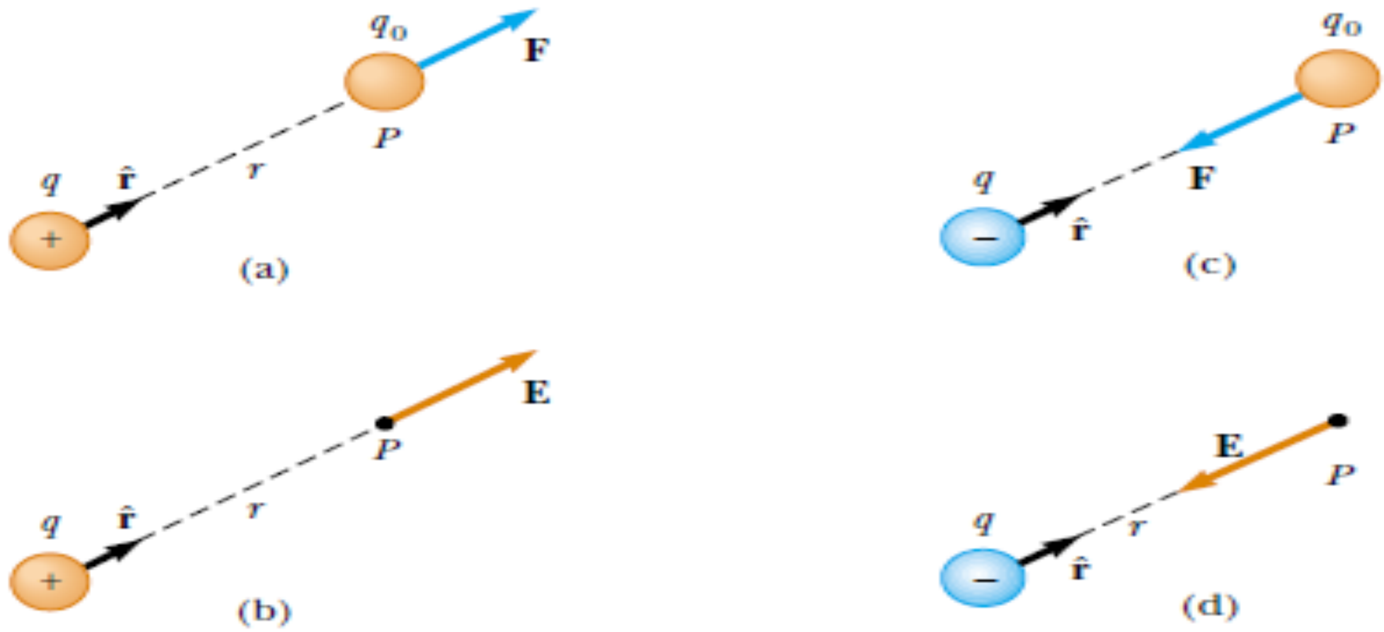
Lecture 2:
The Electric Field and charge distribution

The electric force acting on a positive test charge placed at that point divided by the test charge.

$$\vec{E} = \frac{\vec{F}_e}{q_0} \qquad \text{N/C}$$

$$\vec{F}_e = k_e \frac{q q_0}{r^2} \hat{r}$$

$$\vec{E} = \frac{\vec{F}_e}{q_0} = k_e \frac{q}{r^2} \hat{r}$$



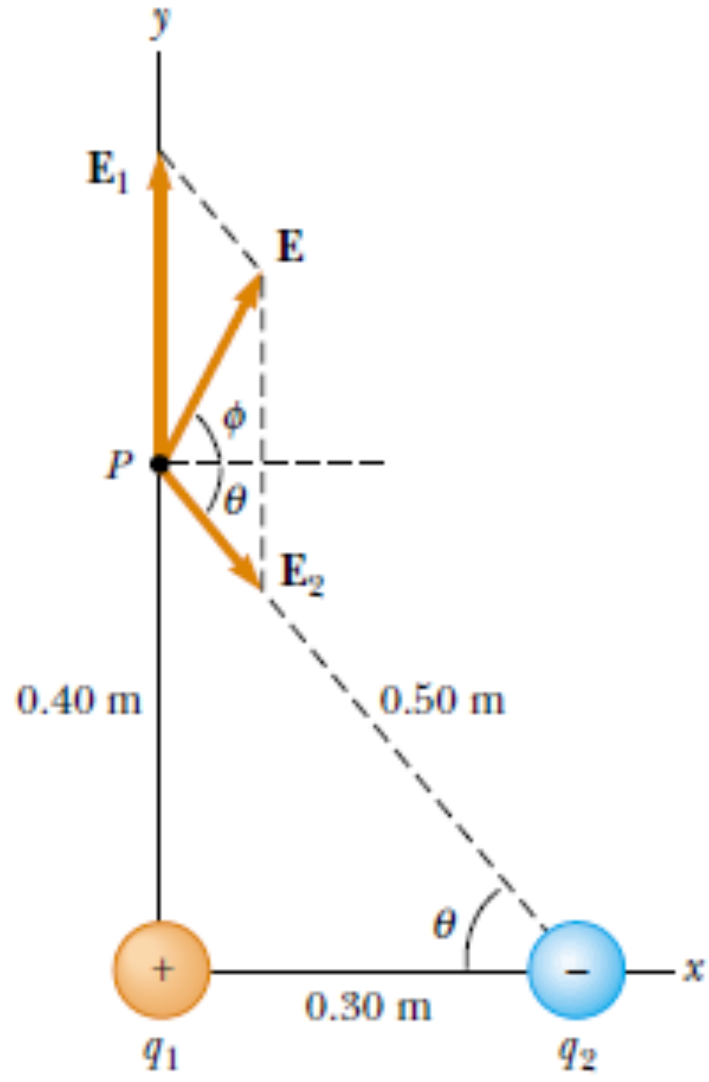
The Electric Field

The total electric field in a point denoted P is equal to the vector sum of the electric fields of all the charges.

$$\vec{E} = k_e \sum_i \frac{q_i}{r_i^{\,2}} \hat{r}_i$$

EXAMPLE 1:

A charge q1=7.0 μC is located at the origin, and a second charge q2=-5.0 μC is located on the x axis, 0.30 m from the origin as shown in the figure. Find the electric field at the point P, which has coordinates



Example Electric Field Due to Two Charges

A charge $q_1 = 7.0\ \mu C$ is located at the origin, and a second charge $q_2 = -5.0\ \mu C$ is located on the x axis, 0.30 m from the origin (Fig. 23.14). Find the electric field at the point P, which has coordinates (0, 0.40) m.

