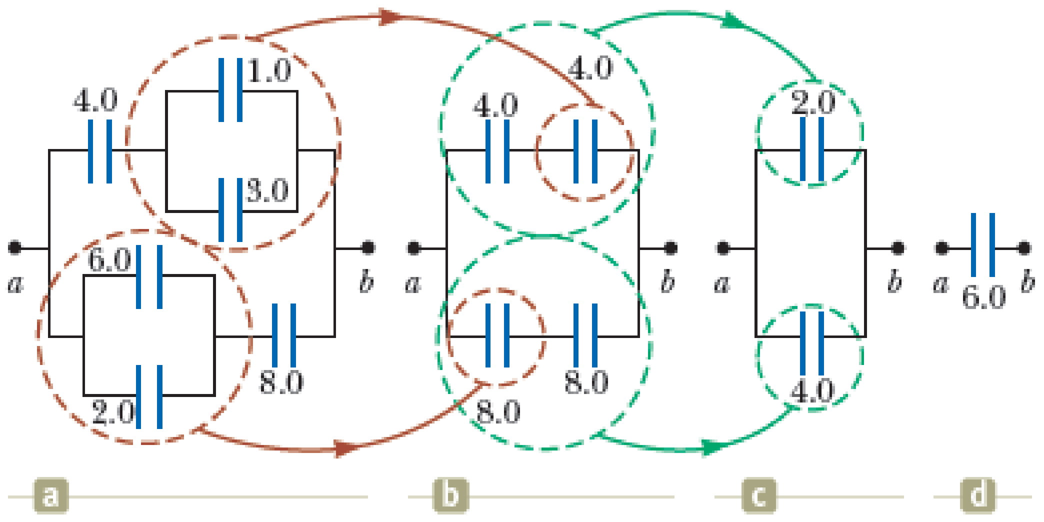
- Series Combination

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$



Example 26.3 Equivalent Capacitance

Find the equivalent capacitance between a and b for the combination of capacitors shown in Figure 26.9a. All capacitances are in microfarads.



in parallel

$$C_{eq} = C_1 + C_2 = 4.0\ \mu F$$

$$C_{eq} = C_1 + C_2 = 8.0\ \mu F$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4.0\ \mu F} + \frac{1}{4.0\ \mu F} = \frac{1}{2.0\ \mu F}$$

$$C_{eq} = 2.0\ \mu F$$

in series.

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{8.0\ \mu F} + \frac{1}{8.0\ \mu F} = \frac{1}{4.0\ \mu F}$$

$$C_{eq} = 4.0\ \mu F$$

$$C_{eq} = C_1 + C_2 = 6.0\ \mu F$$

Energy stored in a charged capacitor

- The potential energy stored in a charged capacitor has the following forms:

$$U_E = \frac{Q^2}{2C} = \tfrac{1}{2}Q\,\Delta V = \tfrac{1}{2}C(\Delta V)^2$$

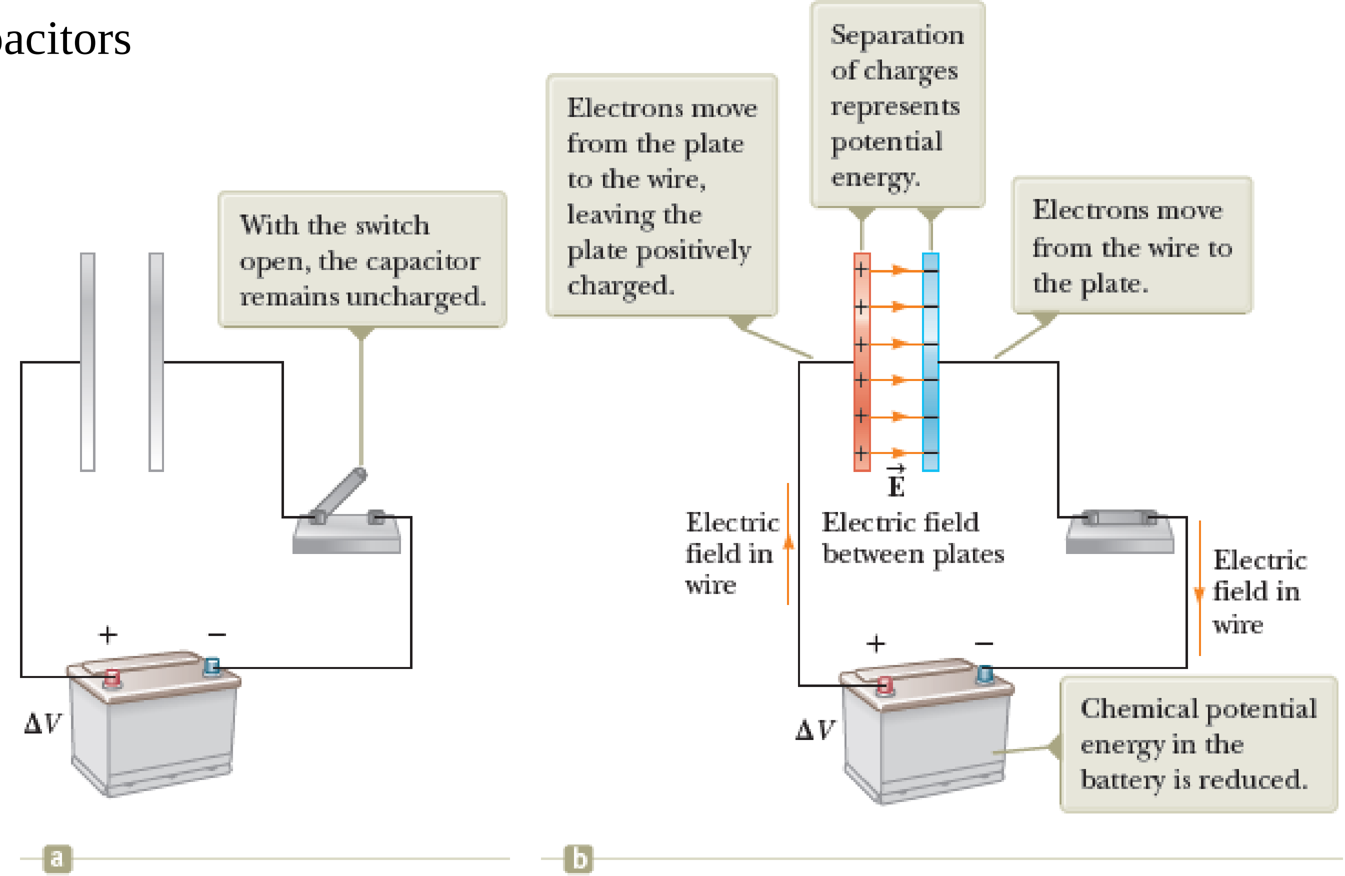
- The potential energy stored in a Parallel-Plate Capacitors

$$U_E = \tfrac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)(Ed)^2 = \tfrac{1}{2}(\epsilon_0 A d)E^2$$

- and energy density is:

$$u_E = \tfrac{1}{2}\epsilon_0 E^2$$

That is, the energy density in any electric field is proportional to the square of the magnitude of the electric field at a given point. Regardless of the source of the electric field.



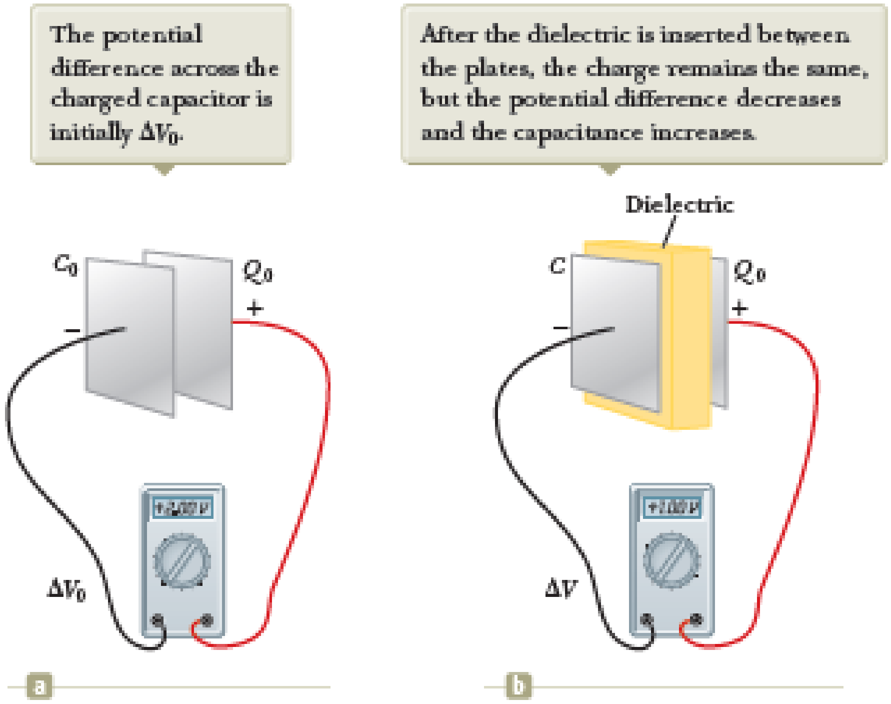
Capacitors with dielectrics

- A **dielectric** is a non-conducting material such as rubber, glass, or waxed paper. We can perform the following experiment to illustrate the effect of a dielectric in a capacitor.

