LECTURE 3:
ELECTRIC POTENTIAL

Outlines

- Electric Potential
- Potential Difference
- Potential Differences in a Uniform Electric Field

Electric Potential

- The potential energy per unit charge U/q_0 is independent of the value of q_0 and has a value at every point in an electric field

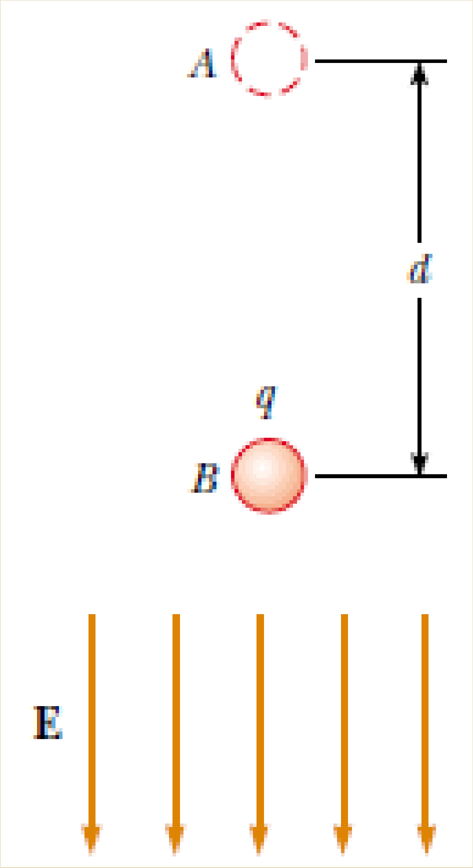
$$V = \frac{U}{q_0}$$

Potential Difference

- The change in potential energy of the system when a test charge is moved between the points divided by the test charge q_0

$$\Delta V = V_B - V_A = \frac{\Delta U}{q_0} = -\int_A^B \vec{E} \cdot d\vec{s}$$

Potential Differences in a Uniform Electric Field



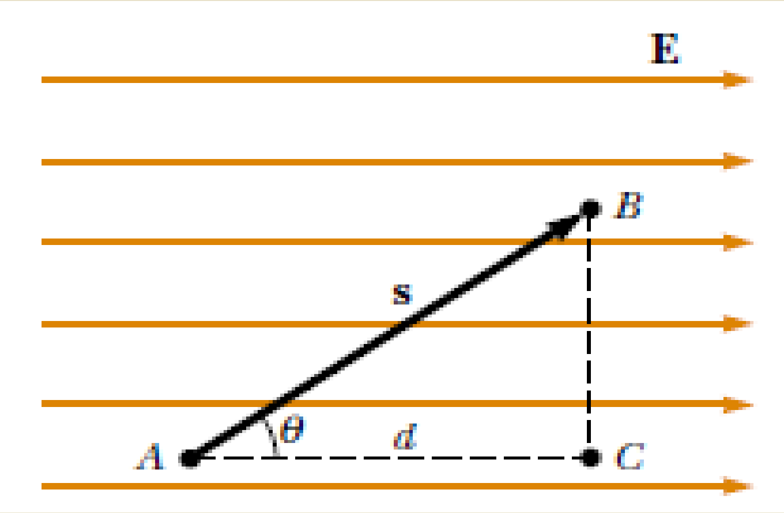
$$\Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s} = -E\int_A^B ds = -Ed$$

$$V_B < V_A$$

- The negative sign indicates that the electric potential at point B is lower than at point A; that is, $V_B < V_A$.

- Electric field lines always point in the direction of decreasing electric potential, as shown in Figure

Example 1



$$V_B - V_A = ?$$

$$V_C - V_A = ?$$

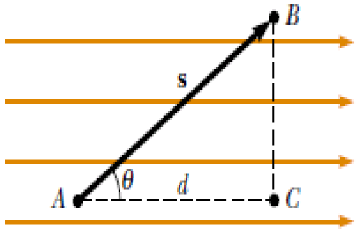


Figure 25.3 A uniform electric field directed along the positive *x* axis. Point *B* is at a lower electric potential than point *A*. Points *B* and *C* are at the *same* electric potential.

