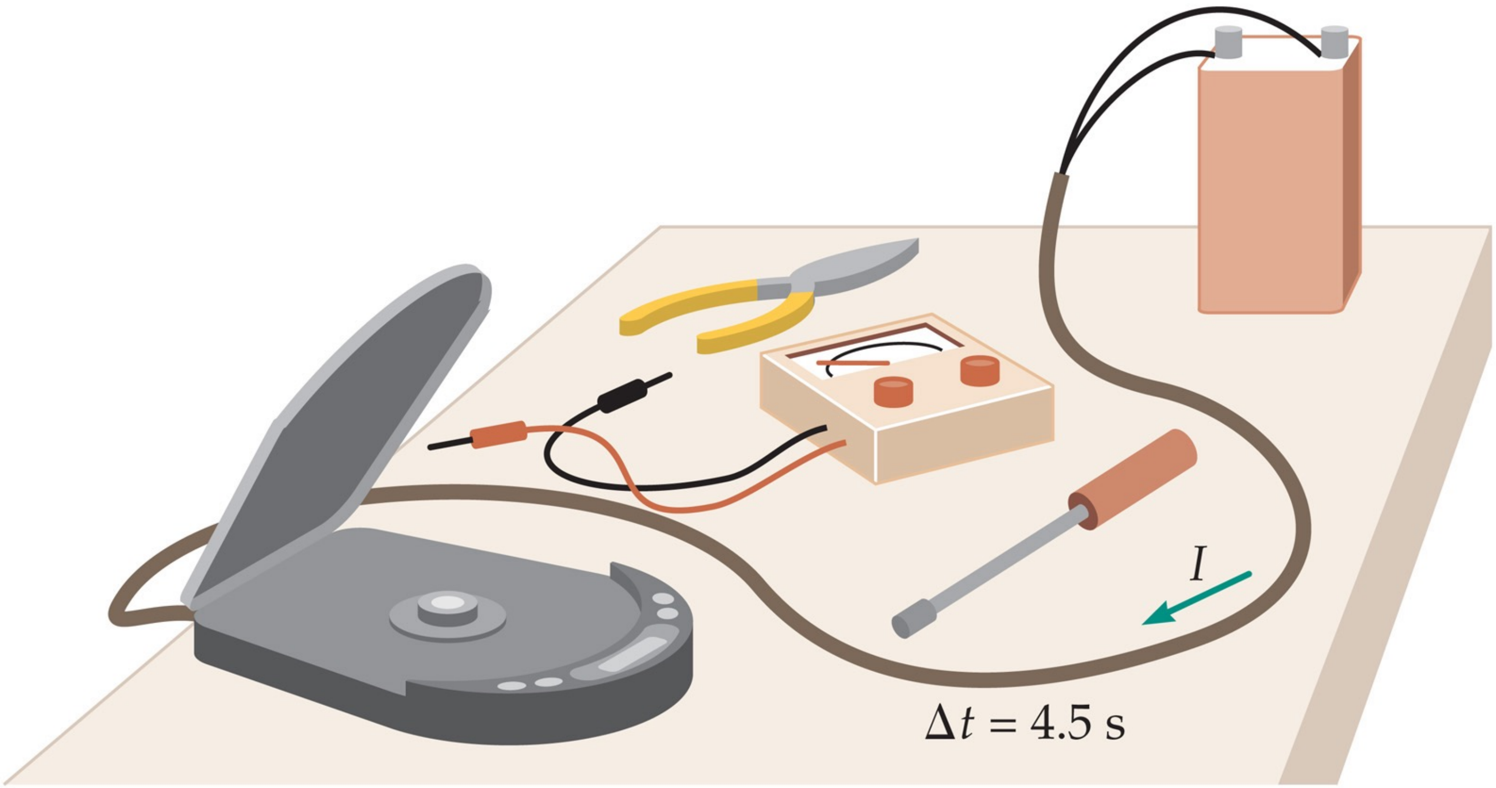
Chapter 21

Electric Current and Direct-Current Circuits



Units of Chapter 21

- ***21.1*** Electric Current
- ***21.2*** Resistance and Ohm’s Law
- ***21.3*** Energy and Power in Electric Circuits
- ***21.4*** Resistors in Series and Parallel
- ***21.5*** Kirchhoff’s Rules
- ***21.6*** Circuits Containing Capacitors
- ***21.7*** RC Circuits

21.1 Electric Current

- ***Electric current*** : is the ***flow of electric charge*** from one place to another

Definition of Electric Current, *I*

$$I = \frac{\Delta Q}{\Delta t}$$

SI unit: coulomb per second, C/s = ampere, A

- ***Electric circuit*** *: A* ***closed path through which charge can flow****, returning to its starting point.*
- ***Direct current circuit*** (dc circuits): are that kind of circuits in which the ***current always flows in the same direction .***

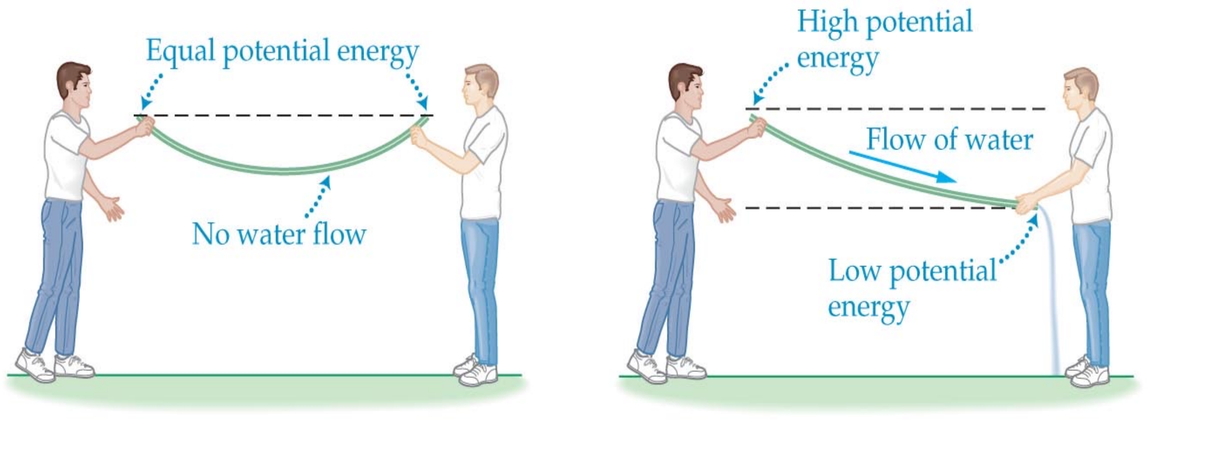
❑ ***Note***: *although* ***electrons move freely*** *in metal wires,* ***they don't flow unless*** *the wires are* ***connected to a source of electrical energy U*** .

WHERE:

$$\Delta U = q\ \Delta v$$

Compare this to the flow of the water in a garden hose

(look at the figure)

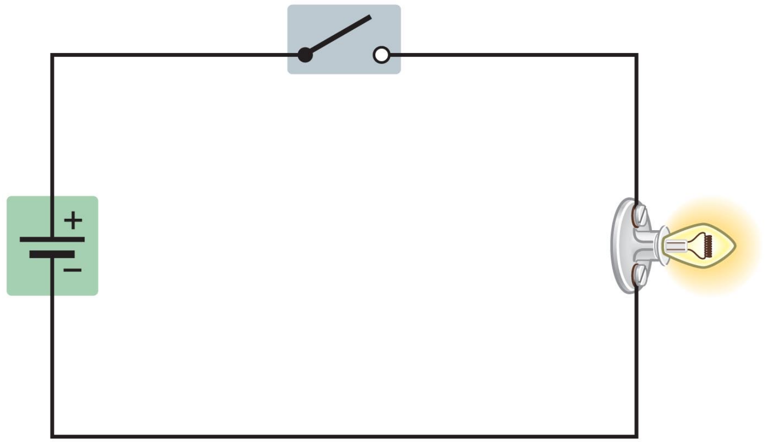


(a) Equal potential energy → no flow

(b) Water flows from high potential energy to low

The same concept applies in the two terminals of a battery

(look at the figure)



(b) Circuit diagram for flashlight

Direction of flow of electrons in a circuit :

when the battery is connected to the circuit, ***electrons move*** in a closed path ***from the – ve terminal through the circuit to the +ve terminal.***

The electromotive force:

A battery that is ***disconnected*** from any circuit has an ***electric potential difference between its terminals*** that is called the electromotive force or ($emf\ (\mathcal{E})$)

Despite its name, the ***emf is an electric potential (Measured in volts)***, ***NOT*** *a* ***force***.