- Consider an insolated, charged capacitor :
- Notice that the potential difference ***decreases*** ($k = V_0/V$)
- Since charge stayed the same ($Q=Q_0$) $\rightarrow$ capacitance ***increases***

$$C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0/\kappa} = \kappa \frac{Q_0}{\Delta V_0}$$

$$\boxed{C = \kappa C_0}$$



- Dielectric constant (k) is a material property

- Capacitance is multiplied by a factor *k* when the dielectric fills the region between the plates completely
- E.g., for a parallel-plate capacitor

$$C = \kappa \varepsilon_0 \frac{A}{d}$$

- The capacitance is limited from above by the electric discharge that can occur through the dielectric material separating the plates.
- In other words, there exists a maximum of the electric field, sometimes called dielectric strength, that can be produced in the dielectric before it breaks down.
- Therefore, a dielectric provides the following advantages:

• An increase in capacitance

• An increase in maximum operating voltage

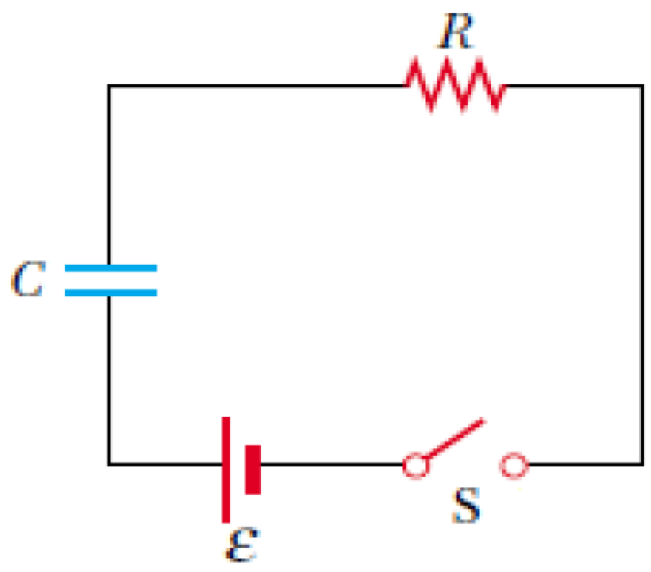
• Possible mechanical support between the plates, which allows the plates to be close together without touching, thereby decreasing d and increasing C

Table 26.1 **Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature**

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

RC Circuits

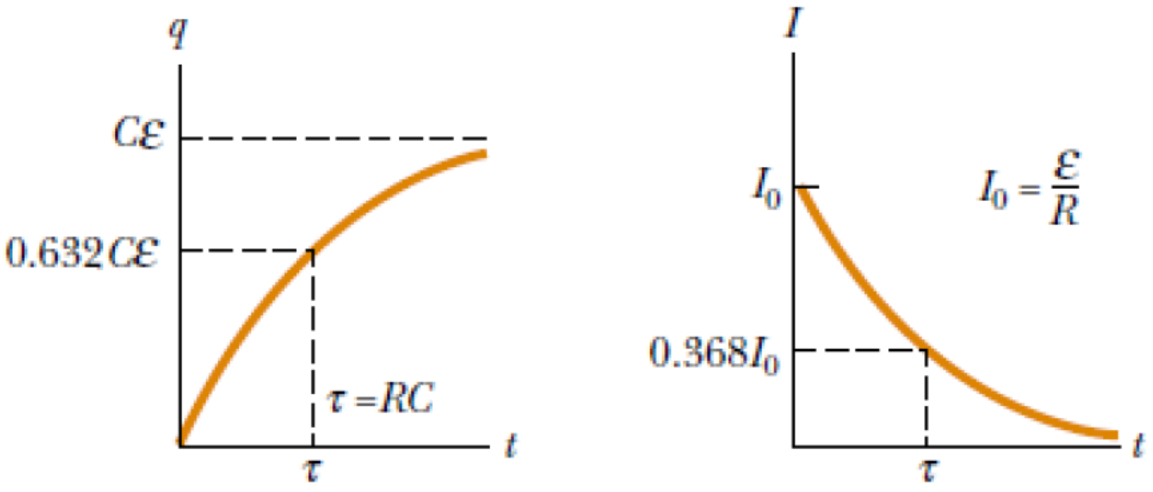
In DC circuits containing capacitors, the current is always in the same direction but may vary in time. A circuit containing a series combination of a resistor and a capacitor is called an RC circuit.



✓ **Charging a capacitor** At *t= 0 switch* ***S is closed,***

$$q(t) = C\mathcal{E}(1 - e^{-t/RC}) = Q(1 - e^{-t/RC})$$

$$I(t) = \frac{\mathcal{E}}{R} e^{-t/RC}$$



✓ **Discharging a capacitor** .At *t= 0 switch* ***S is open,***

$$q(t) = Qe^{-t/RC}$$

$$I(t) = \frac{dq}{dt} = \frac{d}{dt}\left(Qe^{-t/RC}\right) = -\frac{Q}{RC}e^{-t/RC}$$

Where the constant τ is the time constant of the RC circuit: $\tau = RC$

Time constant: is the time interval during which the current decreases to $\frac{1}{e}$ of its initial value.

Example 28.9 Charging a Capacitor in an *RC* Circuit

An uncharged capacitor and a resistor are connected in series to a battery as shown in Figure 28.16, where $\mathcal{E} = 12.0$ V, $C = 5.00\ \mu$F, and $R = 8.00 \times 10^5\ \Omega$. The switch is thrown to position *a*. Find the time constant of the circuit, the maximum charge on the capacitor, the maximum current in the circuit, and the charge and current as functions of time.

$$\tau = RC = (8.00 \times 10^5\ \Omega)(5.00 \times 10^{-6}\ \text{F}) = \boxed{4.00\ \text{s}}$$

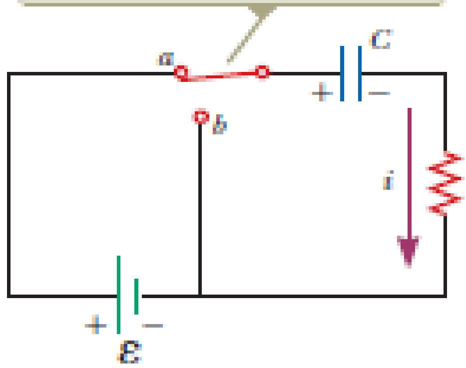
$$Q_{\text{max}} = C\mathcal{E} = (5.00\ \mu\text{F})(12.0\ \text{V}) = \boxed{60.0\ \mu\text{C}}$$

$$I_i = \frac{\mathcal{E}}{R} = \frac{12.0\ \text{V}}{8.00 \times 10^5\ \Omega} = \boxed{15.0\ \mu\text{A}}$$

$$(1)\quad q(t) = \boxed{60.0(1 - e^{-t/4.00})}$$

$$(2)\quad i(t) = \boxed{15.0e^{-t/4.00}}$$

When the switch is thrown to position *a*, the capacitor begins to charge up.



Example 28.10 Discharging a Capacitor in an *RC* Circuit

Consider a capacitor of capacitance *C* that is being discharged through a resistor of resistance *R* as shown in Figure 28.16c.

(A) After how many time constants is the charge on the capacitor one-fourth its initial value?

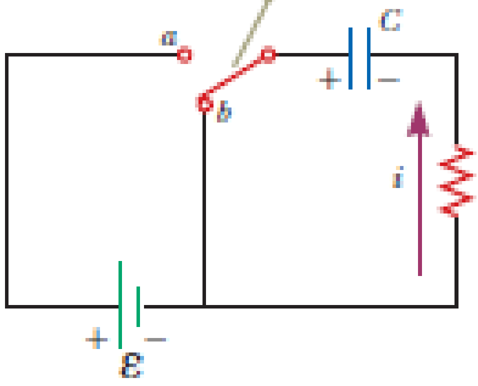
$$\frac{Q_i}{4} = Q_i e^{-t/RC}$$

$$\tfrac{1}{4} = e^{-t/RC}$$

$$-\ln 4 = -\frac{t}{RC}$$

$$t = RC \ln 4 = 1.39RC = \boxed{1.39\tau}$$

When the switch is thrown to position *b*, the capacitor discharges.