If q_0 is negative, then ΔU in Equation 25.7 is positive and the situation is reversed: **A system consisting of a negative charge and an electric field gains electric potential energy when the charge moves in the direction of the field.** If a negative charge is released from rest in an electric field, it accelerates in a direction opposite the direction of the field. In order for the negative charge to move in the direction of the field, an external agent must apply a force and do positive work on the charge.

Now consider the more general case of a charged particle that moves between A and B in a uniform electric field such that the vector $\mathbf{s}$ is not parallel to the field lines, as shown in Figure 25.3. In this case, Equation 25.3 gives

$$\Delta V = -\int_A^B \mathbf{E}\cdot d\mathbf{s} = -\mathbf{E}\cdot\int_A^B d\mathbf{s} = -\mathbf{E}\cdot\mathbf{s} \qquad (25.8)$$

where again we are able to remove $\mathbf{E}$ from the integral because it is constant. The change in potential energy of the charge–field system is

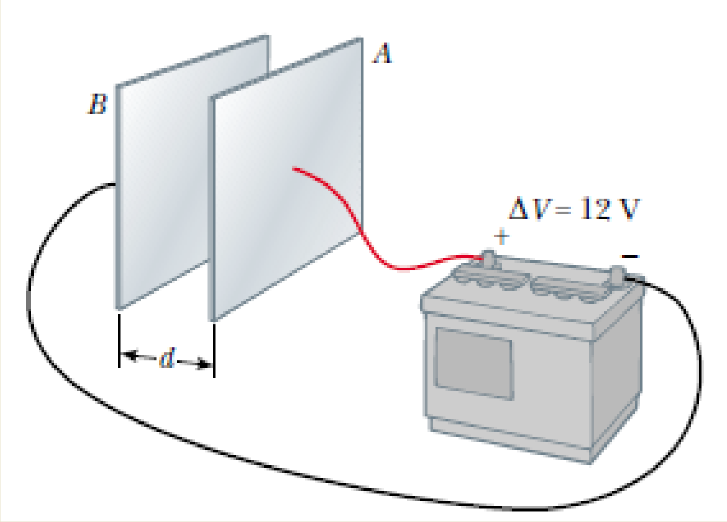
Change in potential energy when a charged particle is moved in a uniform electric field

$$\Delta U = q_0\,\Delta V = -q_0\,\mathbf{E}\cdot\mathbf{s} \qquad (25.9)$$

Finally, we conclude from Equation 25.8 that all points in a plane perpendicular to a uniform electric field are at the same electric potential. We can see this in Figure 25.3, where the potential difference $V_B - V_A$ is equal to the potential difference $V_C - V_A$. (Prove this to yourself by working out the dot product $\mathbf{E}\cdot\mathbf{s}$ for $\mathbf{s}_{A\to B}$, where the angle θ between $\mathbf{E}$ and $\mathbf{s}$ is arbitrary as shown in Figure 25.3, and the dot product for $\mathbf{s}_{A\to C}$, where $\theta = 0$.) Therefore, $V_B = V_C$. **The name equipotential surface is given to any surface consisting of a continuous distribution of points having the same electric potential.**

The equipotential surfaces of a uniform electric field consist of a family of parallel planes that are all perpendicular to the field. Equipotential surfaces for fields with other symmetries are described in later sections.

Example 2



- A battery produces a specified potential difference ΔV between conductors attached to the battery terminals. A 12-V battery is connected between two parallel plates, as shown in the figure. The separation between the plates is $d = 0.30$ cm, and we assume the electric field between the plates to be uniform. Find the magnitude

  of the electric field between the plates.

Example 25.1 The Electric Field Between Two Parallel Plates of Opposite Charge

A battery produces a specified potential difference ΔV between conductors attached to the battery terminals. A 12-V battery is connected between two parallel plates, as shown in Figure 25.5. The separation between the plates is $d = 0.30$ cm, and we assume the electric field between the plates to be uniform. (This assumption is reasonable if the plate separation is small relative to the plate dimensions and if we do not consider locations near the plate edges.) Find the magnitude of the electric field between the plates.