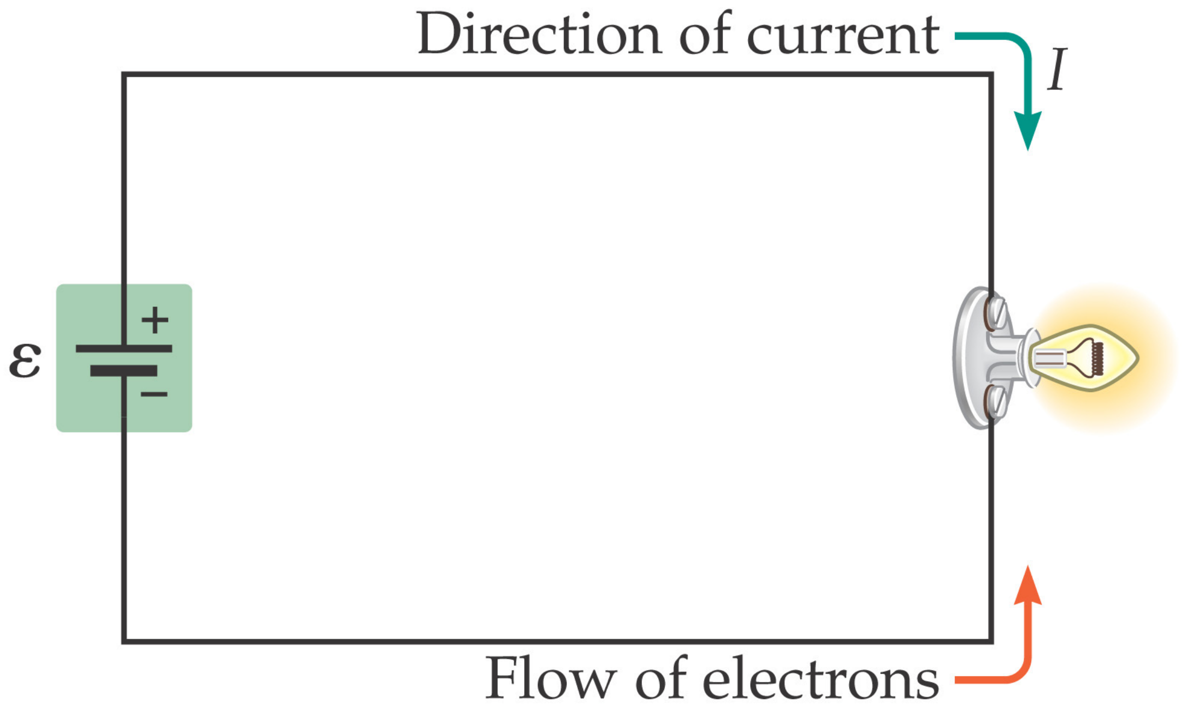
The amount of work it takes to move a charge ΔQ from one terminal to the other is:

$$W = \Delta Q\, \mathcal{E}$$

The direction of ***conventional current*** flow – ***from the positive terminal*** to the ***negative one***. It was decided before it was realized that electrons are negatively charged. Therefore, current flows around a circuit in the direction of a positive charge

electrons move the other way.

However, this does not matter in most circuits.



Exercise 2, page 845

A flash bulb carries a current of (0.18 A) for (78 sec).

(a) How much charge flows through the bulb in this time?

(b) How many electrons?

Answers: (a) 14 C (b) 8.8 x 10^{19} electrons

Exercise 4, page 845

A car battery does 260 J of work on the charge passing through it as it starts an engine.

(a) If the emf of the battery is 12v, how much charge passes through the battery during the start?

(b) If the emf is doubled to (24 v), does the amount of charge passing through the battery increase or decrease? By what factor?

Answers: (a) 22 C (b) Decrease by a factor of 2 (1/2)

21.2 *Resistance and Ohm's Law*

- Wires present some ***resistance to the motion of electrons***. Ohm's law relates the voltage to the current:

Ohm's Law

$$V = IR$$

SI unit: volt, V

Be careful – Ohm's law is not a universal law and is only useful for certain materials (which include most metallic conductors)

- So, the resistance of the wire will be

$$R = \frac{V}{I}$$

The units of resistance, volts per ampere, are called ohms (**Ω**)

$$1\,\Omega = 1\,\mathrm{V/A}$$

Resistivity (ρ)

Question: Does the resistance change with material?

Two wires of the same length and diameter will have different resistances if they are ***made of different materials***. This ***property of a material is called the resistivity.***

Definition of Resistivity, ρ

$$R = \rho\left(\frac{L}{A}\right)$$

The difference between insulators, semiconductors, and conductors can be clearly seen in their resistivity.

TABLE 21–1 Resistivities

Substance	Resistivity, $\rho\ (\Omega \cdot m)$
Insulators	
Quartz (fused)	7.5×10^{17}
Rubber	1 to 100×10^{13}
Glass	1 to $10{,}000 \times 10^{9}$
Semiconductors	
Silicon*	0.10 to 60
Germanium*	0.001 to 0.5
Conductors	
Lead	22×10^{-8}
Iron	9.71×10^{-8}
Tungsten	5.6×10^{-8}
Aluminum	2.65×10^{-8}
Gold	2.20×10^{-8}
Copper	1.68×10^{-8}
Silver	1.59×10^{-8}

*The resistivity of a semiconductor varies greatly with the type and amount of impurities it contains. This property makes them particularly useful in electronic applications.

✓ Conceptual checkpoint 1 page 819

Wire 1 has a length ***L*** and a circular cross sectional of diameter ***D***

Wire 2 is constructed from the ***same material*** as wire 1 and has the same shape, but its length is ***2L*** and its diameter is ***2D.***

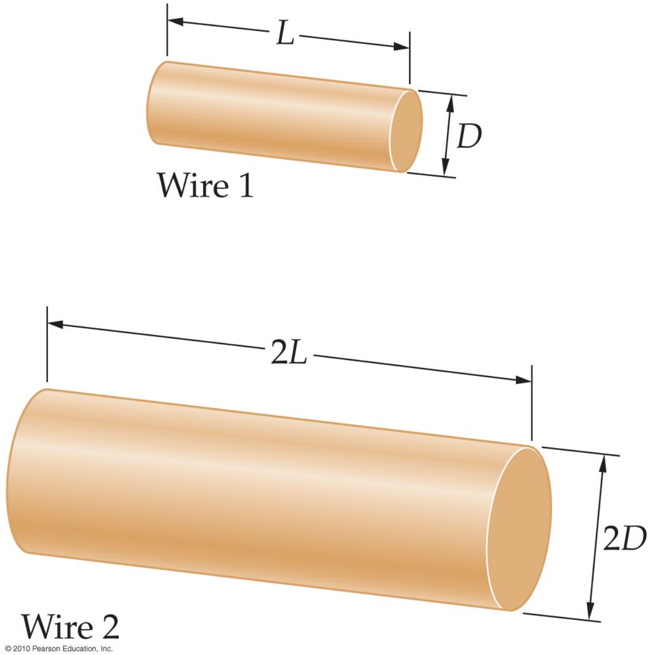
Is the ***resistance of wire 2*** :

(a) The same as that of wire 1 ($R_2 = R_1$)

(b) Twice that of wire 1 ($R_2 = 2 R_1$)

(c) Half that of wire 1 ($R_2 = R_1/2$)

Answer: (c)



Exercise 8, page 845

A conducting wire is quadrupled in length and tripled in diameter.

(a) does its resistance increase, decrease, or stay the same?

(b) by what factor does its resistance change?