Chapter 5 :Sources of the Magnetic Field

Outlines:

- ✓ *The Biot-Savart's law,*
- ✓ *the magnetic force between two parallel conductors,*
- ✓ *Ampere's law,*
- ✓ *the magnetic field of a solenoid,*
- ✓ *magnetic flux,*
- ✓ *Gauss's law in magnetism*

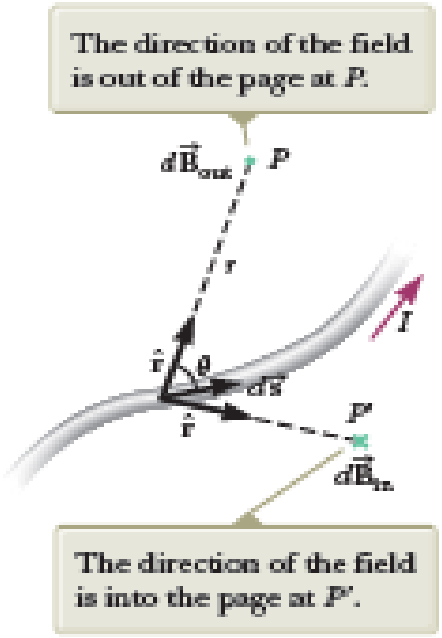
The Biot-Savart's law,

- Oersted's discovery in 1819 that a compass needle is deflected by a current-carrying conductor.
- From the experimental results for Biot and Savart, they arrived at a mathematical expression that gives the magnetic field at some point in space in terms of the current that produces the field.

$$\left.\begin{aligned} dB &\propto I \\ dB &\propto ds \\ dB &\propto \frac{1}{r^2} \end{aligned}\right\}$$

$$\boxed{d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}}$$

Tesla (T)



- Where μ_0 permeability of free space, $\mu_0 = 4\pi \times 10^{-7}$ T.m/ A

The vector $\overrightarrow{dB}$ is perpendicular both to d and to the unit vector $\hat{r}$

Example 30.1 Magnetic Field Surrounding a Thin, Straight Conductor

Consider a thin, straight wire of finite length carrying a constant current I and placed along the x axis as shown in Figure 30.3. Determine the magnitude and direction of the magnetic field at point P due to this current.

$$d\vec{\mathbf{s}} \times \hat{\mathbf{r}} = |d\vec{\mathbf{s}} \times \hat{\mathbf{r}}|\hat{\mathbf{k}} = \left[dx \sin\left(\frac{\pi}{2} - \theta\right)\right]\hat{\mathbf{k}} = (dx \cos\theta)\hat{\mathbf{k}}$$

$$(1) \quad d\vec{\mathbf{B}} = (dB)\hat{\mathbf{k}} = \frac{\mu_0 I}{4\pi}\frac{dx\cos\theta}{r^2}\hat{\mathbf{k}}$$

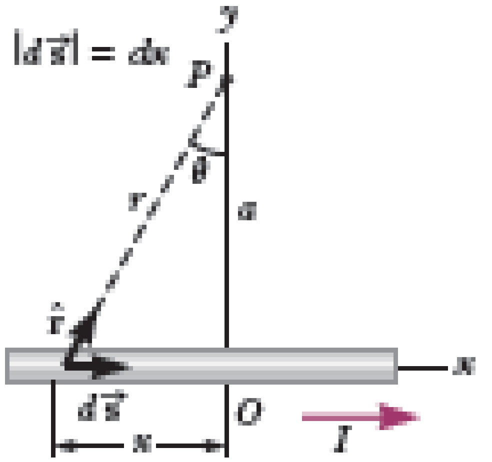
$$(2) \quad r = \frac{a}{\cos\theta}$$

$$x = -a\tan\theta$$

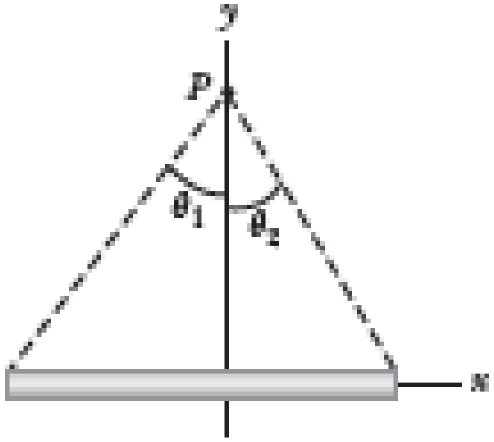
$$(3) \quad dx = -a\sec^2\theta\, d\theta = -\frac{a\, d\theta}{\cos^2\theta}$$

$$(4) \quad dB = -\frac{\mu_0 I}{4\pi}\left(\frac{a\, d\theta}{\cos^2\theta}\right)\left(\frac{\cos^2\theta}{a^2}\right)\cos\theta = -\frac{\mu_0 I}{4\pi a}\cos\theta\, d\theta$$

$$B = -\frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2}\cos\theta\, d\theta = \boxed{\frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)}$$



a



b

Example 30.2 Magnetic Field Due to a Curved Wire Segment

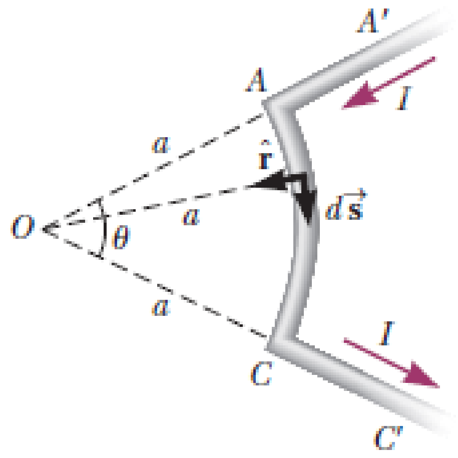
Calculate the magnetic field at point O for the current-carrying wire segment shown in Figure 30.4. The wire consists of two straight portions and a circular arc of radius a, which subtends an angle θ.

Analyze Each length element $d\vec{\mathbf{s}}$ along path AC is at the same distance a from O, and the current in each contributes a field element $d\vec{\mathbf{B}}$ directed into the page at O. Furthermore, at every point on AC, $d\vec{\mathbf{s}}$ is perpendicular to $\hat{\mathbf{r}}$; hence, $|d\vec{\mathbf{s}} \times \hat{\mathbf{r}}| = ds$.

$$dB = \frac{\mu_0}{4\pi}\frac{I\,ds}{a^2}$$

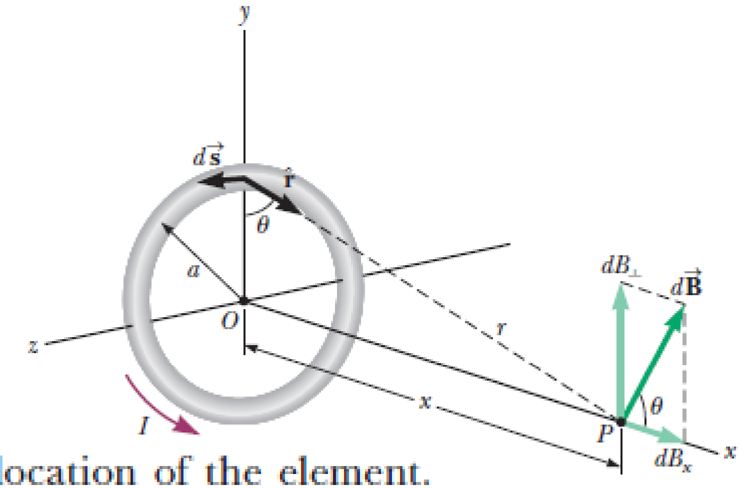
$$B = \frac{\mu_0 I}{4\pi a^2}\int ds = \frac{\mu_0 I}{4\pi a^2}\,s$$

$$B = \frac{\mu_0 I}{4\pi a^2}(a\theta) = \boxed{\frac{\mu_0 I}{4\pi a}\,\theta}$$



Example 30.3 Magnetic Field on the Axis of a Circular Current Loop

Consider a circular wire loop of radius a located in the yz plane and carrying a steady current I as in Figure 30.5. Calculate the magnetic field at an axial point P a distance x from the center of the loop.



Analyze In this situation, every length element $d\vec{s}$ is perpendicular to the vector $\hat{r}$ at the location of the element. Therefore, for any element, $|d\vec{s} \times \hat{r}| = (ds)(1)\sin 90° = ds$. Furthermore, all length elements around the loop are at the same distance r from P, where $r^2 = a^2 + x^2$.

$$dB = \frac{\mu_0 I}{4\pi}\frac{|d\vec{s} \times \hat{r}|}{r^2} = \frac{\mu_0 I}{4\pi}\frac{ds}{(a^2 + x^2)}$$

$$dB_x = \frac{\mu_0 I}{4\pi}\frac{ds}{(a^2 + x^2)}\cos\theta$$

$$B_x = \oint dB_x = \frac{\mu_0 I}{4\pi}\oint \frac{ds\cos\theta}{a^2 + x^2}$$

$$\cos\theta = \frac{a}{(a^2 + x^2)^{1/2}}$$

$$B_x = \frac{\mu_0 I}{4\pi}\oint \frac{ds}{a^2 + x^2}\left[\frac{a}{(a^2 + x^2)^{1/2}}\right] = \frac{\mu_0 I}{4\pi}\frac{a}{(a^2 + x^2)^{3/2}}\oint ds \qquad B_x = \frac{\mu_0 I}{4\pi}\frac{a}{(a^2 + x^2)^{3/2}}(2\pi a) = \boxed{\frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}}$$

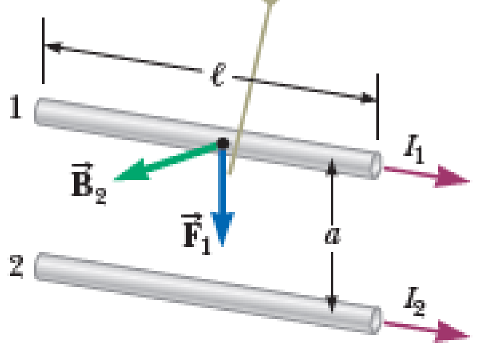
The Magnetic Force Between Two Parallel Conductors

- Consider two long, straight, parallel wires separated by a distance *a* and carrying currents I_1 and I_2 in the same direction.
- We can determine the force exerted on one wire due to the magnetic field set up by the other wire.

$$F_1 = I_1 l\, B_2 = I_1 l \frac{\mu_\circ I_2}{2\pi a}$$

The direction of $\vec{F}_1$ is toward wire 2.