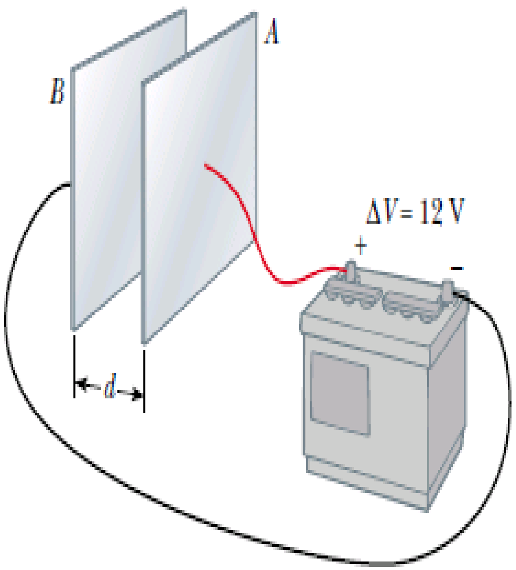


Figure 25.5 (Example 25.1) A 12-V battery connected to two parallel plates. The electric field between the plates has a magnitude given by the potential difference ΔV divided by the plate separation d.

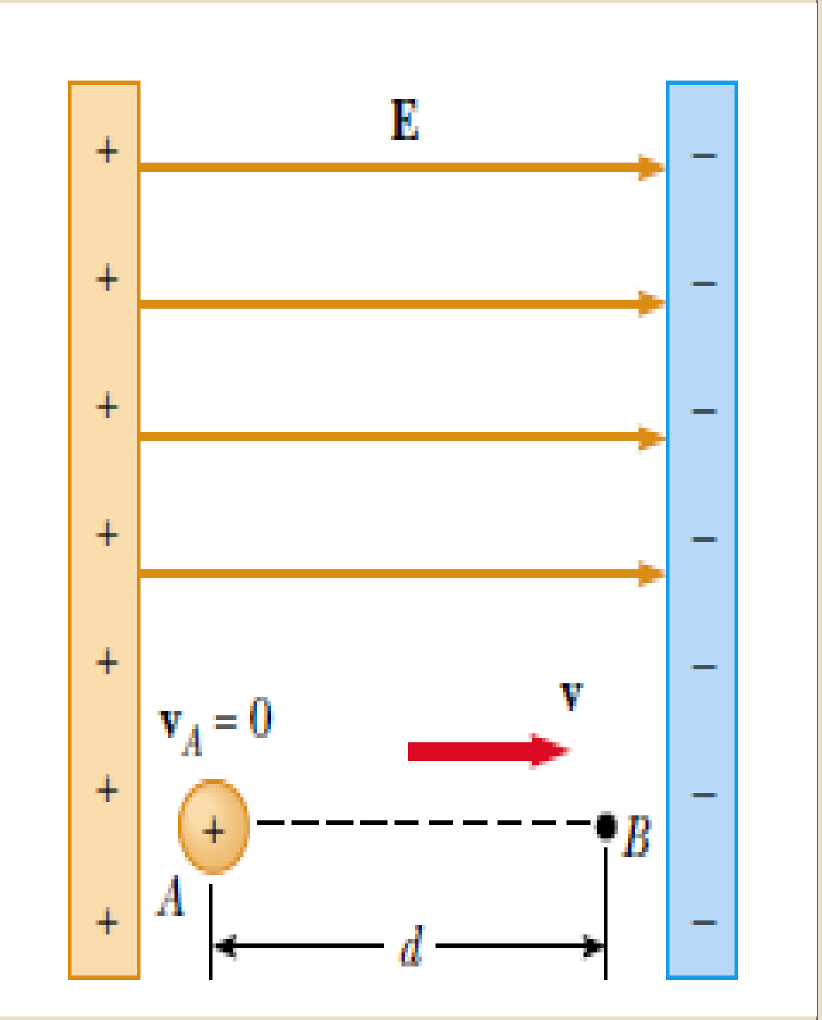
Solution The electric field is directed from the positive plate (A) to the negative one (B), and the positive plate is at a higher electric potential than the negative plate is. The potential difference between the plates must equal the potential difference between the battery terminals. We can understand this by noting that all points on a conductor in equilibrium are at the same electric potential[1]; no potential difference exists between a terminal and any portion of the plate to which it is connected. Therefore, the magnitude of the electric field between the plates is, from Equation 25.6,

$$E = \frac{|V_B - V_A|}{d} = \frac{12\ \text{V}}{0.30 \times 10^{-2}\ \text{m}} = \boxed{4.0 \times 10^3\ \text{V/m}}$$

The configuration of plates in Figure 25.5 is called a *parallel-plate capacitor*, and is examined in greater detail in Chapter 26.