Answer: Decrease (4/9)

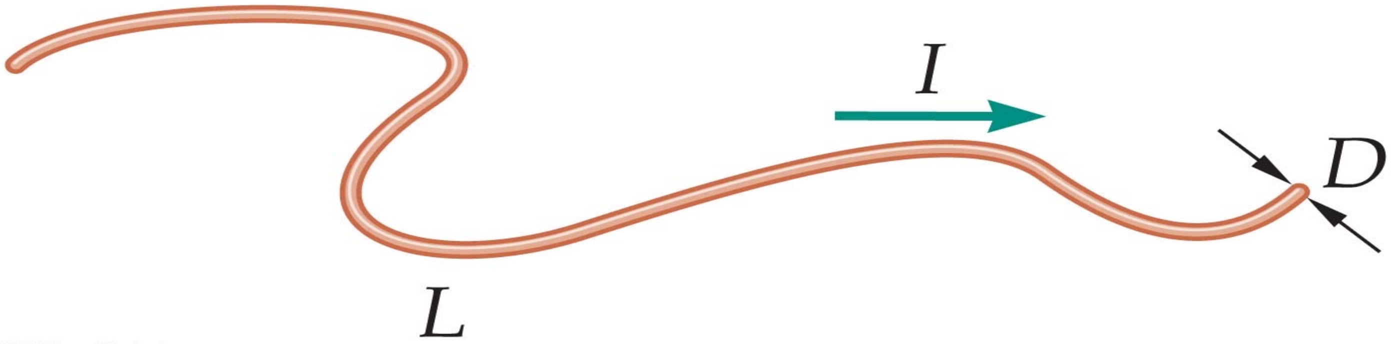
Example 2, page 820

A current of (1.82 A) flows through a copper wire (1.75 m) long and (1.1 mm) in diameter.

Find the **potential difference** between the ends of the wire.

(ρ for copper = 1.68×10^{-8} Ω.m)

Answer: 0.0577 V



21.3 Energy and Power in Electric Circuits

- When a charge moves across a potential difference, its potential energy changes:

$$\Delta U = (\Delta Q)V$$

- Therefore, the ***power*** it takes to do this is

$$P = \frac{\Delta U}{\Delta t} = \frac{(\Delta Q)V}{\Delta t}$$

Electrical Power

$$P = IV$$

SI unit: watt, W

- In materials for which Ohm's law holds, the ***power*** can be written as:

$$P = IV = I(IR) = I^2R$$

$$P = IV = \left(\frac{V}{R}\right)V = \frac{V^2}{R}$$

- This **power** mostly **becomes heat** inside the **resistive material (dissipated power)**

- ***Energy usage:*** When the electric company sends you a bill, your usage is quoted in kilowatt-hours (kWh). *They are charging you for energy use*, and kWh are a measure ***of energy***.

$$\begin{aligned} 1 \text{ kilowatt-hour} &= (1000 \text{ W})(3600 \text{ s}) = (1000 \text{ J/s})(3600 \text{ s}) \\ &= 3.6 \times 10^6 \text{ J} \end{aligned}$$

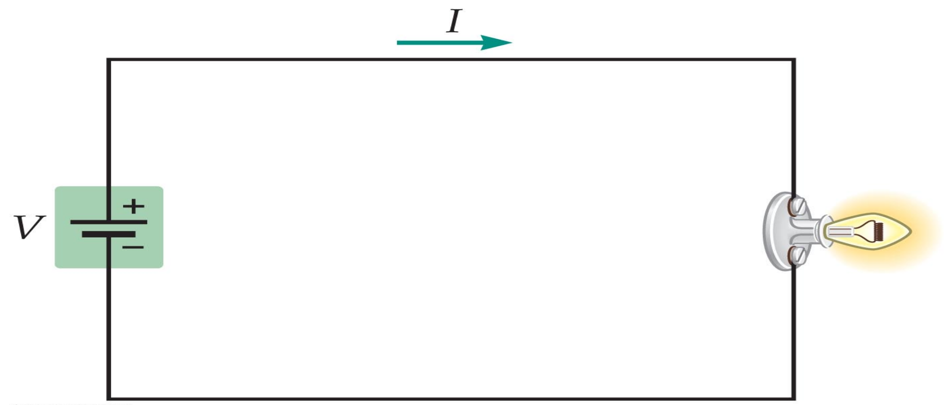
Conceptual checkpoint 2, page 822

A battery that produces a potential difference **V** is connected to a **(5 W)** light bulb. Later, the 5 W light bulb is replaced with a (**10 W)** light bulb.

(a) In which case does the battery supply more current?

(b) Which light bulb has the greater resistance?

Answers: (a) 10 Watt (b) 5 Watt



Example 3, page 822

A battery with an emf of 12 V is connected to a 545 Ω resistor. How much energy is dissipated ΔU in the resistor in 65 Sec? **Answer:** ……

Exercise 30, page 847

A current in a 120 V reading lamp is (2.6 A) if the cost of electrical energy is (0.075 $ per kWh). How much does it cost to operate the light for an hour? **Answer**: 0.023 $

Solution: Exercise 30, page 847

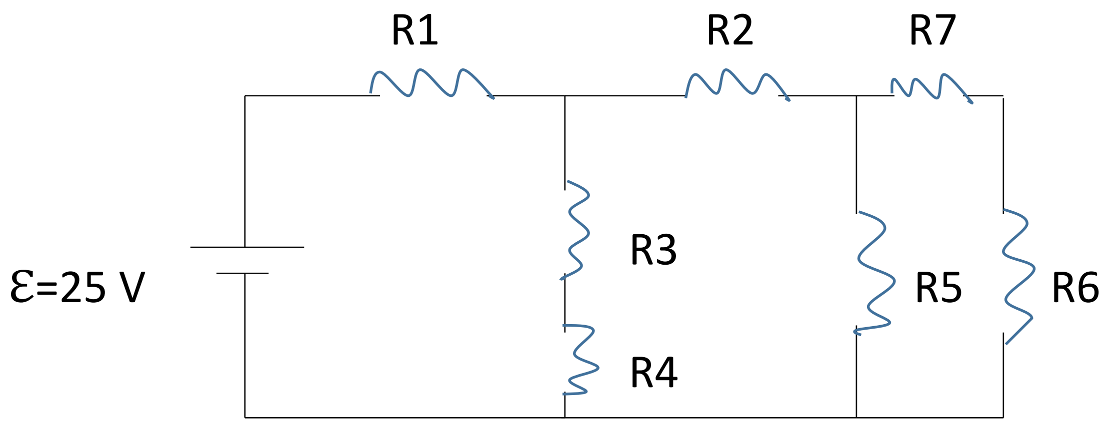
Power = IV

= (120)(2.6)= 312 W

Energy = P Δt = (0.312 KW).(1 hr) = 0.312 KW.h

Cost = 0.312 x 0.075 = 0.023 $

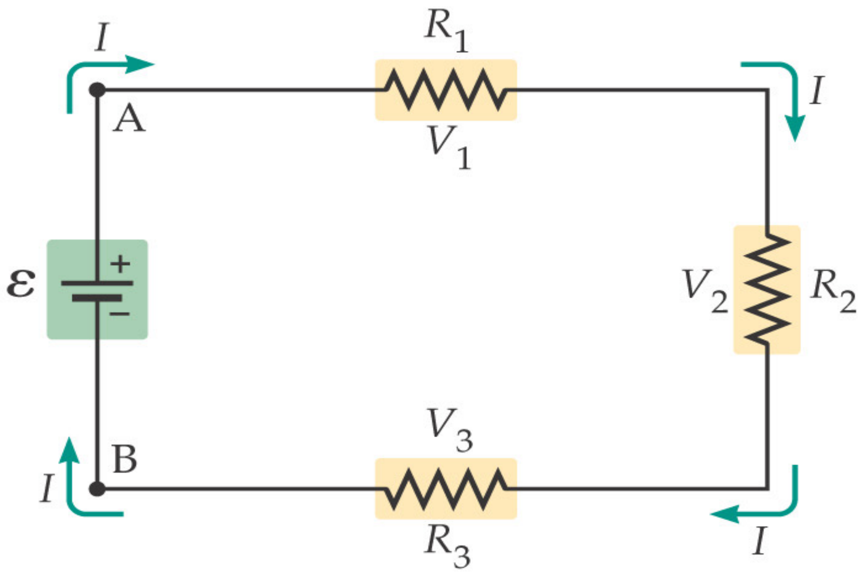
EXTRA PRACTICE



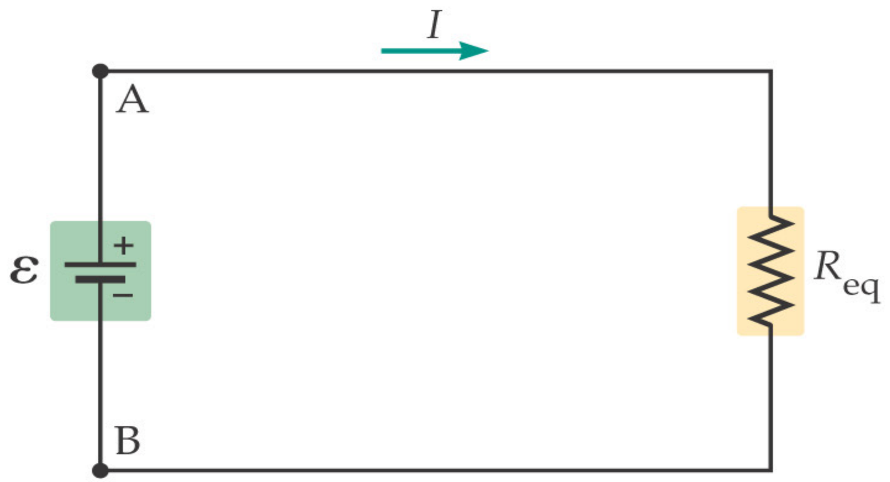
21.4 Resistors in Series and Parallel

(A) Resistors in Series

Resistors connected **end to end** are said to be **in series**. They can be replaced by a **single equivalent resistance without changing** the **current** in the circuit.