The field $\vec{\mathbf{B}}_2$ due to the current in wire 2 exerts a magnetic force of magnitude $F_1 = I_1 \ell B_2$ on wire 1.



- If the field set up at wire 2 by wire 1 is calculated, the $\vec{F}_2$ force acting on wire 2 is found to be equal in

magnitude and opposite in direction to $\vec{F}_1$.

$$\boldsymbol{\vec{F}_1 = -\vec{F}_2}$$

$$\vec{F} = I\,\vec{l} \times \vec{B}$$

- We can rewrite this magnitude in terms of the force per unit length:
- *The force between two parallel wires is used to define the* ***ampere*** *as follows:*

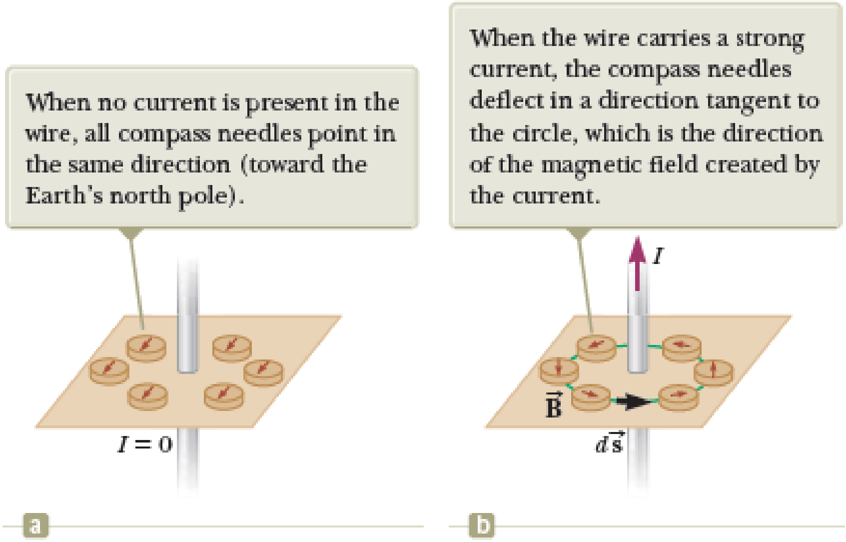
$$\frac{F_B}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

When the magnitude of the force per unit length between two long, parallel wires that carry identical currents and are separated by 1 m is 2×10^{-7} N/m, the current in each wire is defined to be 1 A.

Ampere's law

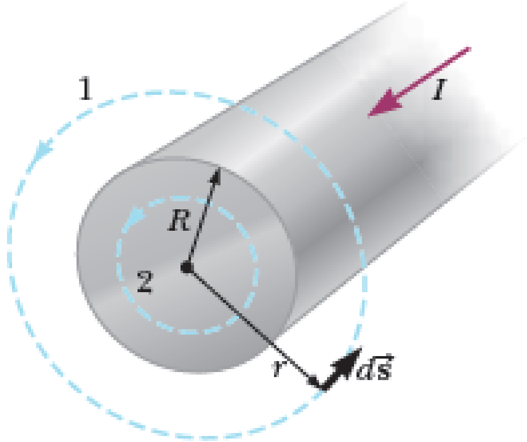
The line integral $\vec{B}.\overrightarrow{ds}$ of around any closed path equals $\mu_0 I$ where I is the total steady current passing through any surface bounded by the closed path

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$



Example 30.5 The Magnetic Field Created by a Long Current-Carrying Wire

A long, straight wire of radius R carries a steady current I that is uniformly distributed through the cross section of the wire (Fig. 30.13). Calculate the magnetic field a distance r from the center of the wire in the regions $r \geq R$ and $r < R$.



Note that the total current passing through the plane of the circle is I and apply Ampère's law:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \oint ds = B(2\pi r) = \mu_0 I$$

Solve for B:

$$B = \boxed{\frac{\mu_0 I}{2\pi r}} \quad (\text{for } r \geq R) \qquad \textbf{(30.14)}$$

Now consider the interior of the wire, where $r < R$. Here the current I' passing through the plane of circle 2 is less than the total current I.

Set the ratio of the current I' enclosed by circle 2 to the entire current I equal to the ratio of the area πr^2 enclosed by circle 2 to the cross-sectional area πR^2 of the wire:

$$\frac{I'}{I} = \frac{\pi r^2}{\pi R^2}$$

Solve for I':

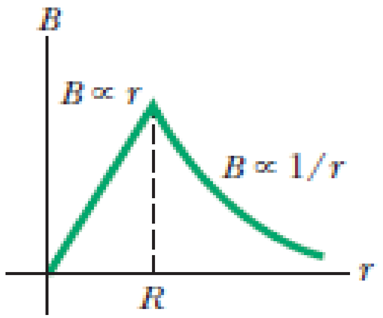
$$I' = \frac{r^2}{R^2} I$$

Apply Ampère's law to circle 2:

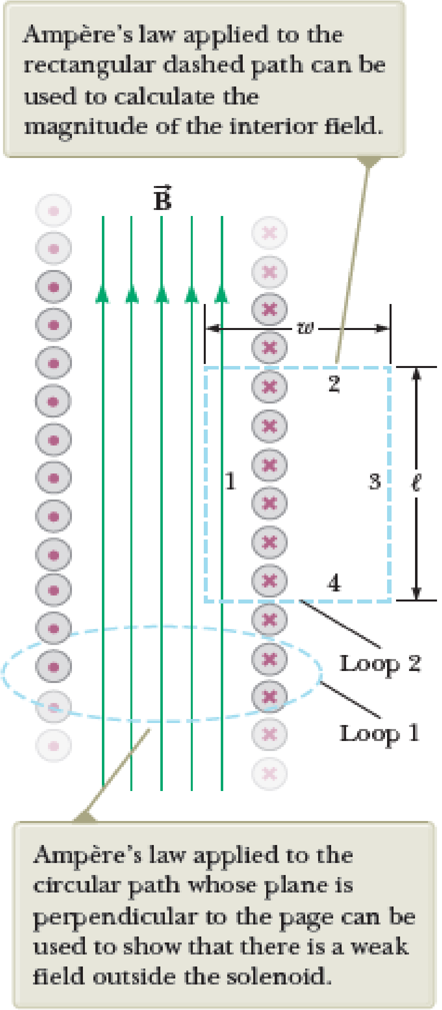
$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r) = \mu_0 I' = \mu_0\left(\frac{r^2}{R^2} I\right)$$

Solve for B:

$$B = \boxed{\left(\frac{\mu_0 I}{2\pi R^2}\right) r} \quad (\text{for } r < R) \qquad \textbf{(30.15)}$$



The Magnetic Field of a Solenoid



A solenoid is a long wire wound in the form of a helix. With this configuration, a reasonably uniform magnetic field can be produced in the space surrounded by the turns of wire(which we shall call the *interior* of the solenoid) when the solenoid carries a current. When the turns are closely spaced, each can be approximated as a circular loop, and the net magnetic field is the vector sum of the fields resulting from all the turns.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\ell = \mu_0 NI$$

$$\boxed{B = \mu_0 \frac{N}{\ell} I = \mu_0 n I}$$

where $n = N / l$, is the number of turns per unit length.

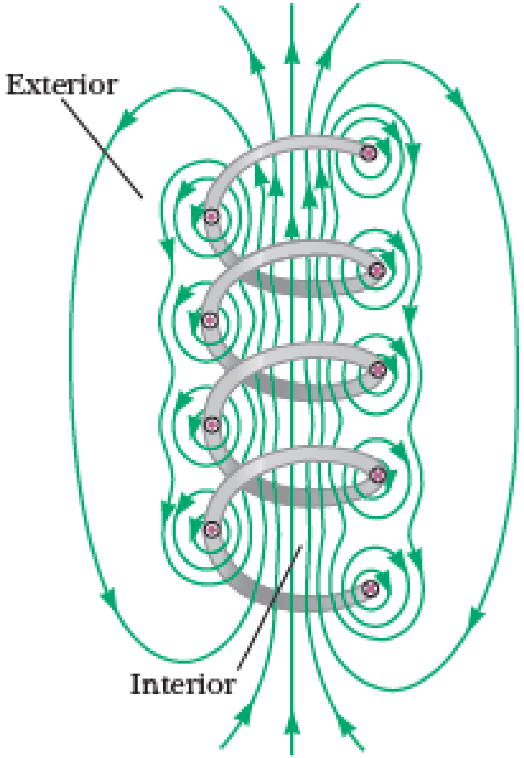
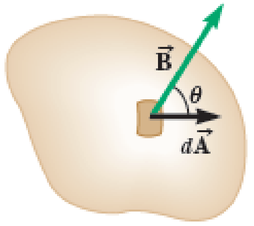


Figure 30.16 The magnetic field lines for a loosely wound solenoid.