Example 3

- A proton is released from rest in a uniform electric field that has a magnitude of 8.0 % 104 V/m. The proton undergoes a displacement of 0.50 m in the direction of E.
- **(A)** Find the change in electric potential between points *A* and *B*.
- **(B)** Find the change in potential energy of the proton–field system for this displacement.
- **(c)** Find the speed of the proton after completing the 0.50 m displacement in the electric field.



Example 25.2 Motion of a Proton in a Uniform Electric Field

Interactive

A proton is released from rest in a uniform electric field that has a magnitude of 8.0×10^4 V/m (Fig. 25.6). The proton undergoes a displacement of 0.50 m in the direction of **E**.

(A) Find the change in electric potential between points *A* and *B*.

Solution Because the positively charged proton moves in the direction of the field, we expect it to move to a position of lower electric potential. From Equation 25.6, we have

$$\Delta V = -Ed = -(8.0 \times 10^4 \text{ V/m})(0.50 \text{ m}) = \boxed{-4.0 \times 10^4 \text{ V}}$$

(B) Find the change in potential energy of the proton–field system for this displacement.

Solution Using Equation 25.3,

$$\begin{aligned}\Delta U &= q_0 \, \Delta V = e \, \Delta V \\ &= (1.6 \times 10^{-19} \text{ C})(-4.0 \times 10^4 \text{ V}) \\ &= \boxed{-6.4 \times 10^{-15} \text{ J}}\end{aligned}$$

The negative sign means the potential energy of the system decreases as the proton moves in the direction of the electric field. As the proton accelerates in the direction of the field, it gains kinetic energy and at the same time the system loses electric potential energy.

(C) Find the speed of the proton after completing the 0.50 m displacement in the electric field.

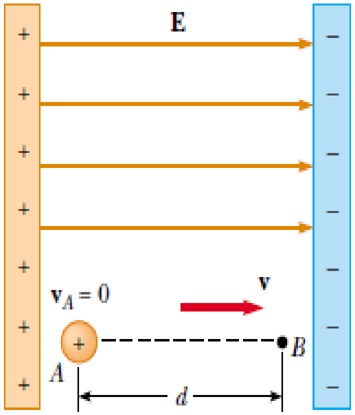


Figure 25.6 (Example 25.2) A proton accelerates from *A* to *B* in the direction of the electric field.

Solution The charge–field system is isolated, so the mechanical energy of the system is conserved:

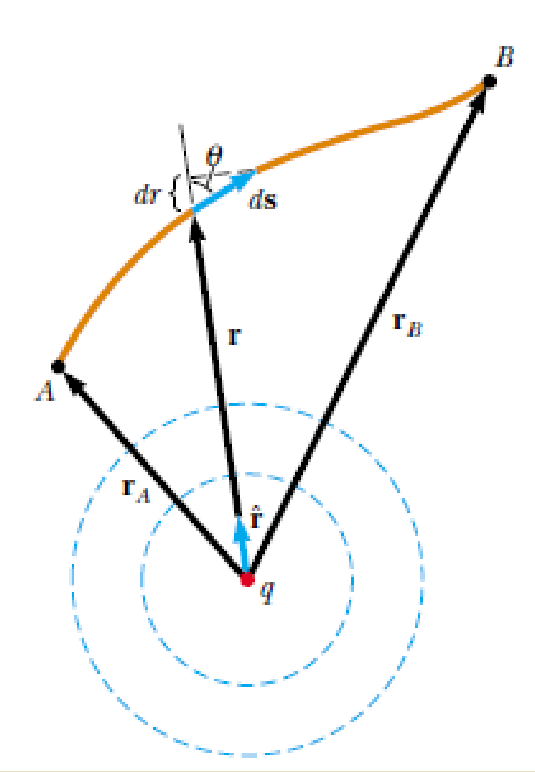
$$\Delta K + \Delta U = 0$$

$$(\tfrac{1}{2}mv^2 - 0) + e \, \Delta V = 0$$

$$\begin{aligned}v &= \sqrt{\frac{-(2e\,\Delta V)}{m}} \\ &= \sqrt{\frac{-2(1.6 \times 10^{-19} \text{ C})(-4.0 \times 10^4 \text{ V})}{1.67 \times 10^{-27} \text{ kg}}} \\ &= \boxed{2.8 \times 10^6 \text{ m/s}}\end{aligned}$$

What If? What if the situation is exactly the same as that shown in Figure 25.6, but no proton is present? Could both parts (A) and (B) of this example still be answered?