(a)



(b)

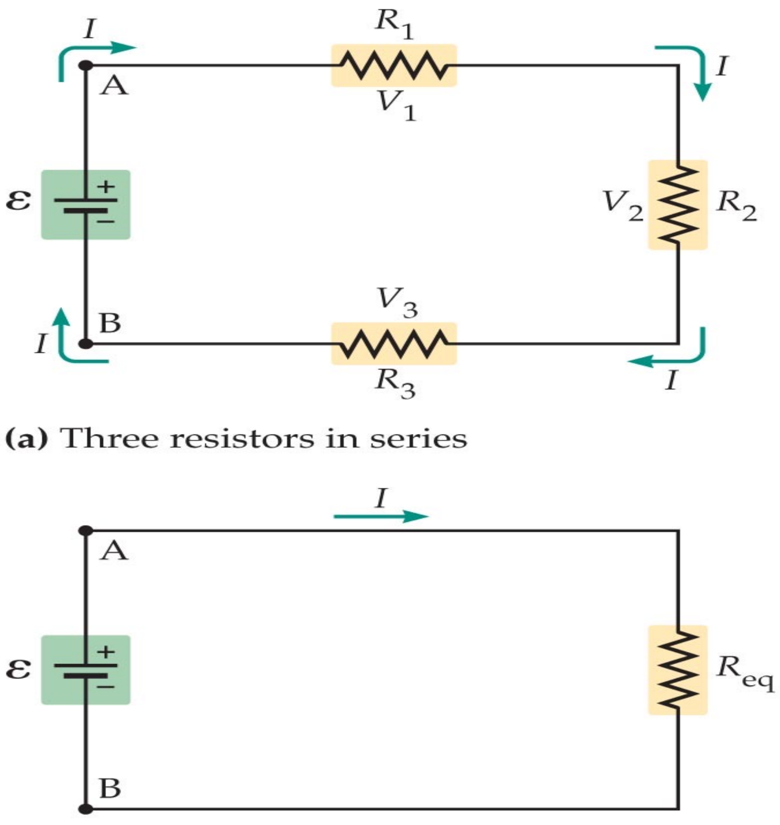
Since the **current through the series resistors** must be **the same** in each, and the total **potential difference** is **the sum of the potential differences across each resistor**, we find that the equivalent resistance is:

Equivalent Resistance for Resistors in Series

$$R_{eq} = R_1 + R_2 + R_3 + \cdots = \sum R$$

SI unit: ohm, Ω

$$R_{eq} = \frac{\varepsilon}{I}$$



(b) Equivalent resistance has the same current

Example 5, page 825

A circuit consists of three resistors connected in series to a (24 V) battery. The current in the circuit is (0.032 A).

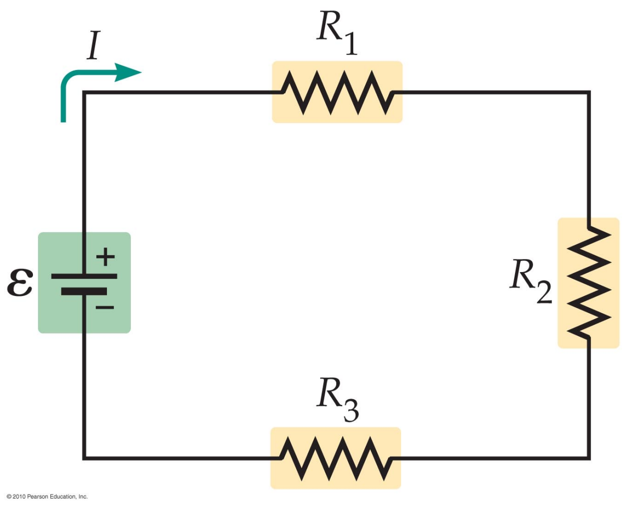
Given that $R_1 = 250\ \Omega$ and $R_2 = 150\ \Omega$

(a) Find the value of R_3

(b) Find the potential difference across each resistor

Answers: (a) $R_3 = 350\ \Omega$

(b) $V_1 = 8$ V $V_2 = 4.8$ V $V_3 = 11.2$ V



Exercise 36 page 847

A circuit consists of three resistors, $R_1 < R_2 < R_3$, connected in series to a battery. Rank these resistors in order of increasing

(a) The current through them

(b) The potential difference across them

(c) The power dissipated across them

Answers: (a) $I_1 = I_2 = I_3$ (b) $V_1 < V_2 < V_3$

(c) P = I V = I2 R , and so $P_1 < P_2 < P_3$ (the bigger R, the more power dissipated in it)

Exercise 41, page 847

A toaster has a power cord with a resistance of (0.02 Ω) connected in series with a (9.6 Ω) heating element.

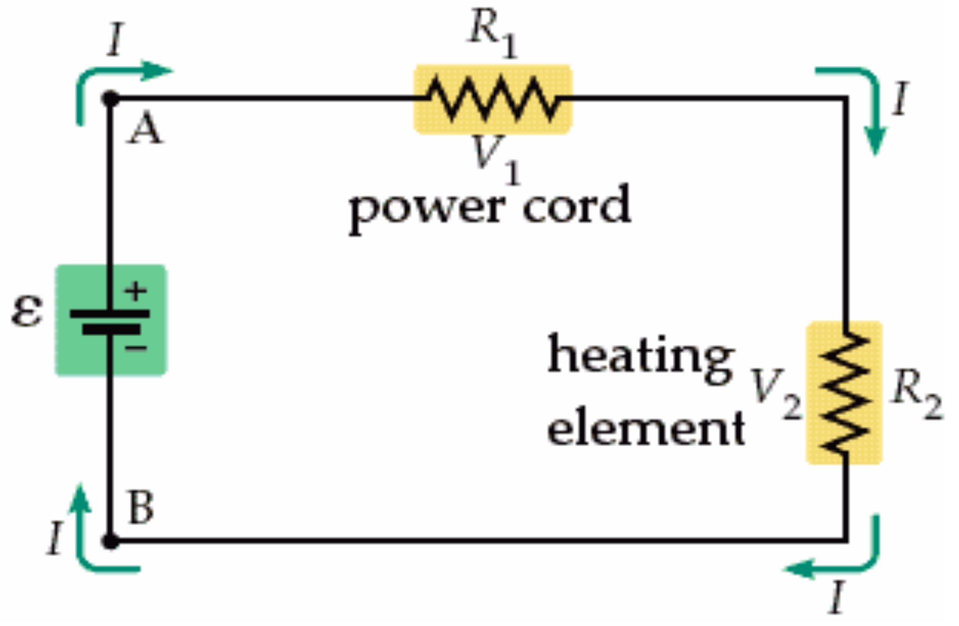
If the potential difference between the terminals of the toaster is (120 V) , how much power is dissipated in