Gauss's law in magnetism

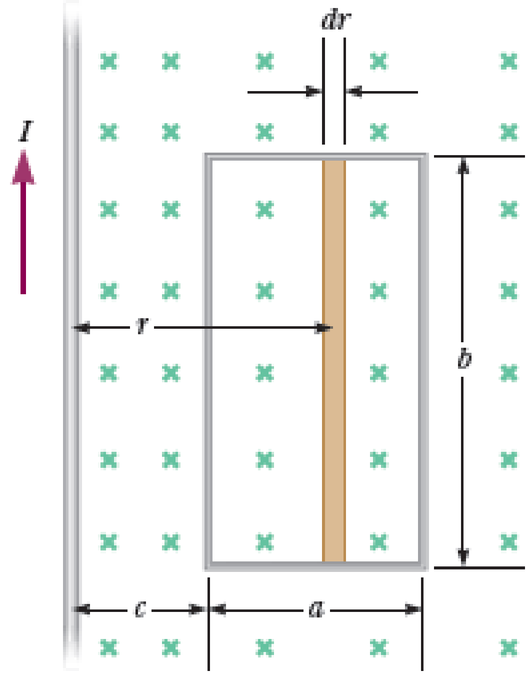
Consider an element of area dA on an arbitrarily shaped surface. If the magnetic field at this element is $\vec{B}$, the magnetic flux through the element is $\vec{B}.\overrightarrow{dA}$, where dA is a vector that is perpendicular to the surface and has a magnitude equal to the area dA. Therefore, the total magnetic flux through the surface is:



$$\Phi_B \equiv \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

Example 30.7 Magnetic Flux Through a Rectangular Loop

A rectangular loop of width a and length b is located near a long wire carrying a current I (Fig. 30.21). The distance between the wire and the closest side of the loop is c. The wire is parallel to the long side of the loop. Find the total magnetic flux through the loop due to the current in the wire.



SOLUTION

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \int B\,dA = \int \frac{\mu_0 I}{2\pi r}\,dA$$

$$\Phi_B = \int \frac{\mu_0 I}{2\pi r}\,b\,dr = \frac{\mu_0 I b}{2\pi} \int \frac{dr}{r}$$

$$\Phi_B = \frac{\mu_0 I b}{2\pi} \int_c^{a+c} \frac{dr}{r} = \frac{\mu_0 I b}{2\pi}\ \ln r \Big|_c^{a+c}$$

$$= \frac{\mu_0 I b}{2\pi} \ln\left(\frac{a+c}{c}\right) = \boxed{\frac{\mu_0 I b}{2\pi} \ln\left(1 + \frac{a}{c}\right)}$$

Gauss's Law in Magnetism

The net magnetic flux through any closed surface is always zero:

$$\oint \vec{B}.\,d\vec{A} = 0$$

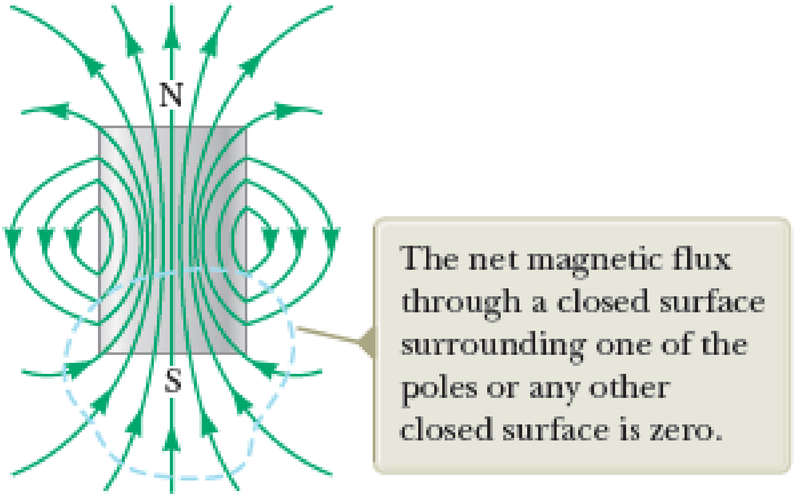


Figure 30.22 The magnetic field lines of a bar magnet form closed loops. (The dashed line represents the intersection of a closed surface with the page.)

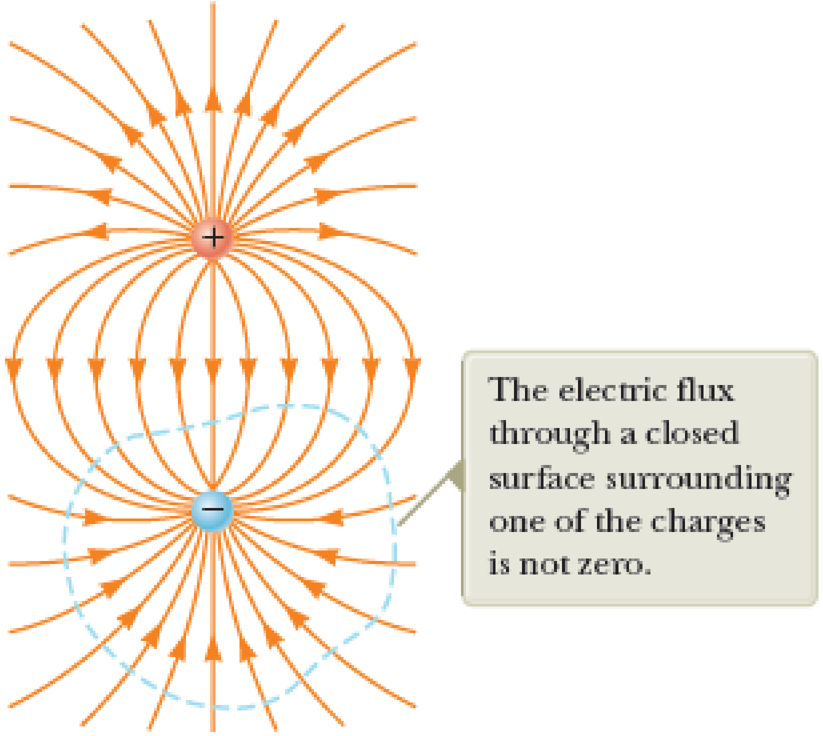
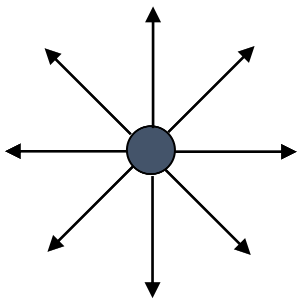
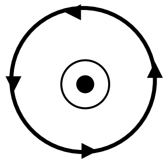


Figure 30.23 The electric field lines surrounding an electric dipole begin on the positive charge and terminate on the negative charge.

Comparison:
Electric Field vs. *Magnetic* Field

	Electric	Magnetic
Source	Charges	Moving Charges
Acts on	Charges	Moving Charges
Force	$F = Eq$	$F = q\ v\ B\ \sin(\theta)$
Direction	Parallel E	Perpendicular to v,B
Field Lines		
Opposites	Charges Attract	Currents Repel

Chapter 6 :Faraday's law

Outlines:

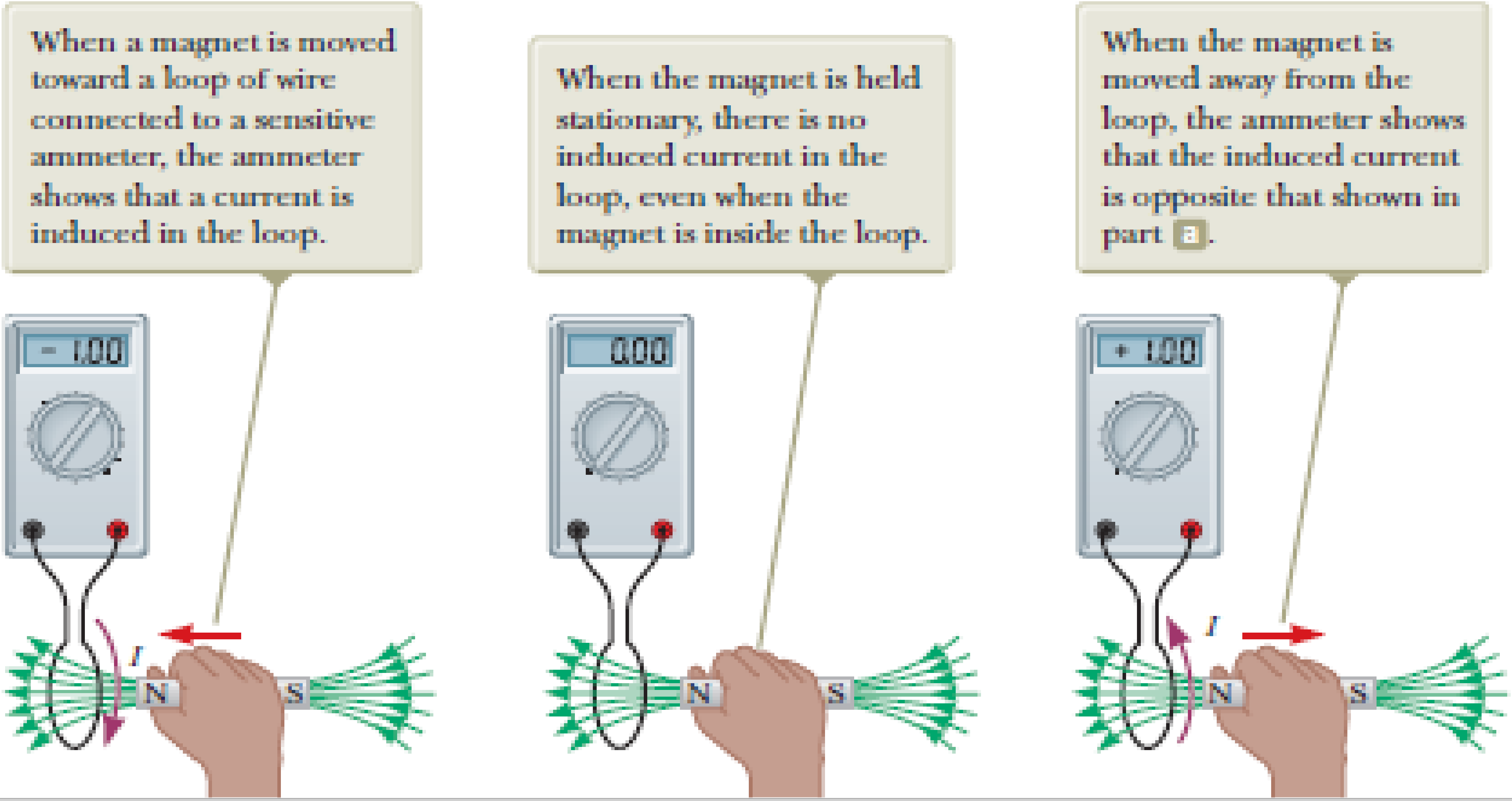
- ✓ *Faraday's law of induction,*
- ✓ *motional emf,*
- ✓ *Lenz's law,*
- ✓ *induced emfs and electric fields,*
- ✓ *generators and motors*

Faraday's law of induction

- Experiments conducted by Faraday in 1831 and by Henry showed that an electric motive force (emf) can be induced in a circuit by a changing magnetic field.

- The results of these experiments led to a very basic and important law of electromagnetism known as ***Faraday's law of induction.***

- emf (and therefore an induced current) can be induced in various processes that involve a *change in a magnetic flux* by : Changig Magnetic field or Conductor moving through a constant magnetic field (called motional emf).



The current is set up even though no batteries are present in the circuit we call such a current an ***induced current*** and say that it is produced by an ***induced emf***.

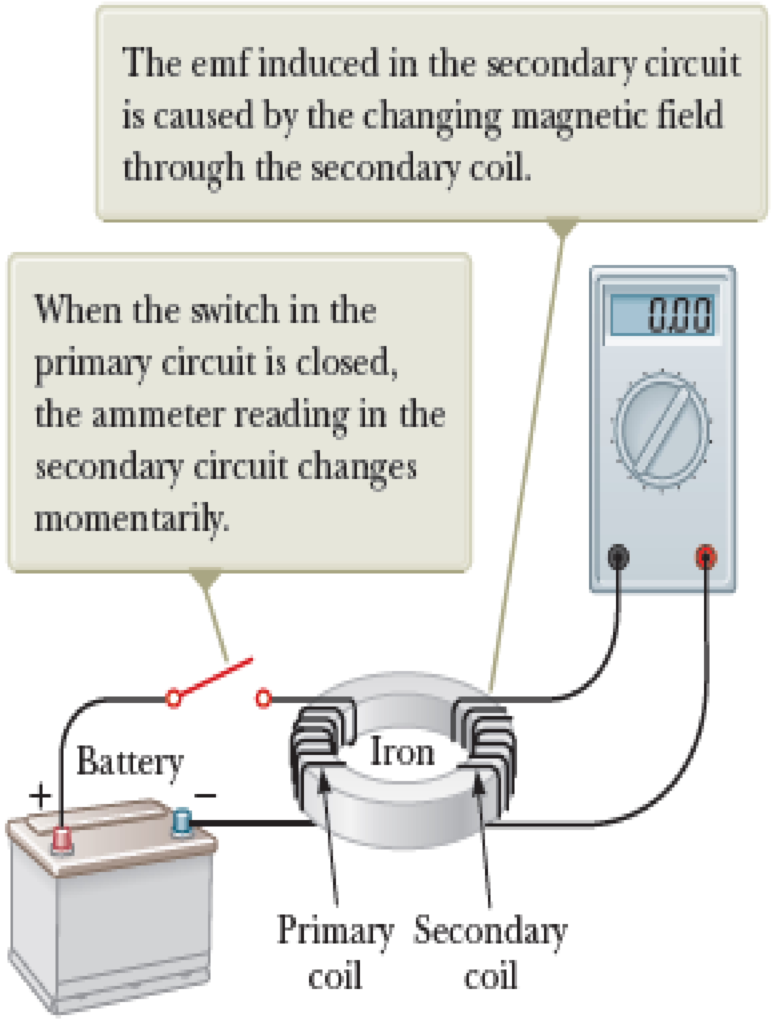
The experiments shown in Figures ,have one thing in common: in each case, an emf is induced in a loop when the magnetic flux through the loop changes with time. In general, this emf is directly proportional to the time rate of change of the magnetic flux through the loop. This statement can be written mathematically as **Faraday's law of induction:**

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

If a coil consists of N loops ,therefore, the total induced emf in the coil is given by

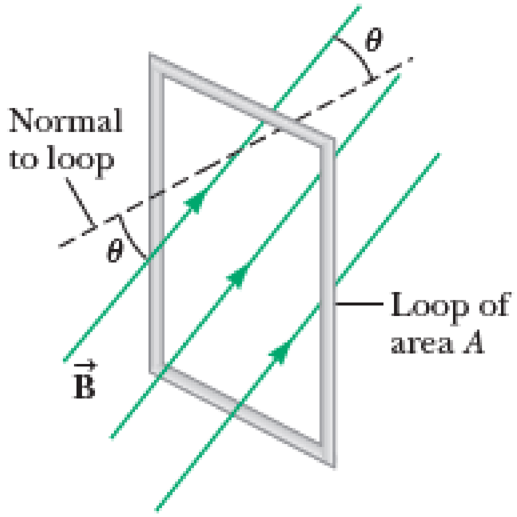
$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$



$$\boxed{\mathcal{E} = -\frac{d}{dt}(BA\cos\theta)}$$

From this expression, we see that an emf can be induced in the circuit in several ways:

- The magnitude of **B** can change with time.
- The area enclosed by the loop can change with time.
- The angle θ between **B** and the normal to the loop can change with time.
- Any combination of the above can occur.



Example 31.1 Inducing an emf in a Coil

A coil consists of 200 turns of wire. Each turn is a square of side d = 18 cm, and a uniform magnetic field directed perpendicular to the plane of the coil is turned on. If the field changes linearly from 0 to 0.50 T in 0.80 s, what is the magnitude of the induced emf in the coil while the field is changing?

$$|\mathcal{E}| = N\frac{\Delta\Phi_B}{\Delta t} = N\frac{\Delta(BA)}{\Delta t} = NA\frac{\Delta B}{\Delta t} = Nd^2\frac{B_f - B_i}{\Delta t}$$

$$|\mathcal{E}| = (200)(0.18\text{ m})^2\frac{(0.50\text{ T} - 0)}{0.80\text{ s}} = \boxed{4.0\text{ V}}$$

WHAT IF? What if you were asked to find the magnitude of the induced current in the coil while the field is changing? Can you answer that question? total resistance of the coil and the circuit is 2Ω

$$\boxed{I = \frac{|\mathcal{E}|}{R} = \frac{4.0\text{ V}}{2.0\ \Omega} = 2.0\text{ A}}$$

Example 31.2 An Exponentially Decaying Magnetic Field

A loop of wire enclosing an area A is placed in a region where the magnetic field is perpendicular to the plane of the loop. The magnitude of $\vec{\mathbf{B}}$ varies in time according to the expression $B = B_{max} e^{-at}$, where a is some constant. That is, at $t = 0$, the field is B_{max}, and for $t > 0$, the field decreases exponentially (Fig. 31.6). Find the induced emf in the loop as a function of time.