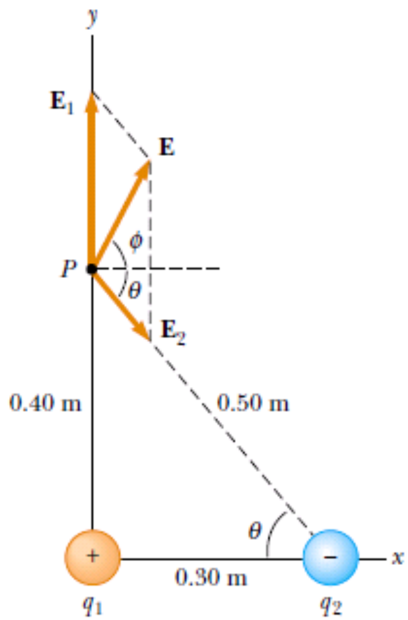


Figure 1 The total electric field **E** at P equals the vector sum $\mathbf{E}_1 + \mathbf{E}_2$, where $\mathbf{E}_1$ is the field due to the positive charge q_1 and $\mathbf{E}_2$ is the field due to the negative charge q_2.

Solution First, let us find the magnitude of the electric field at P due to each charge. The fields $\mathbf{E}_1$ due to the 7.0-μC charge and $\mathbf{E}_2$ due to the -5.0-μC charge are shown in Figure 23.14. Their magnitudes are

$$E_1 = k_e \frac{|q_1|}{r_1^2} = (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{(7.0 \times 10^{-6}\ \mathrm{C})}{(0.40\ \mathrm{m})^2}$$
$$= 3.9 \times 10^5\ \mathrm{N/C}$$

$$E_2 = k_e \frac{|q_2|}{r_2^2} = (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{(5.0 \times 10^{-6}\ \mathrm{C})}{(0.50\ \mathrm{m})^2}$$
$$= 1.8 \times 10^5\ \mathrm{N/C}$$

The vector $\mathbf{E}_1$ has only a y component. The vector $\mathbf{E}_2$ has an x component given by $E_2 \cos\theta = \frac{3}{5}E_2$ and a negative y component given by $-E_2 \sin\theta = -\frac{4}{5}E_2$. Hence, we can express the vectors as

$$\mathbf{E}_1 = 3.9 \times 10^5 \hat{\mathbf{j}}\ \mathrm{N/C}$$
$$\mathbf{E}_2 = (1.1 \times 10^5 \hat{\mathbf{i}} - 1.4 \times 10^5 \hat{\mathbf{j}})\ \mathrm{N/C}$$

The resultant field **E** at P is the superposition of $\mathbf{E}_1$ and $\mathbf{E}_2$:

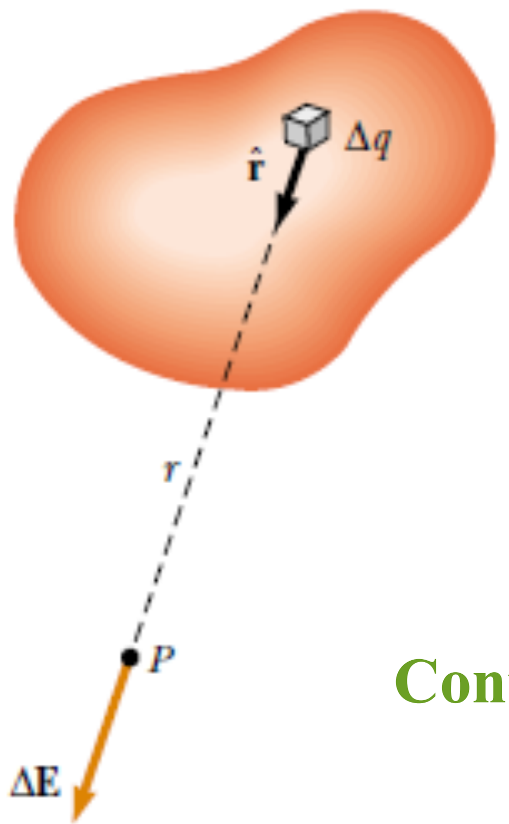
$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = (1.1 \times 10^5 \hat{\mathbf{i}} + 2.5 \times 10^5 \hat{\mathbf{j}})\ \mathrm{N/C}$$

From this result, we find that **E** makes an angle ϕ of 66° with the positive x axis and has a magnitude of 2.7×10^5 N/C.

Electric Field of a Continuous Charge Distribution

The system of charges can be modeled as continuous. Here, the system of closely spaced charges is equivalent to a total charge continuously distributed:

- Along some line
- Over some surface
- Throughout some volume



$$\Delta\vec{E} = k_e \frac{\Delta q}{r^2}\hat{r}$$

Discrete

$$\vec{E} \approx k_e \sum_i \frac{\Delta q_i}{r_i^2}\hat{r}$$

Continuous

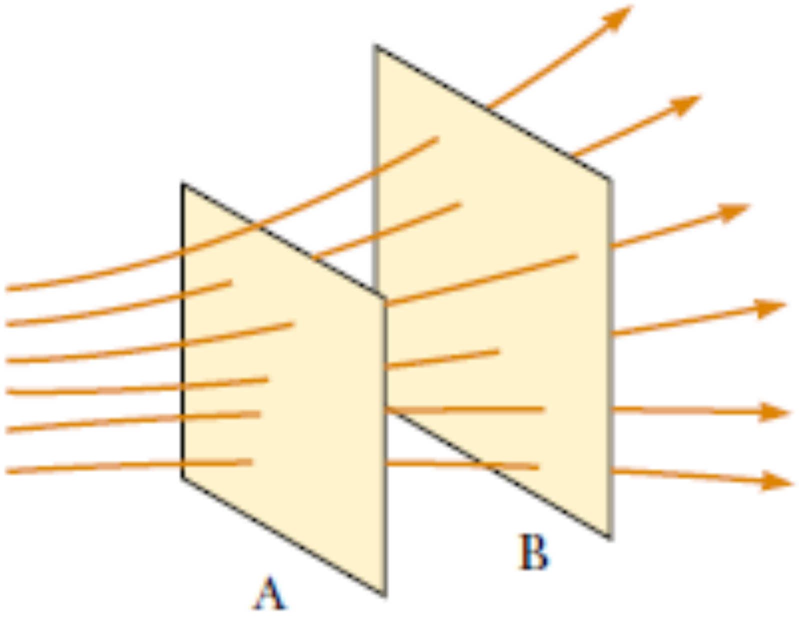
$$\vec{E} = k_e \lim_{\Delta q_i \to 0} \frac{\Delta q_i}{r_i^2}\hat{r}_i = k_e \int \frac{dq}{r^2}$$

Charge density

- The uniformly distribution of charges on a line, a surface, or throughout a volume:
  - linear charge density (λ)
    - a charge Q is uniformly distributed along a line of length l ($\lambda = Q/l$)
  - surface charge density (σ)
    - a charge Q is uniformly distributed on a surface of area A ($\sigma = Q/A$)
  - volume charge density (ρ)
    - a charge Q is uniformly distributed throughout a volume V ($\rho = Q/V$)

ELECTRIC FIELD LINES

Lines that are parallel to the electric field vector at any point in space

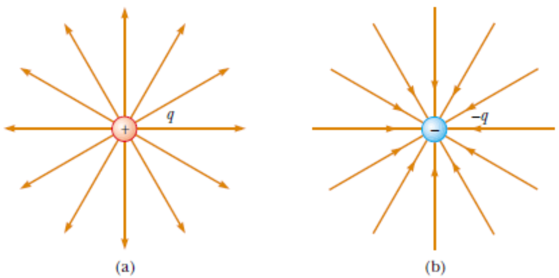


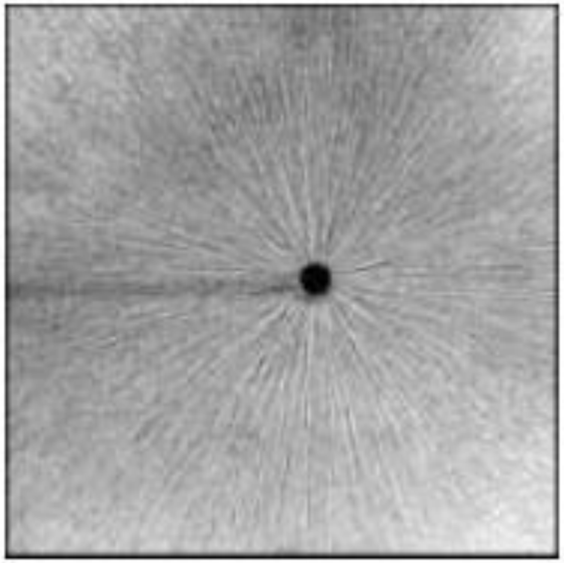
ELECTRIC FIELD LINES

- The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region.

- The field lines are close together where the electric field is strong and far apart where the field is weak.

ELECTRIC FIELD LINES



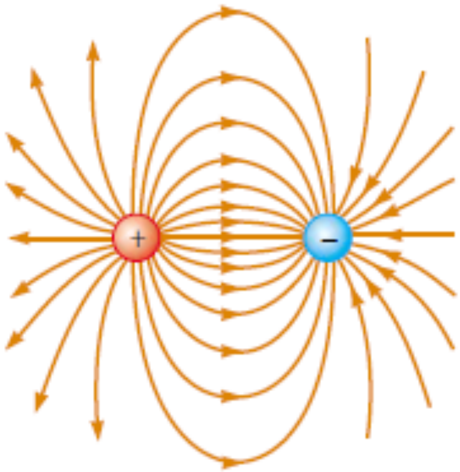


(c)

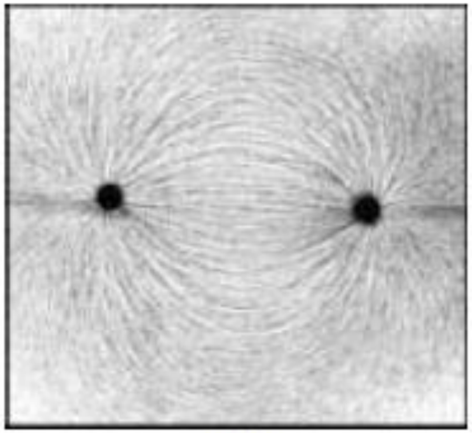
RULES FOR DRAWING ELECTRIC FIELD LINES

- The lines must begin on a positive charge and terminate on a negative charge.
- In the case of an excess of one type of charge, some lines will begin or end infinitely far away.
- The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.
- No two field lines can cross.