Electric Potential and Potential Energy due to Point Charges



$$\Delta V = V_B - V_A = -k_e q \int_A^B \frac{dr}{r^2} = \left.\frac{k_e q}{r}\right|_{r=r_A}^{r=r_B} = k_e q\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

Electric Potential due to Point Charges

- The electric potential created by a point charge at any distance *r* from the charge is

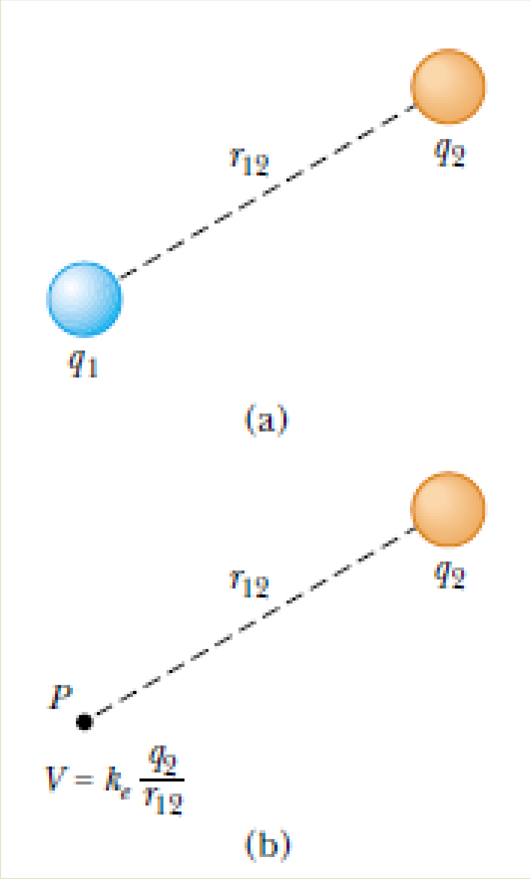
$$V = \frac{k_e q}{r}$$

At $r_A = \infty \rightarrow V_A = 0$

For a group of point charges, we can write the total electric potential at *P in the form*

$$V = k_e \sum_i \frac{q_i}{r_i}$$

Potential Energy due to Point Charges

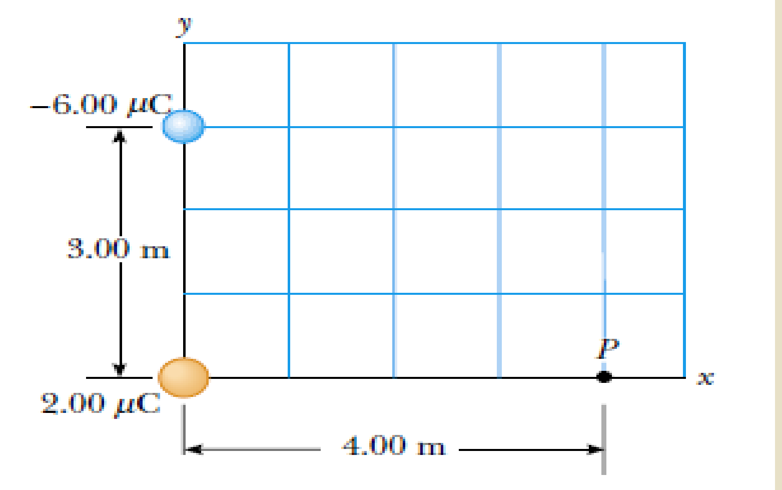


- If two-point charges are separated by a distance r_{12}, the potential energy of the pair of charges is given by