(a) the cord

(b) the heating element

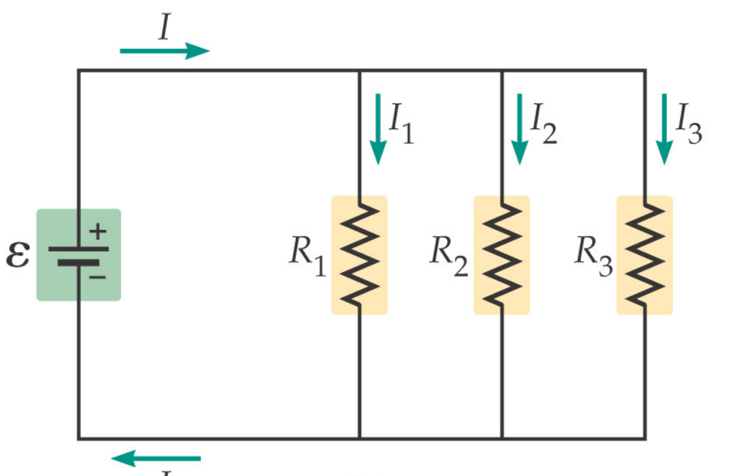
Answers: (a) $P_{\text{power cord}} = 3.1\ W$

(b) $P_{\text{heating element}} = 1500\ W$



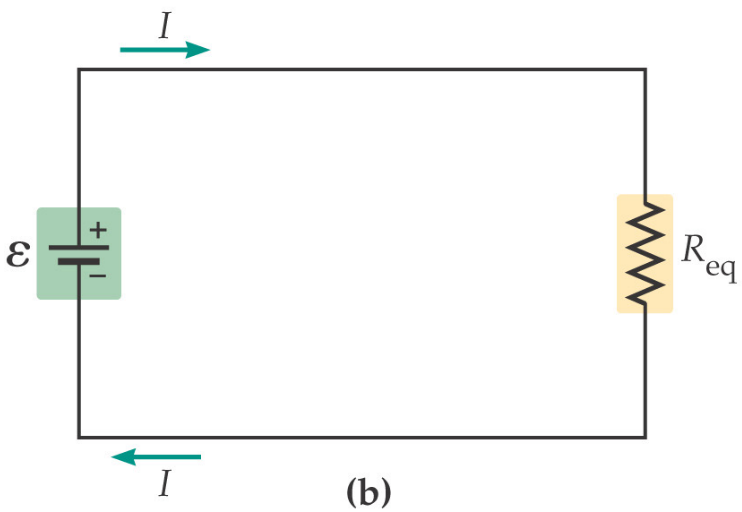
(B) Resistors in Parallel

- Resistors are in parallel when they are ***across the same potential difference***; they can again be replaced by a <u>single equivalent resistance</u> $R_{Equivalent}$



(a)

- Using the fact that the ***potential difference across each resistor is the same***, and the ***total current is the sum of the currents in each resistor***, we find:



(b)

Equivalent Resistance for Resistors in Parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots = \sum \frac{1}{R}$$

SI unit: ohm, Ω

Note that this equation gives you the inverse of the resistance, not the resistance itself!

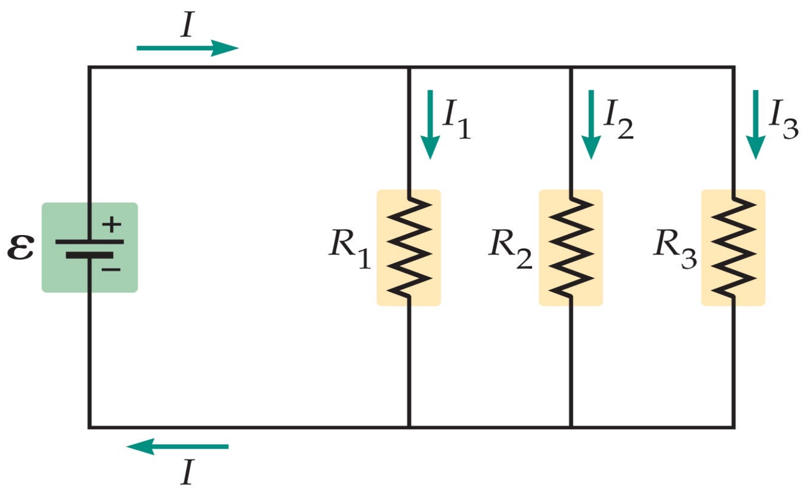
Example 6 page 827

Consider a circuit with three resistors R_1= 250 Ω, R_2= 150 Ω, and R_3= 350 Ω connected in parallel with a 24 V battery. Find:

(a) The total current supplied by the battery

(b) The current through each resistor

Answers: (a) ……………… (b) …………



(a) Three resistors in parallel

Conceptual checkpoint page 828

Two identical light bulbs are connected to a battery either in series or in parallel. Are the bulbs in series:

(a) Brighter than

(b) Dimmer than

(c) The same brightness as the bulbs in parallel

Answer: Conceptual checkpoint page 828

The bulbs connected in series ***are dimmer than*** the bulbs connected in parallel

- The reason is that the

R_{eq} (in series) = twice the resistance of the bulb

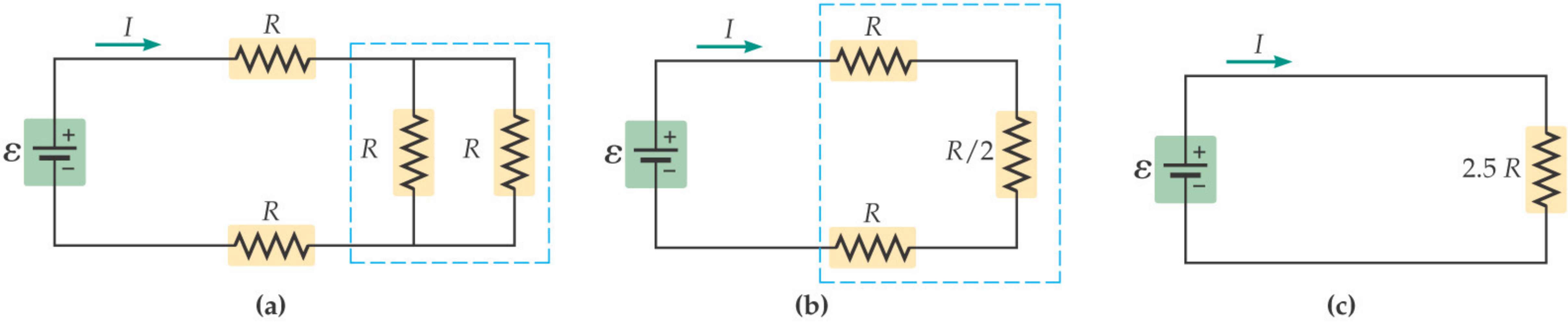
R_{eq} (in parallel)= half the resistance of the bulb

Power = I V = $\frac{V^2}{R_{eq}}$

As a result, more ***power is converted to light*** in the ***parallel circuit***

(C) Combination Circuit

If a circuit is more complex, start with combinations of resistors that are ***either purely in series or in parallel***. Replace these with their equivalent resistances; as you go on you will be able to replace more and more of them.



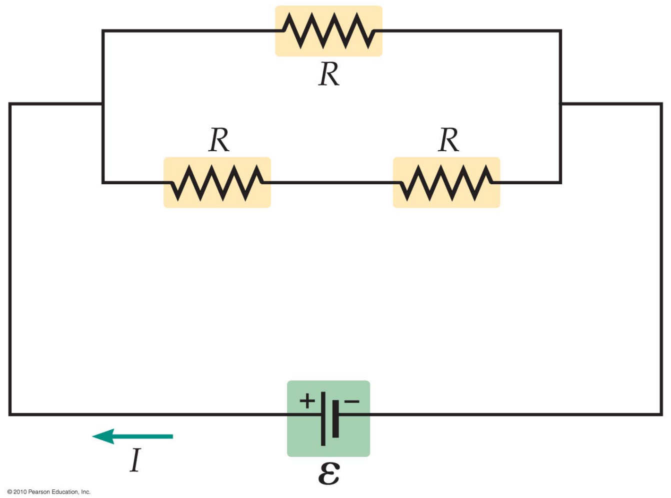
Example 7, page 829

In the circuit shown in the diagram, the emf of the battery is (12 V), and each resistor has a resistance of (200 Ω). Find:

(a) The current supplied by the battery to this circuit.

(b) The current through the lower two resistors.