ELECTRIC FIELD LINES

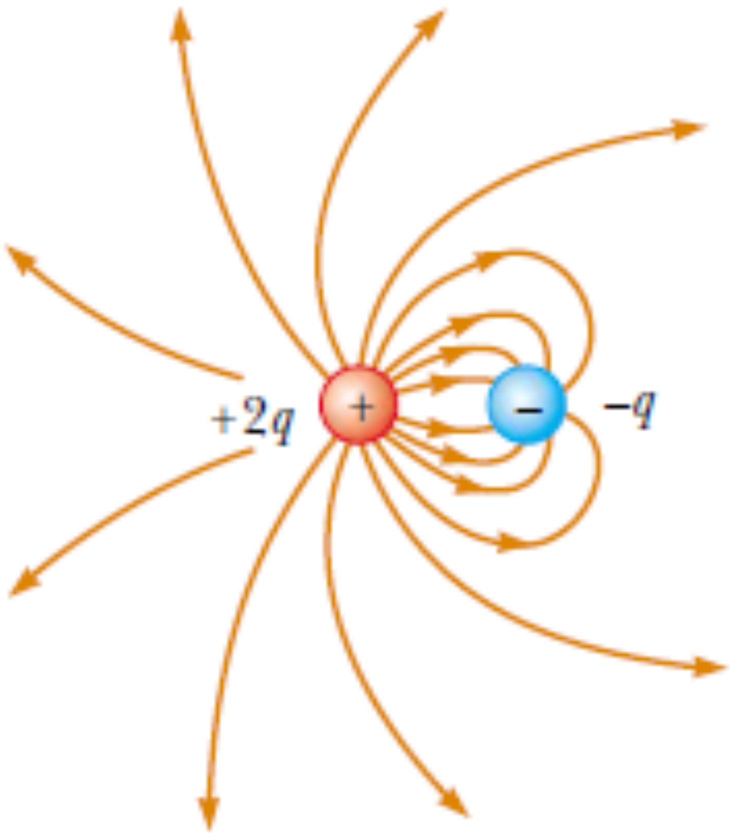


(a)



(b)

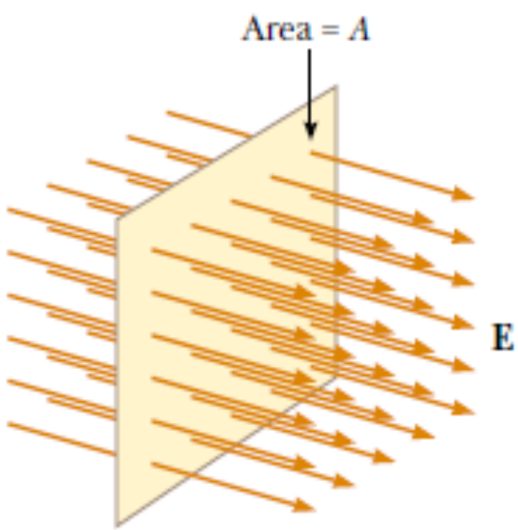
ELECTRIC FIELD LINES



GAUSS'S LAW

Electric Flux (Φ_E)

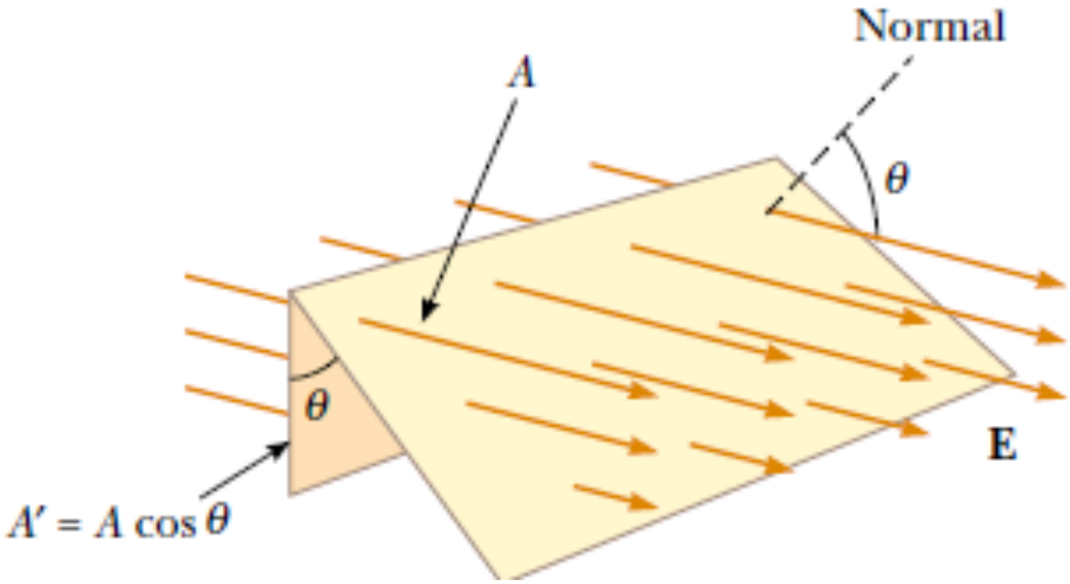
- The number of electric field lines penetrating some surface