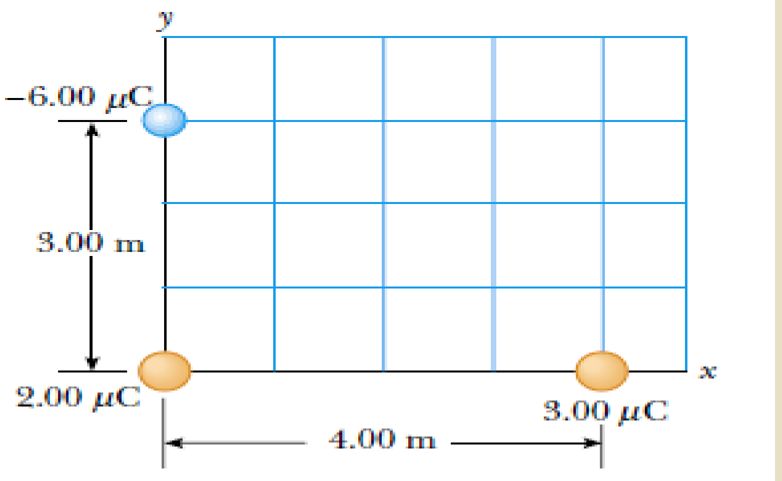
$$U = \frac{k_e q_1 q_2}{r_{12}}$$

Example 4

- charge $q1=2.00\ \mu C$ is located at the origin, and a charge $q2=-6.00\ \mu C$ is located at (0, 3.00) m, as shown in Figure a.
- **(A)** Find the total electric potential due to these charges at the point *P*, whose coordinates are (4.00, 0) m.
- **(B)**Find the change in potential energy of the system of two charges plus a charge $q3 = 3.00\ \mu C$ as the latter



(a)



(b)

Example 25.3 The Electric Potential Due to Two Point Charges Interactive

A charge $q_1 = 2.00\ \mu C$ is located at the origin, and a charge $q_2 = -6.00\ \mu C$ is located at (0, 3.00) m, as shown in Figure 25.12a.

(A) Find the total electric potential due to these charges at the point P, whose coordinates are (4.00, 0) m.

Solution For two charges, the sum in Equation 25.12 gives

$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$V_P = (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) \times \left(\frac{2.00 \times 10^{-6}\ \text{C}}{4.00\ \text{m}} - \frac{6.00 \times 10^{-6}\ \text{C}}{5.00\ \text{m}}\right)$$

$$= -6.29 \times 10^3\ \text{V}$$

(B) Find the change in potential energy of the system of two charges plus a charge $q_3 = 3.00\ \mu C$ as the latter charge moves from infinity to point P (Fig. 25.12b).

Solution When the charge q_3 is at infinity, let us define $U_i = 0$ for the system, and when the charge is at P, $U_f = q_3V_P$; therefore,

$$\Delta U = q_3V_P - 0 = (3.00 \times 10^{-6}\ \text{C})(-6.29 \times 10^3\ \text{V})$$

$$= -1.89 \times 10^{-2}\ \text{J}$$

Therefore, because the potential energy of the system has decreased, positive work would have to be done by an external agent to remove the charge from point P back to infinity.

What If? **You are working through this example with a classmate and she says, "Wait a minute! In part (B), we ignored the potential energy associated with the pair of charges q_1 and q_2!" How would you respond?**

Answer Given the statement of the problem, it is not necessary to include this potential energy, because part (B) asks for the *change* in potential energy of the system as q_3 is brought in from infinity. Because the configuration of charges q_1 and q_2 does not change in the process, there is no ΔU associated with these charges. However, if part (B) had asked to find the change in potential energy when *all three* charges start out infinitely far apart and are then brought to the positions in Figure 25.12b, we would need to calculate the change as follows, using Equation 25.14:

$$U = k_e\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) \times \left(\frac{(2.00 \times 10^{-6}\ \text{C})(-6.00 \times 10^{-6}\ \text{C})}{3.00\ \text{m}} + \frac{(2.00 \times 10^{-6}\ \text{C})(3.00 \times 10^{-6}\ \text{C})}{4.00\ \text{m}} + \frac{(3.00 \times 10^{-6}\ \text{C})(-6.00 \times 10^{-6}\ \text{C})}{5.00\ \text{m}}\right)$$