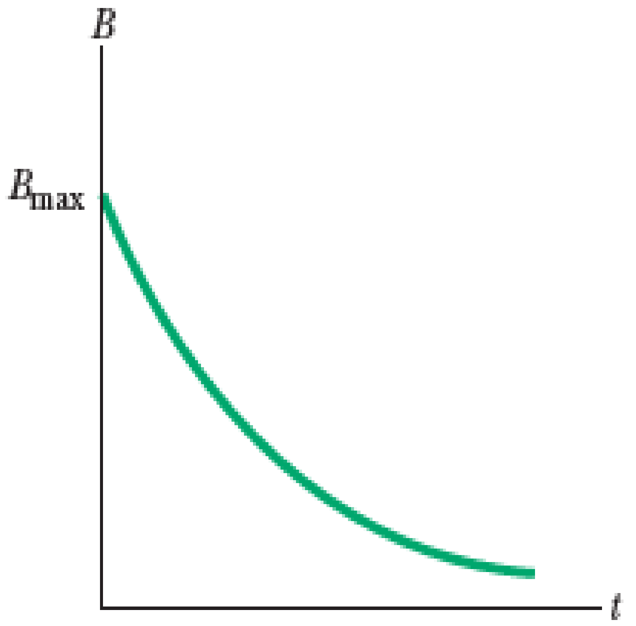
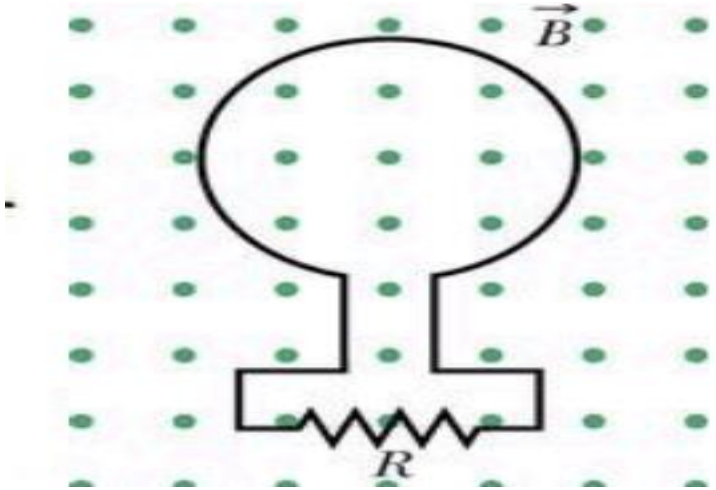


Figure 31.6 (Example 31.2) Exponential decrease in the magnitude of the magnetic field through a loop with time. The induced emf and induced current in a conducting path attached to the loop vary with time in the same way.

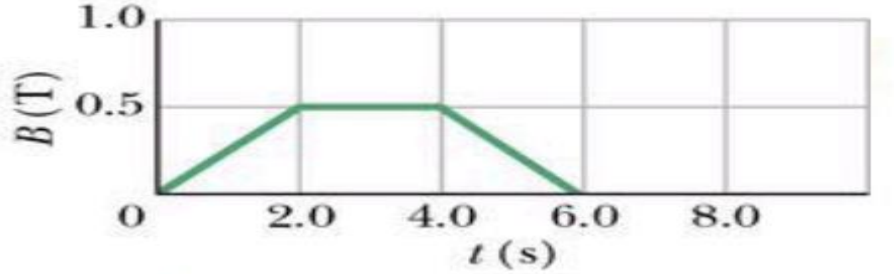
$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(AB_{max}\, e^{-at}\right) = -AB_{max}\frac{d}{dt}e^{-at} = \boxed{aAB_{max}\, e^{-at}}$$

1- A UHF television loop antenna has a diameter of 11 cm. The magnetic field of a TV signal is normal to the plane of the loop and, at one instant of time, its magnitude is changing at the rate 0.16 T/s. The magnetic field is uniform. What emf is induced in the antenna?

2- The magnetic flux through the loop shown in Fig. increases according to the relation $\surd_B = 6.0t^2 + 7.0t$, where $\surd_B$ is in milliwebers and t is in seconds. (a) What is the magnitude of the emf induced in the loop when t = 2.0 s? (b) What is the direction of the current through R?



3- The magnetic field through a single loop of wire, 12 cm in radius and of 8.5 ∧ resistance, changes with time as shown in the figure. Calculate the emf in the loop as a function of time. Consider the time intervals (a) t = 0 to t = 2.0 s, (b) t = 2.0 s to t = 4.0 s, (c) t = 4.0 s to t = 6.0 s. The (uniform) magnetic field is perpendicular to the plane of the loop.



4- A uniform magnetic field is normal to the plane of a circular loop 10 cm in diameter and made of copper wire (of diameter 2.5 mm). (a) Calculate the resistance of the wire. (see copper resistivity in table 1.68 $\times 10^{-8}$ **Ohm.m) (b) At what rate must the magnetic field change with time if an induced current of 10 A is to appear in the loop?**

Motional emf

- Consider a circuit consisting of a conducting bar of length ***l*** sliding along two fixed parallel conducting rails An emf induced in a conductor moving through a constant magnetic field.

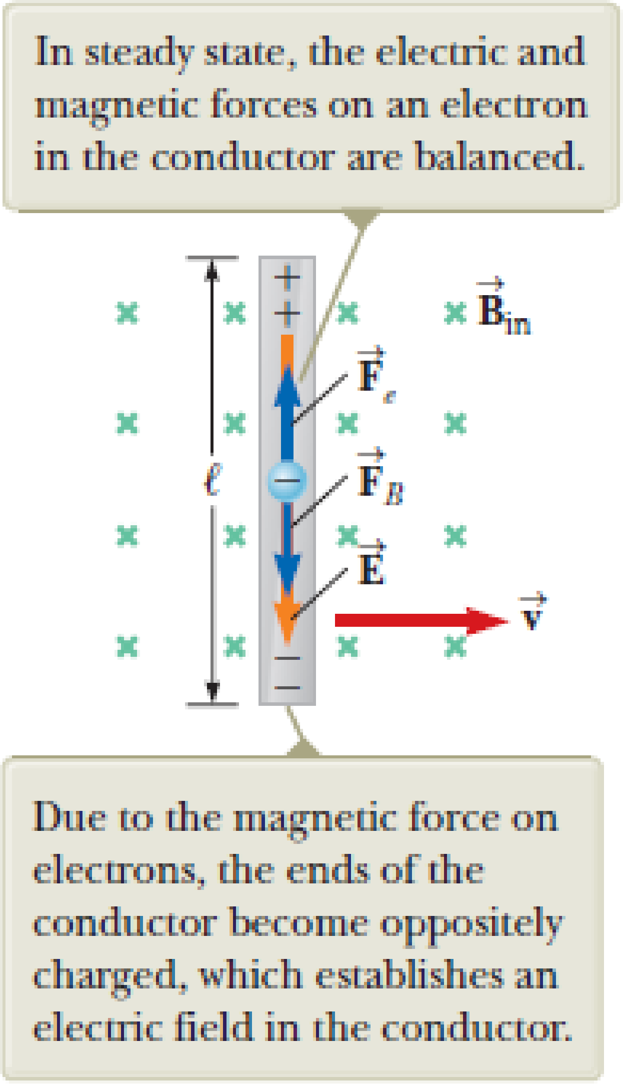
$$\boxed{\varepsilon = -B\, l\, v}$$

- We can find the magnitude of the induced current:

$$\boxed{I = \frac{|\mathcal{E}|}{R} = \frac{B\ell v}{R}}$$

Where, the power (energy per time):

$$\boxed{P = \frac{\varepsilon^2}{R}}$$



Example 31.3 Magnetic Force Acting on a Sliding Bar AM

The conducting bar illustrated in Figure 31.9 (page 942) moves on two frictionless, parallel rails in the presence of a uniform magnetic field directed into the page. The bar has mass m, and its length is ℓ. The bar is given an initial velocity $\vec{\mathbf{v}}_i$ to the right and is released at $t = 0$. Using Newton's laws, find the velocity of the bar as a function of time.