$$\Phi_E = EA$$

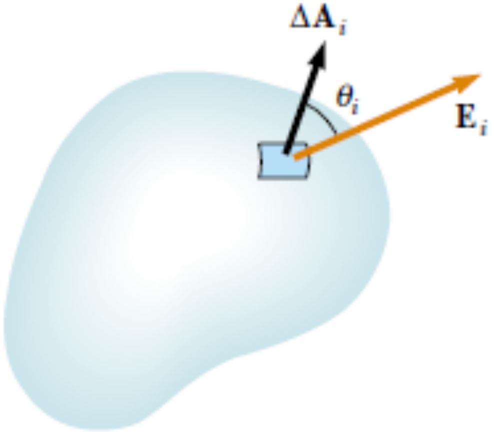
N.m^2/C

ELECTRIC FLUX



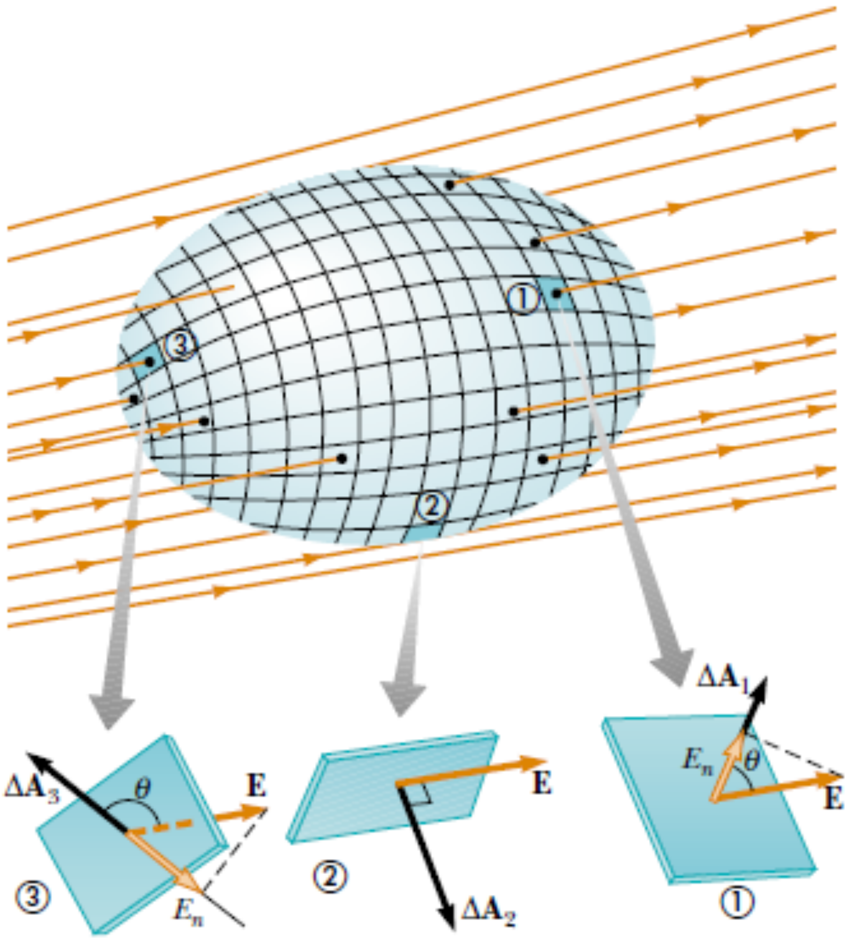
$$\Phi_E = EA' = EA\cos\theta$$

ELECTRIC FLUX



$$\Phi_E = \lim_{\Delta A_i \to 0} \sum_i \vec{E}_i \cdot \Delta\vec{A}_i = \int_{Surface} \vec{E} \cdot d\vec{A}$$

ELECTRIC FLUX



$$\Phi_E = \oint \vec{E} \cdot d\vec{A}$$

EXAMPLE 1

What is the electric flux through a sphere that has a radius of 1.00 m and carries a charge of +1.00 μC at its center?

Example 24.1 Electric Flux Through a Sphere

What is the electric flux through a sphere that has a radius of 1.00 m and carries a charge of $+1.00\ \mu C$ at its center?

Solution The magnitude of the electric field 1.00 m from this charge is found using Equation 23.9:

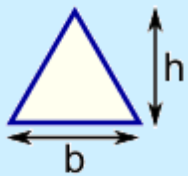
$$E = k_e \frac{q}{r^2} = (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{1.00 \times 10^{-6}\ \mathrm{C}}{(1.00\ \mathrm{m})^2}$$

$$= 8.99 \times 10^3\ \mathrm{N/C}$$

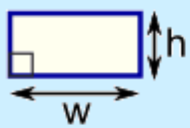
The field points radially outward and is therefore everywhere perpendicular to the surface of the sphere. The flux through the sphere (whose surface area $A = 4\pi r^2 = 12.6\ \mathrm{m^2}$) is thus

$$\Phi_E = EA = (8.99 \times 10^3\ \mathrm{N/C})(12.6\ \mathrm{m^2})$$

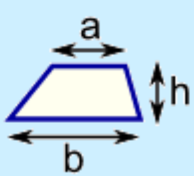
$$= \boxed{1.13 \times 10^5\ \mathrm{N \cdot m^2/C}}$$



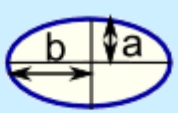
Triangle
Area = ½ × b × h
b = base
h = vertical height



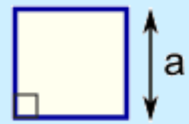
Rectangle
Area = w × h
w = width
h = height



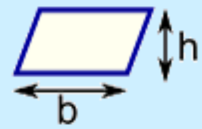
Trapezoid (US)
Trapezium (UK)
Area = ½(a+b) × h
h = vertical height



Ellipse
Area = **π**ab

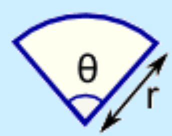


Square
Area = a^2
a = length of side



Parallelogram
Area = b × h
b = base
h = vertical height

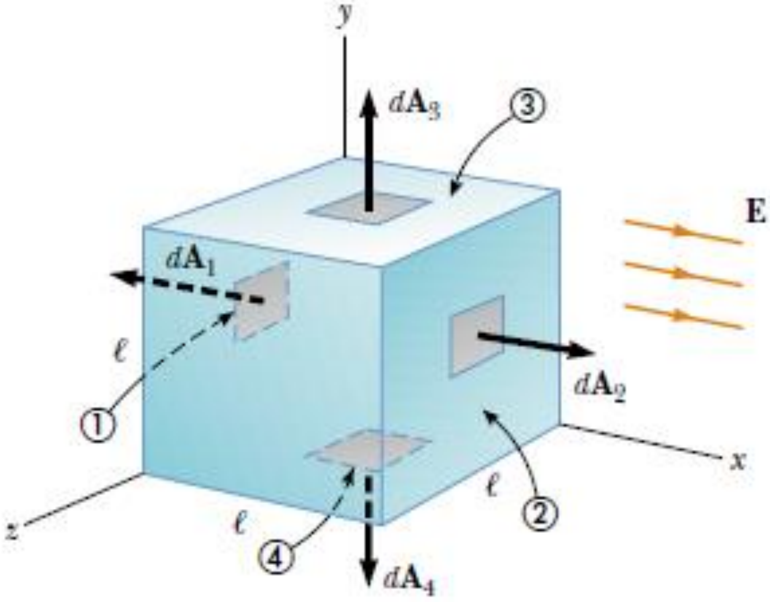
Circle
Area = **π** × r^2
Circumference = 2 × π × r
r = radius



Sector
Area = ½ × r^2 × θ
r = radius
θ = angle in **radians**

EXAMPLE 2

Consider a uniform electric field E oriented in the x direction. Find the net electric flux through the surface of a cube of edge length l, oriented as shown in the figure.



Example 24.2 Flux Through a Cube

Consider a uniform electric field $\mathbf{E}$ oriented in the x direction. Find the net electric flux through the surface of a cube of edge length ℓ, oriented as shown in Figure 24.5.