$$= -5.48 \times 10^{-2}\ \text{J}$$

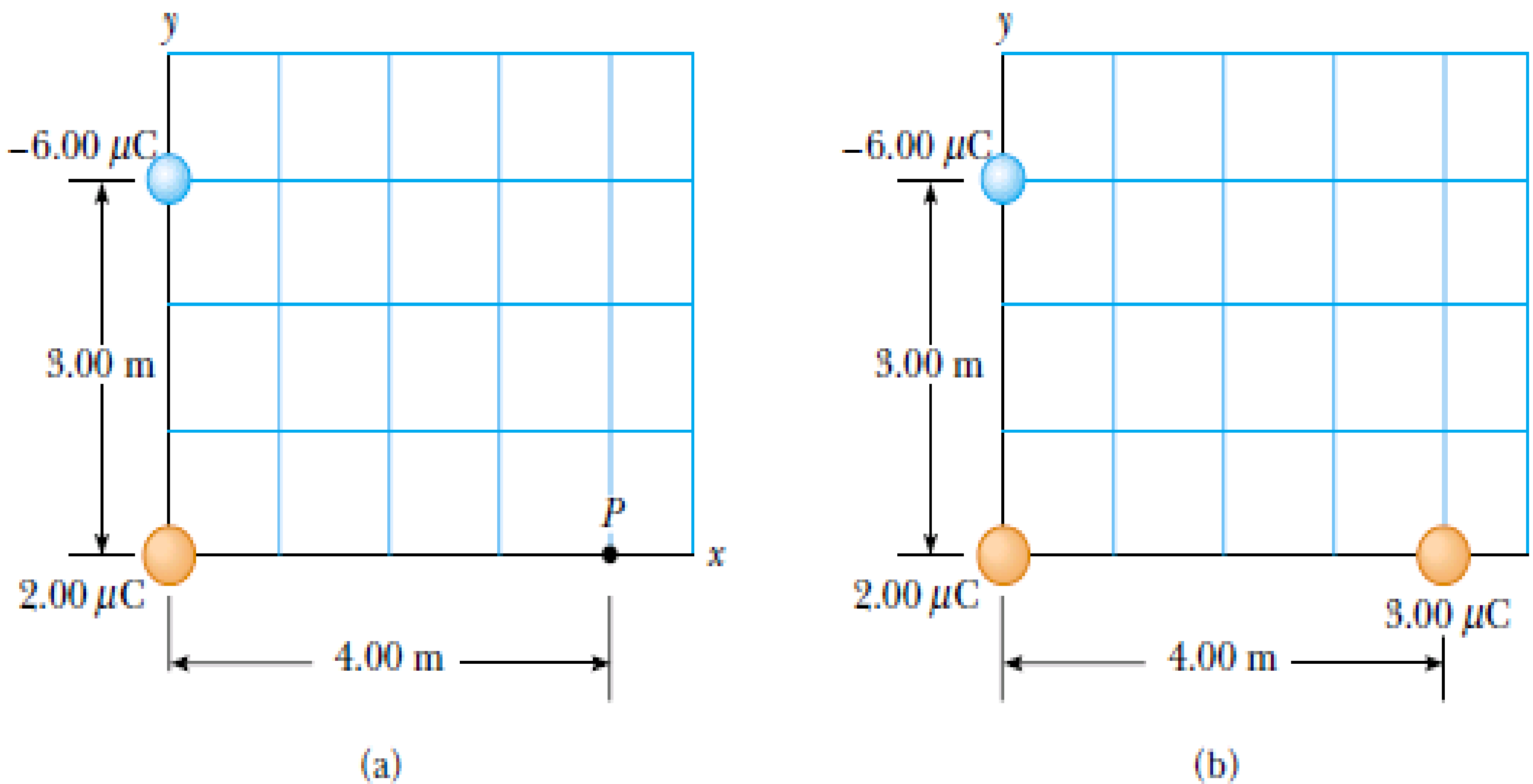


Figure 25.12 (Example 25.3) (a) The electric potential at P due to the two charges q_1 and q_2 is the algebraic sum of the potentials due to the individual charges. (b) A third charge $q_3 = 3.00\ \mu C$ is brought from infinity to a position near the other charges.