$$F_x = ma \quad \rightarrow \quad -I\ell B = m\frac{dv}{dt}$$

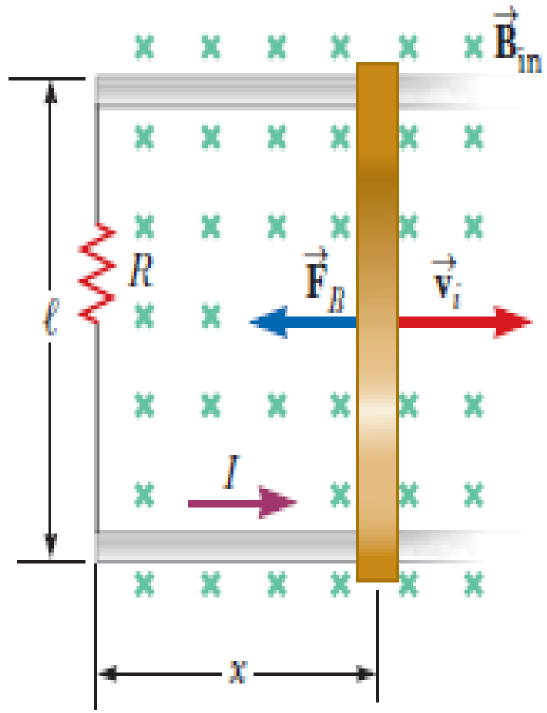
$$m\frac{dv}{dt} = -\frac{B^2\ell^2}{R}v$$

$$\frac{dv}{v} = -\left(\frac{B^2\ell^2}{mR}\right)dt$$

$$\int_{v_i}^{v}\frac{dv}{v} = -\frac{B^2\ell^2}{mR}\int_0^t dt$$

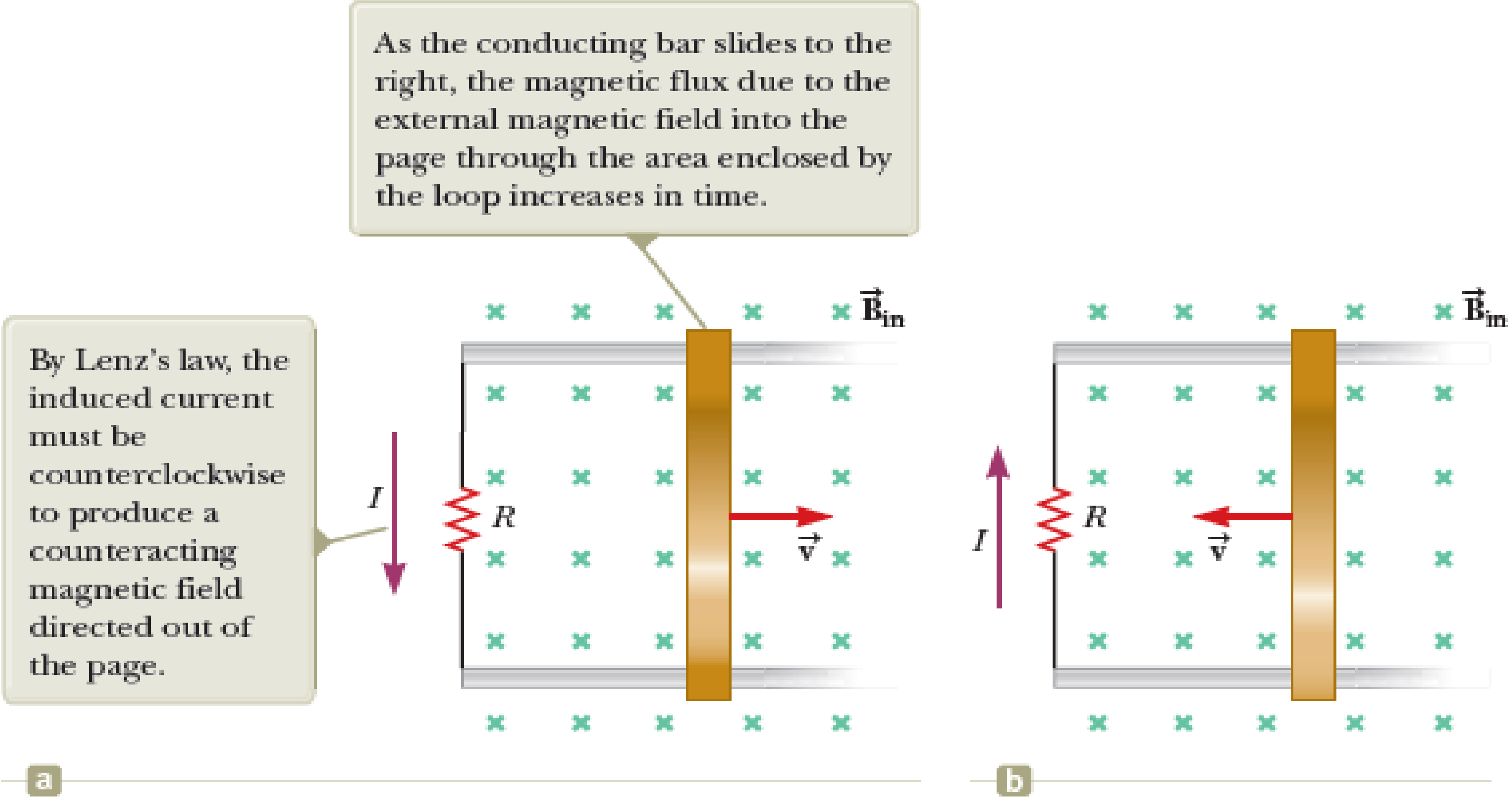
$$\ln\left(\frac{v}{v_i}\right) = -\left(\frac{B^2\ell^2}{mR}\right)t$$

$$(1) \quad v = \boxed{v_i e^{-t/\tau}}$$



Lenz's law,

- The induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop.



Induced emf and Electric Fields

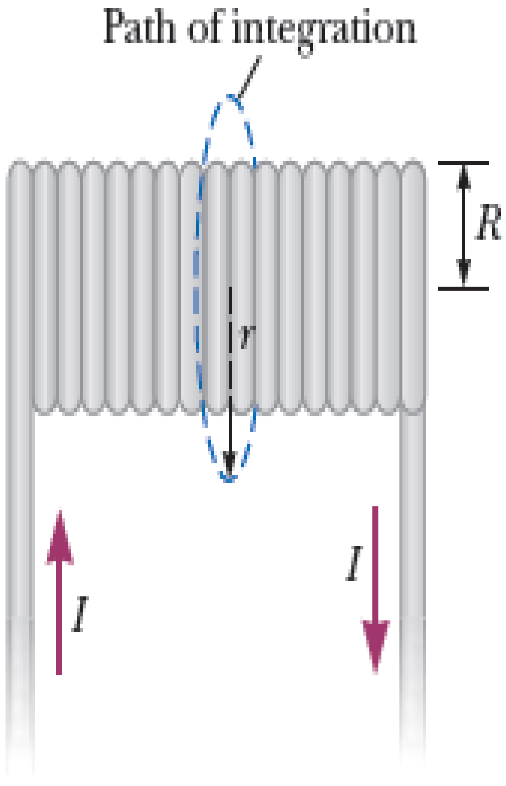
- We have seen that a changing magnetic flux induces an emf and a current in a conducting loop. In our study of electricity, we related a current to an electric field that applies electric forces on charged particles. In the same way, we can relate an induced current in a conducting loop to an electric field.

$$\oint \vec{E}.d\vec{s} = -\frac{d\Phi_B}{dt}$$

Example 31.7 Electric Field Induced by a Changing Magnetic Field in a Solenoid

A long solenoid of radius R has n turns of wire per unit length and carries a time-varying current that varies sinusoidally as $I = I_{max} \cos \omega t$, where I_{max} is the maximum current and ω is the angular frequency of the alternating current source (Fig. 31.16).

(A) Determine the magnitude of the induced electric field outside the solenoid at a distance $r > R$ from its long central axis.



$$(1) \quad -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\pi R^2) = -\pi R^2 \frac{dB}{dt}$$

$$(2) \quad B = \mu_0 n I = \mu_0 n I_{max} \cos \omega t$$

$$(3) \quad -\frac{d\Phi_B}{dt} = -\pi R^2 \mu_0 n I_{\max} \frac{d}{dt}(\cos \omega t) = \pi R^2 \mu_0 n I_{\max} \omega \sin \omega t$$

$$(4) \quad \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E(2\pi r)$$

$$E(2\pi r) = \pi R^2 \mu_0 n I_{\max} \omega \sin \omega t$$

$$E = \boxed{\frac{\mu_0 n I_{\max} \omega R^2}{2r} \sin \omega t} \quad (\text{for } r > R)$$

(B) What is the magnitude of the induced electric field inside the solenoid, a distance r from its axis?

For an interior point ($r < R$), the magnetic flux through an integration loop is given by $\Phi_B = B\pi r^2$.

$$(5) \quad -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\pi r^2) = -\pi r^2 \frac{dB}{dt}$$

$$(6) \quad -\frac{d\Phi_B}{dt} = -\pi r^2 \mu_0 n I_{\max} \frac{d}{dt}(\cos \omega t) = \pi r^2 \mu_0 n I_{\max} \omega \sin \omega t$$

$$E(2\pi r) = \pi r^2 \mu_0 n I_{\max} \omega \sin \omega t$$

$$E = \boxed{\frac{\mu_0 n I_{\max} \omega}{2} r \sin \omega t} \quad (\text{for } r < R)$$

✓ *generators and motors*

- ✓ Electric generators are devices that take in energy by work and transfer it out by electrical transmission. To understand how they operate, let us consider the **alternating current (AC) generator.** In its simplest form, it consists of a loop of wire rotated by some external means in a magnetic field
- ✓ the magnetic flux through the coil at any time *t* is

$$\Phi_B = BA\cos\theta = BA\cos\omega t$$

- ✓ The induced emf in the coil is

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -NBA\frac{d}{dt}(\cos\omega t) = NBA\omega\sin\omega t$$

- ✓ the maximum emf has the value , which occurs when ωt = 90 or 270.

$$\mathcal{E}_{max} = NBA\omega$$