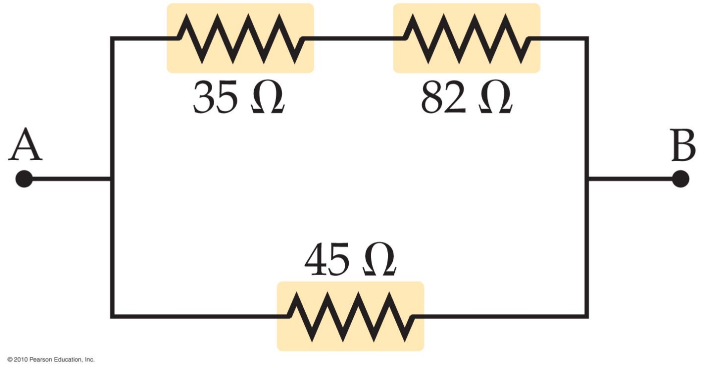
Answers: (a) ….   (b) ….



Exercise 38 page 847

Find the equivalent resistance between points A and B for the group of resistors shown.

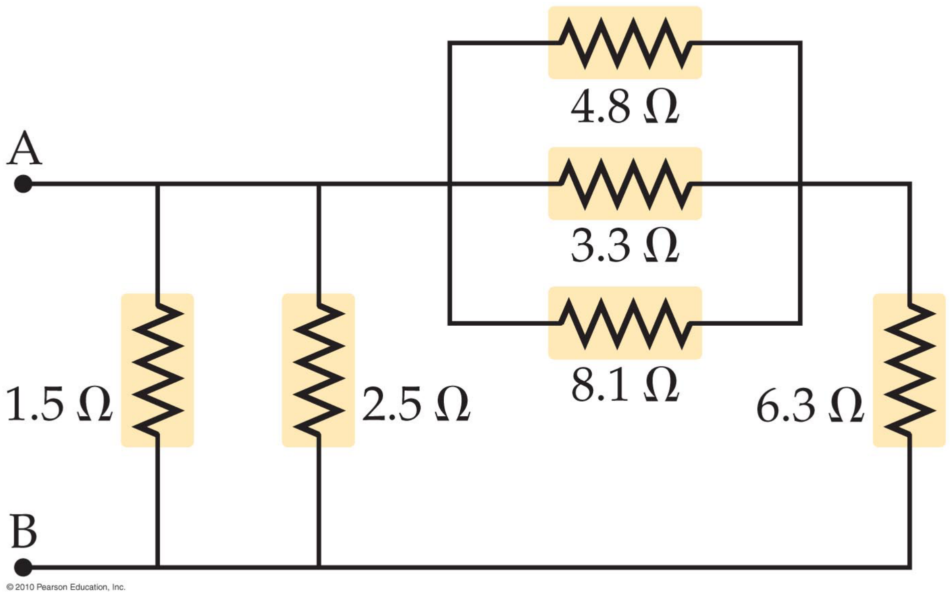
Answer: 32.6 Ω



Exercise 49 page 848

Find the equivalent resistance between points A and B shown in the figure.

Answer: 0.84 Ω



(D) Kirchhoff's Rules

- ✓ Kirchhoff's rules are two rules introduced by the German physicist Gustav Kirchhoff.
- ✓ These rules are simply ways to find ***currents*** and ***voltages*** in an electric circuit, and they are:

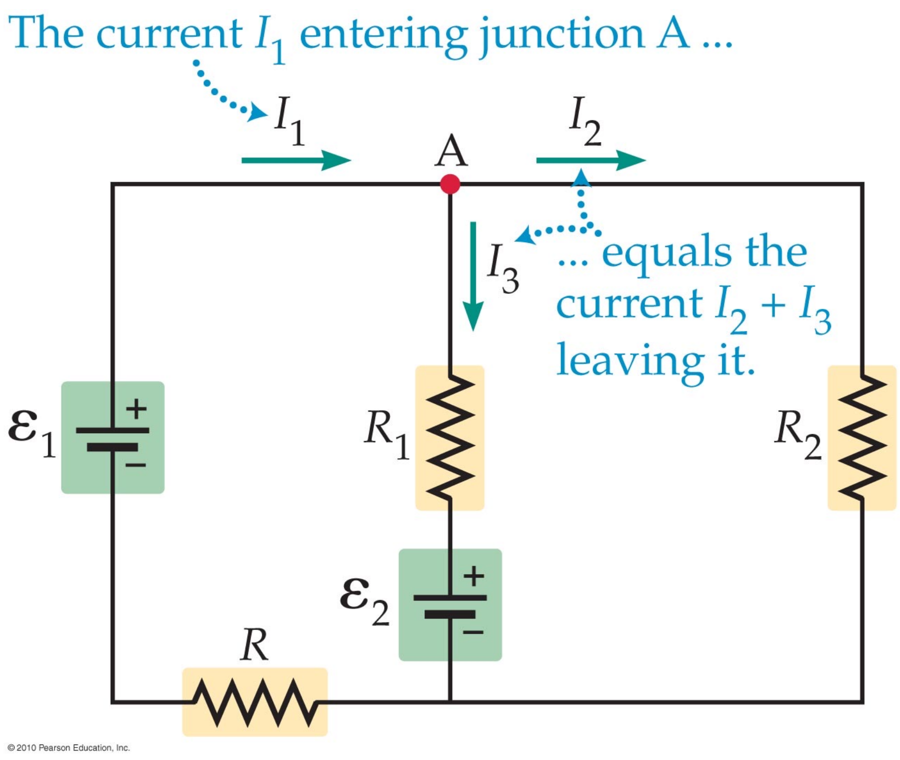
(1) The Junction rule

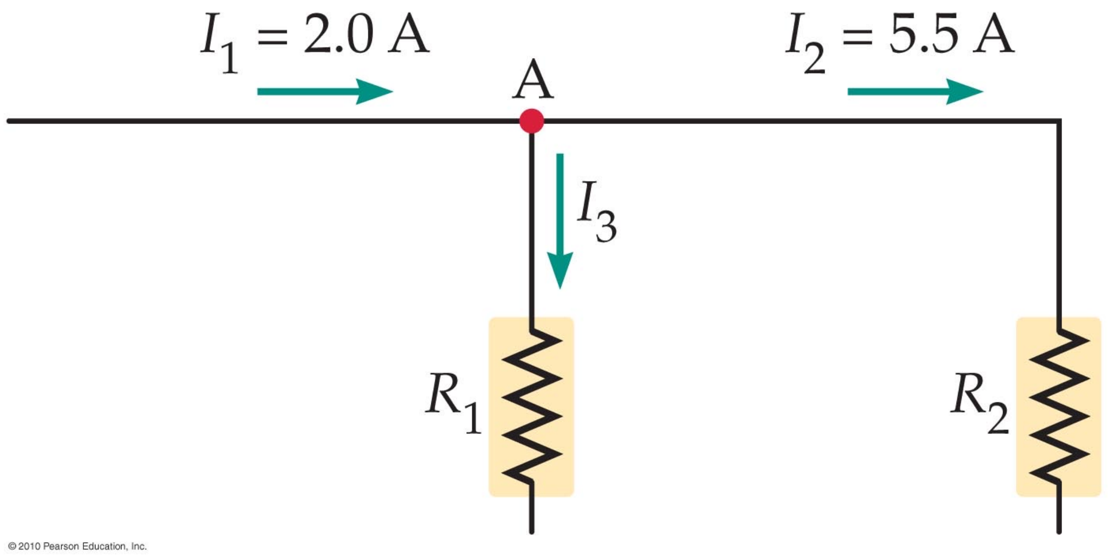
(2) The loop rule

<u>*(A) The junction rule*</u>

"The algebraic sum **<u>of all currents</u>** meeting **<u>at any junction</u>** in a circuit must **<u>equal zero</u>**

in other words, The sum of currents entering a junction= sum of currents leaving the junction"