Solution The net flux is the sum of the fluxes through all faces of the cube. First, note that the flux through four of the faces (③, ④, and the unnumbered ones) is zero because $\mathbf{E}$ is perpendicular to $d\mathbf{A}$ on these faces.

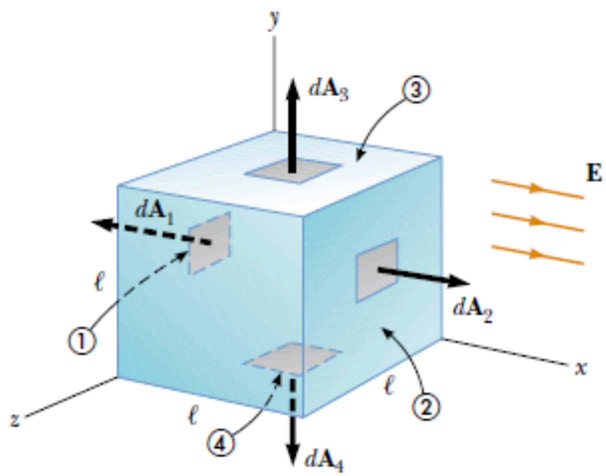


Figure 24.5 (Example 24.2) A closed surface in the shape of a cube in a uniform electric field oriented parallel to the x axis. Side ④ is the bottom of the cube, and side ① is opposite side ②.

The net flux through faces ① and ② is

$$\Phi_E = \int_1 \mathbf{E}\cdot d\mathbf{A} + \int_2 \mathbf{E}\cdot d\mathbf{A}$$

For face ①, $\mathbf{E}$ is constant and directed inward but $d\mathbf{A}_1$ is directed outward ($\theta = 180°$); thus, the flux through this face is

$$\int_1 \mathbf{E}\cdot d\mathbf{A} = \int_1 E(\cos 180°)\, dA = -E\int_1 dA = -EA = -E\ell^2$$

because the area of each face is $A = \ell^2$.

For face ②, $\mathbf{E}$ is constant and outward and in the same direction as $d\mathbf{A}_2$ ($\theta = 0°$); hence, the flux through this face is

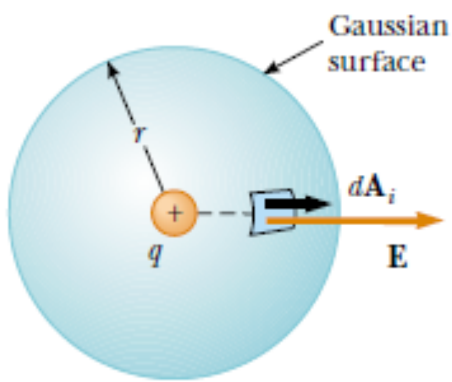
$$\int_2 \mathbf{E}\cdot d\mathbf{A} = \int_2 E(\cos 0°)\, dA = E\int_2 dA = +EA = E\ell^2$$

Therefore, the net flux over all six faces is

$$\Phi_E = -E\ell^2 + E\ell^2 + 0 + 0 + 0 + 0 = \boxed{0}$$

GAUSS'S LAW

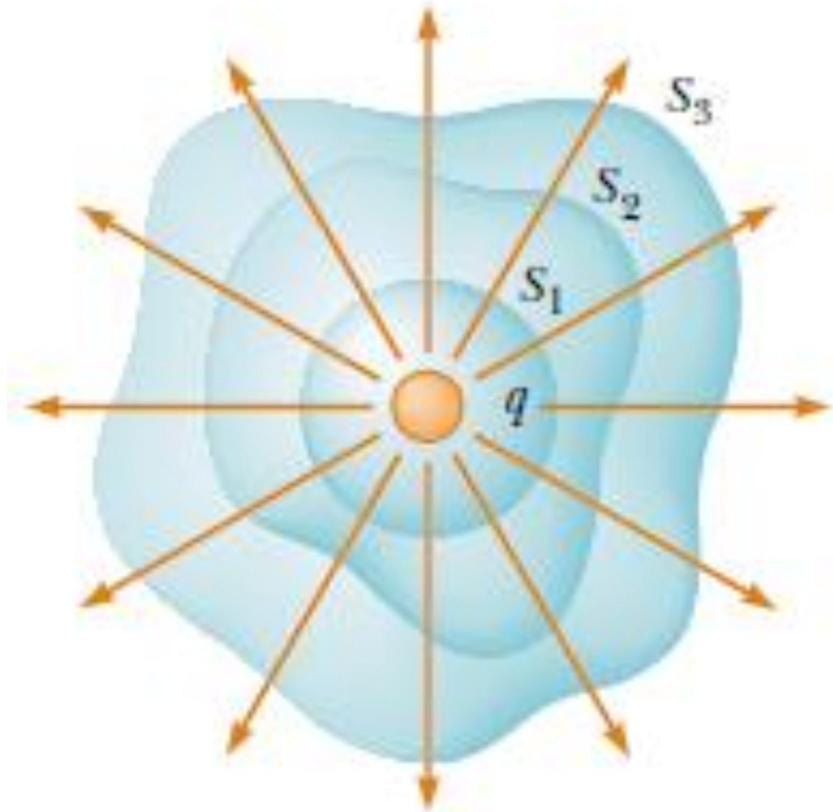
A general relationship between the net electric flux through a closed surface (often called a *Gaussian surface*) and the charge enclosed by the surface



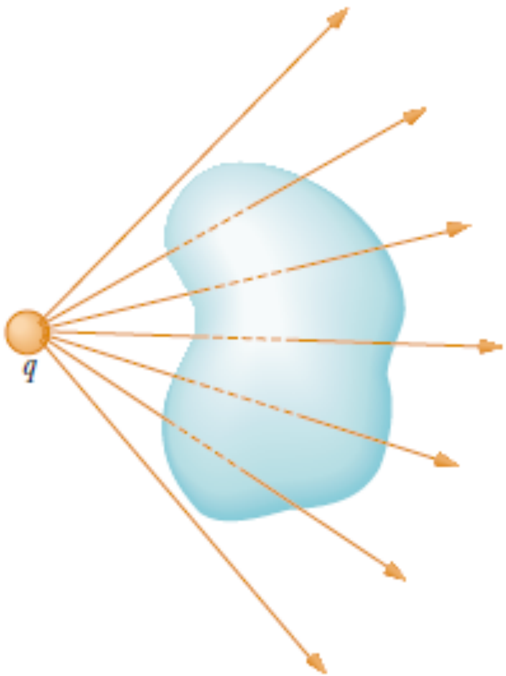
$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = E \oint dA = \frac{q}{\varepsilon_0}$$

GAUSS'S LAW

The net flux through any closed surface surrounding a point charge q is given by q/ε_0 and is independent of the shape of that surface



GAUSS'S LAW



The net electric flux through a closed surface that surrounds no charge is zero

The electric field due to many charges is the vector sum of the electric fields produced by the individual charges

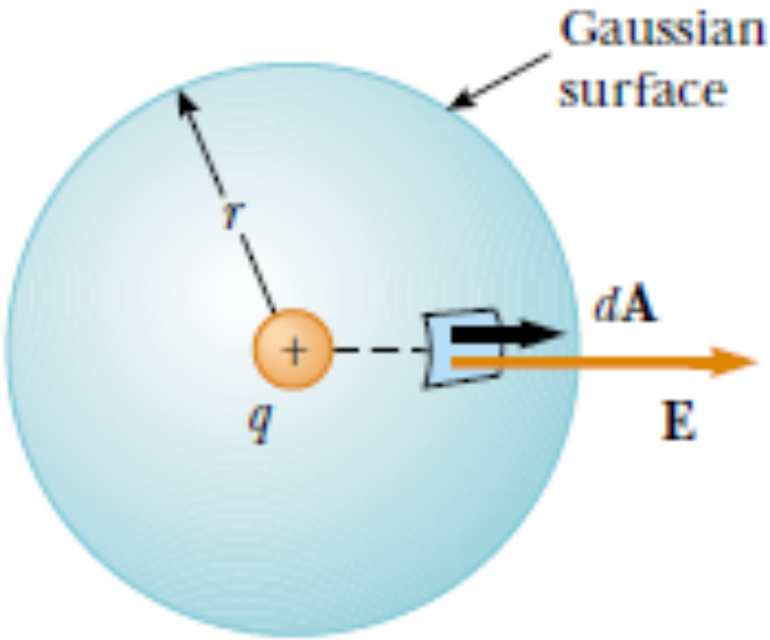
$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint (\vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots).d\vec{A}$$

GAUSS'S LAW

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

EXAMPLE 3

Starting with Gauss's law, calculate the electric field due to an isolated point charge q



Text

Example 24.4 The Electric Field Due to a Point Charge

Starting with Gauss's law, calculate the electric field due to an isolated point charge q.

Solution A single charge represents the simplest possible charge distribution, and we use this familiar case to show how to solve for the electric field with Gauss's law. Figure 24.10 and our discussion of the electric field due to a point charge in Chapter 23 help us to conceptualize the physical situation. Because the space around the single charge has spherical symmetry, we categorize this problem as one in which there is enough symmetry to apply Gauss's law. To analyze any Gauss's law problem, we consider the details of the electric field and choose a gaussian surface that satisfies some or all of the conditions that we have listed above. We choose a spherical gaussian surface of radius r centered on the point charge, as shown in Figure 24.10. The electric field due to a positive point charge is directed radially outward by symmetry and is therefore normal to the surface at every point. Thus, as in condition (2), $\mathbf{E}$ is parallel to $d\mathbf{A}$ at each point. Therefore, $\mathbf{E} \cdot d\mathbf{A} = E\,dA$ and Gauss's law gives

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \oint E\,dA = \frac{q}{\epsilon_0}$$

By symmetry, E is constant everywhere on the surface, which satisfies condition (1), so it can be removed from the integral. Therefore,

$$\oint E\,dA = E \oint dA = E(4\pi r^2) = \frac{q}{\epsilon_0}$$

where we have used the fact that the surface area of a sphere is $4\pi r^2$. Now, we solve for the electric field:

$$E = \frac{q}{4\pi\epsilon_0 r^2} = \boxed{k_e \frac{q}{r^2}}$$

To finalize this problem, note that this is the familiar electric field due to a point charge that we developed from Coulomb's law in Chapter 23.

What If? **What if the charge in Figure 24.10 were not at the center of the spherical gaussian surface?**

Answer In this case, while Gauss's law would still be valid, the situation would not possess enough symmetry to evaluate the electric field. Because the charge is not at the center, the magnitude of $\mathbf{E}$ would vary over the surface of the sphere and the vector $\mathbf{E}$ would not be everywhere perpendicular to the surface.

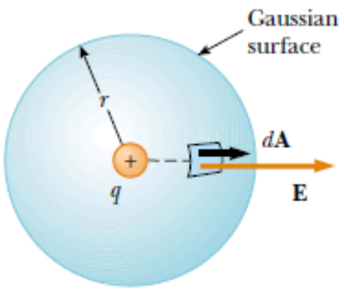
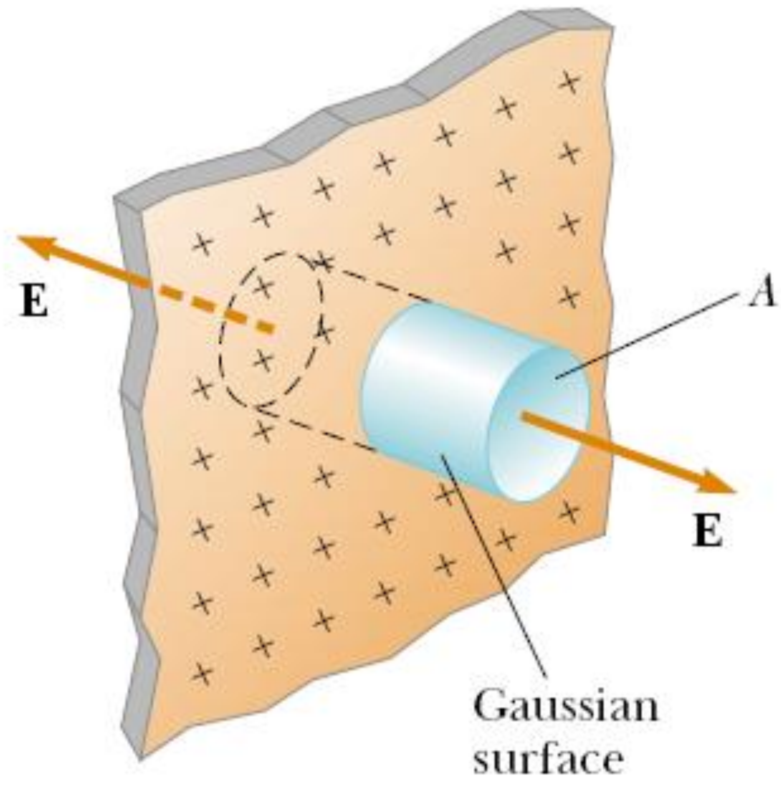


Figure 24.10 (Example 24.4) The point charge q is at the center of the spherical gaussian surface, and $\mathbf{E}$ is parallel to $d\mathbf{A}$ at every point on the surface.

EXAMPLE 4

Find the electric field due to an infinite plane of positive charge with uniform surface charge density σ.



Example 24.8 A Plane of Charge

Find the electric field due to an infinite plane of positive charge with uniform surface charge density σ.

Solution By symmetry, **E** must be perpendicular to the plane and must have the same magnitude at all points equidistant from the plane. The fact that the direction of **E** is away from positive charges indicates that the direction of **E** on one side of the plane must be opposite its direction on the other side, as shown in Figure 24.15. A gaussian surface that reflects the symmetry is a small cylinder whose axis is perpendicular to the plane and whose ends each have an area A and are equidistant from the plane. Because **E** is parallel to the curved surface—and, therefore, perpendicular to $d\mathbf{A}$ everywhere on the surface—condition (3) is satisfied and there is no contribution to the surface integral from this surface. For the flat ends of the cylinder, conditions (1) and (2) are satisfied. The flux through each end of the cylinder is EA; hence, the total flux through the entire gaussian surface is just that through the ends, $\Phi_E = 2EA$.

Noting that the total charge inside the surface is $q_{\text{in}} = \sigma A$, we use Gauss's law and find that the total flux through the gaussian surface is

A plane of charge

$$\Phi_E = 2EA = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

leading to

$$E = \frac{\sigma}{2\epsilon_0} \qquad (24.8)$$

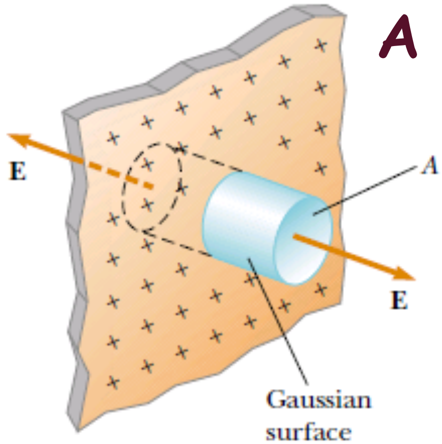


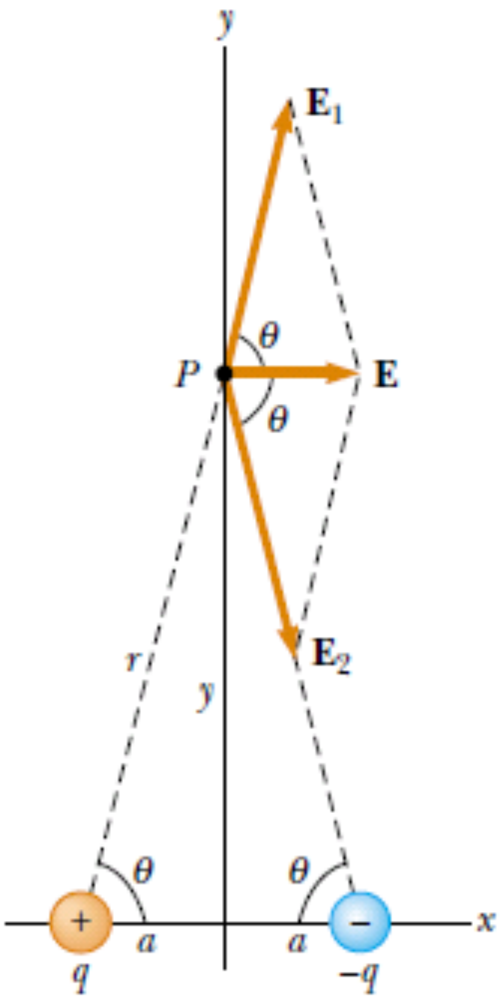
Figure 24.15 (Example 24.8) A cylindrical gaussian surface penetrating an infinite plane of charge. The flux is EA through each end of the gaussian surface and zero through its curved surface.

Because the distance from each flat end of the cylinder to the plane does not appear in Equation 24.8, we conclude that $E = \sigma/2\epsilon_0$ at *any* distance from the plane. That is, the field is uniform everywhere.

What If? **Suppose we place two infinite planes of charge parallel to each other, one positively charged and the other negatively charged. Both planes have the same surface charge density. What does the electric field look like now?**

HOMEWORK 1:

An **electric dipole** is defined as a positive charge q and a negative charge $-q$ separated by a distance $2a$. For the dipole shown in Figure 1, find the electric field $\mathbf{E}$ at P due to the dipole, where P is a distance $y \gg a$ from the origin.



HOMEWORK 2:

Draw the electric field lines for the following cases:

1- An electron (right side) separated by 5 cm from a proton (left side).

2- A proton (right side) separated by 6 cm from a proton (left side).

3- An electron (right side) is separated by 6 cm from an electron (left side).

HOMEWORK 3:

A rod of length ℓ has a uniform positive charge per unit length λ and a total charge Q. Calculate the electric field at a point P that is located along the long axis of the rod and a distance a from one end .