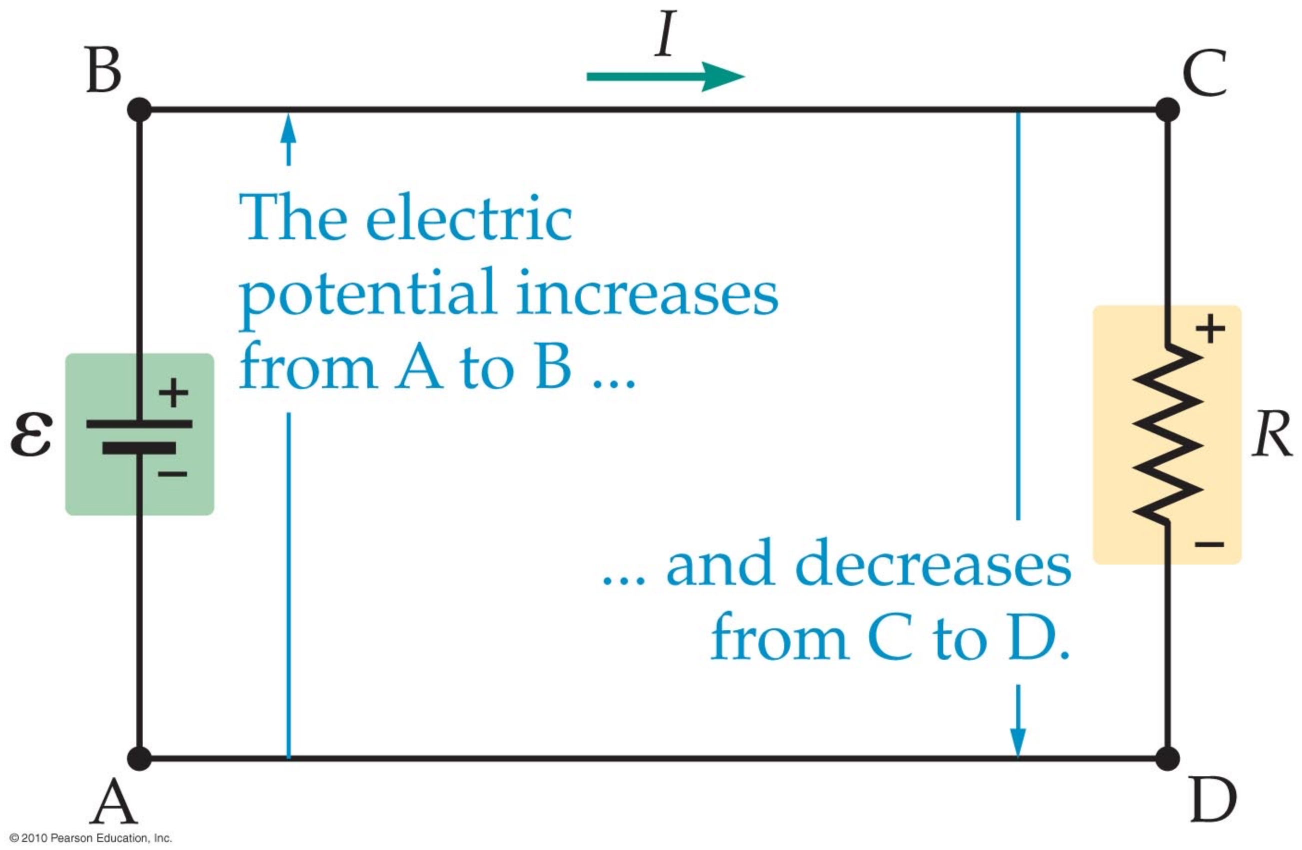


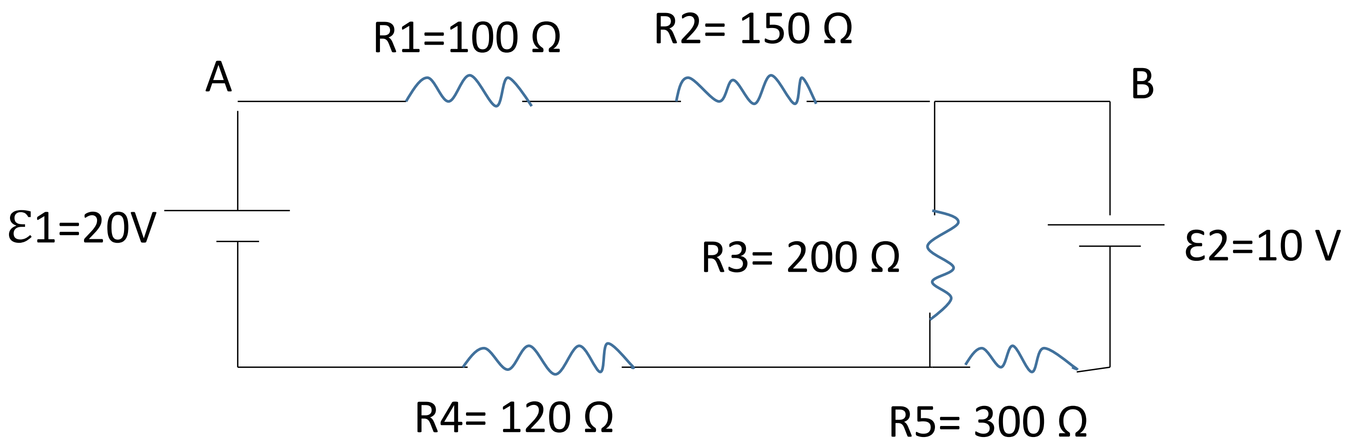


(B) The Loop Rule

"The algebraic **sum of all potential** differences around any **closed loop** in a circuit **is zero**"

The ***potential increases*** as One moves from the ***– to +*** Plate of a battery, and ***Decreases*** as one moves Through a ***resistor in the direction of the current***.





Solution

So apply Kirchhoff's 2nd law, and starting from point A:

$$\varepsilon - V_R = 0$$

And so,

$$V_R = \varepsilon$$

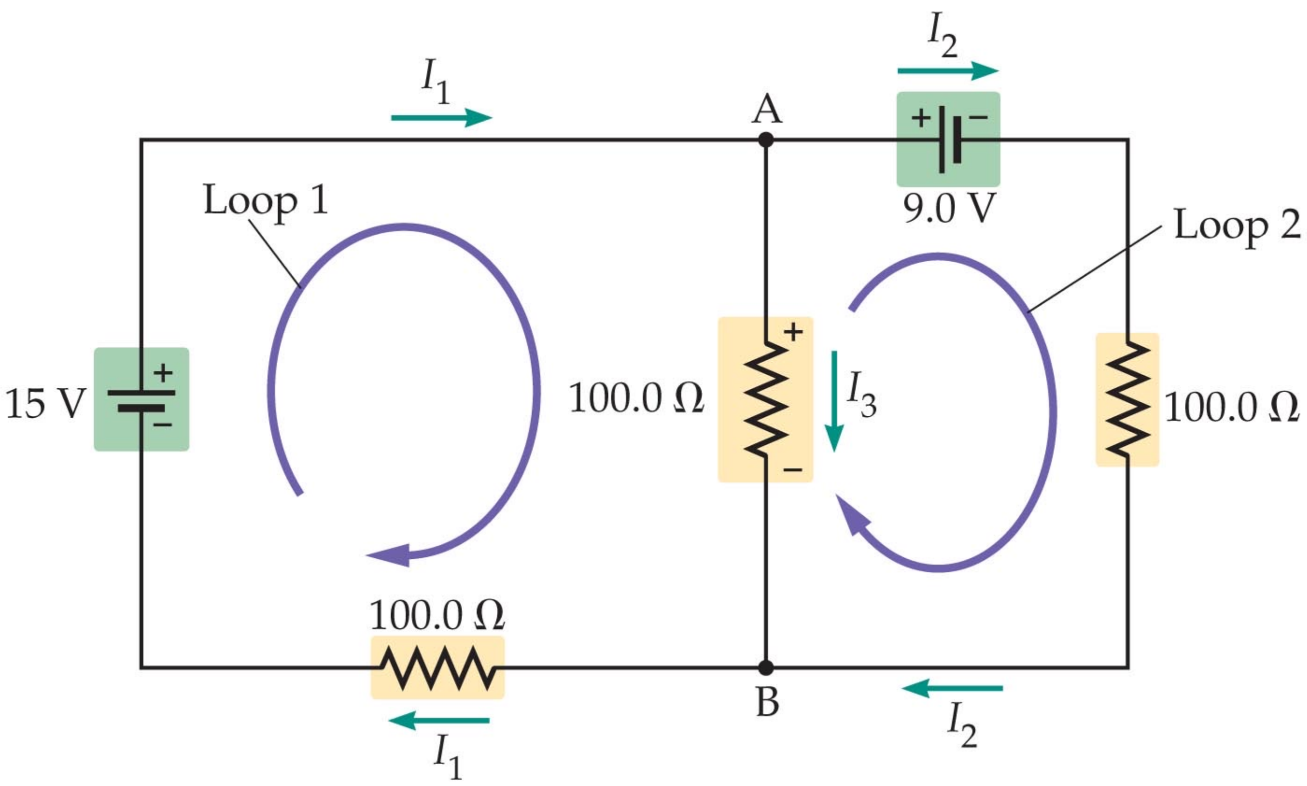
Or

$$I = \frac{\varepsilon}{R} \qquad \text{as in Ohm's Law}$$

Active Example 2, page 833

Find the currents in the circuit shown

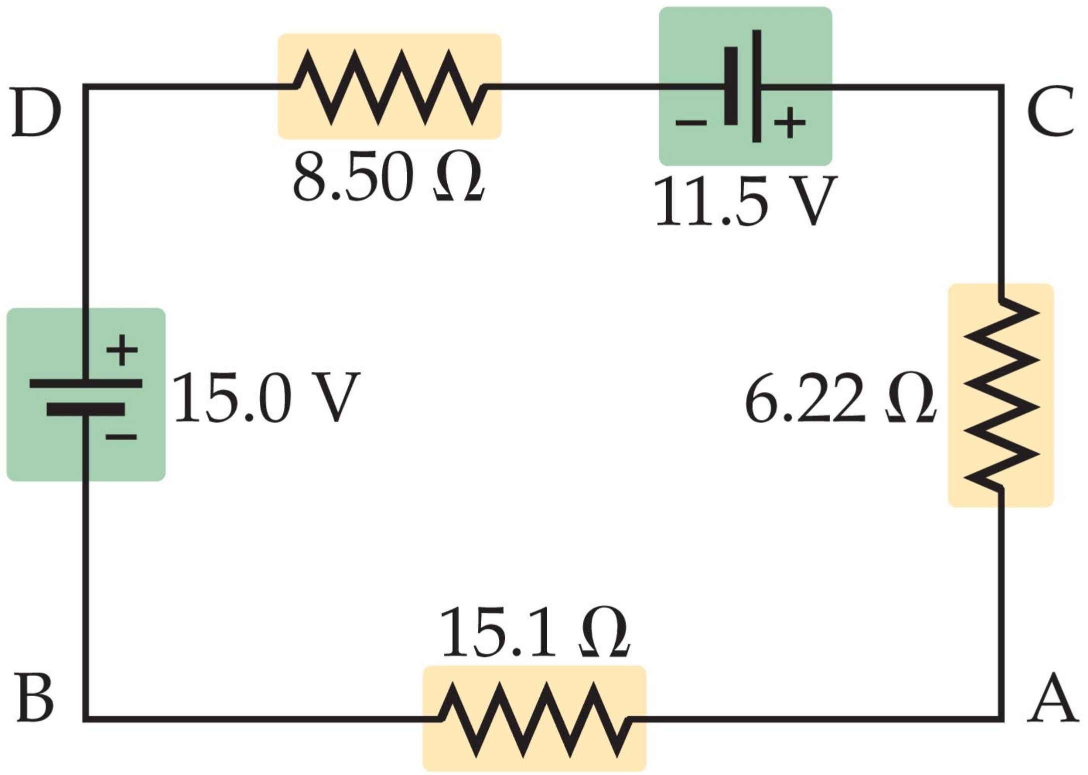
Answers: (a) I_1= 0.07 A (b) I_2= - 0.01 A (c) I_3= 0.08 A



Exercise 58, Page 848

Find the ***magnitude*** and ***direction*** (clockwise or counterclockwise) of the ***current*** in the circuit shown.

Answer: (a) I= 0.89 A (b) CW



Exercise 59, Page 848

For the same circuit in the previous exercise, if the polarity of the (11.5 v) battery is reversed.

(a) Do you expect this to increase or decrease the amount of current flowing in the circuit?

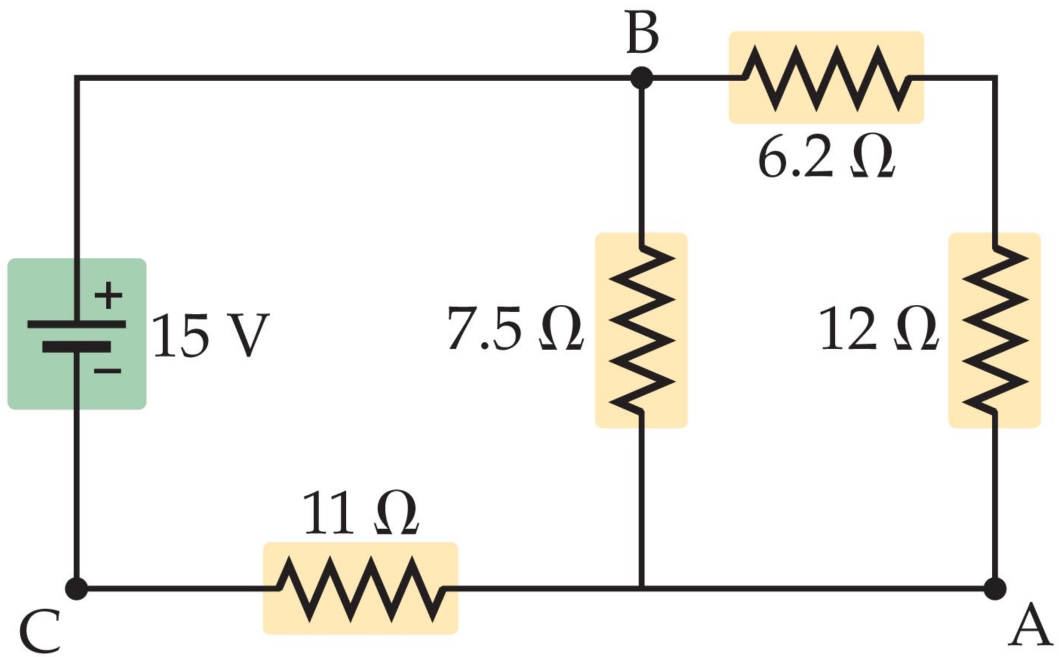
(b) Calculate the ***<u>magnitude</u>*** and ***<u>direction</u>*** (clockwise or counterclockwise) of the current in this case.

Answers: (a) ……….. (b) 0.12 A and CW

Exercise 61, Page 848

For the circuit shown, find the current through each resistor using Kirchhoff's laws.

Answer: I_1= 0.92 A, I_2= 0.27 A, I_3= 0.65 A



Exercise 63, page 849

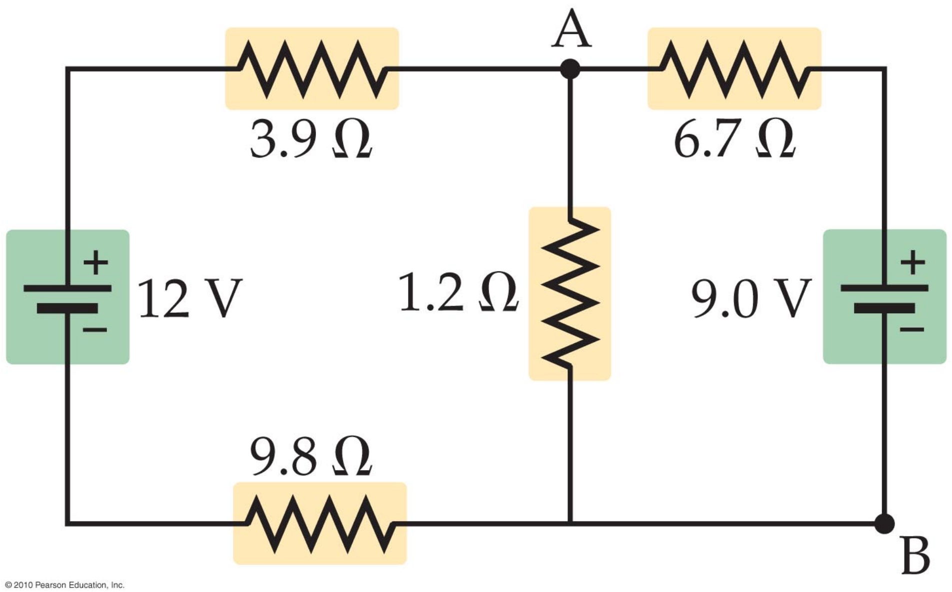
For the circuit shown find:

(a) The current through each resistor

(b) Determine the potential difference between the points A and B

Answers: (a) $I_1 = 0.72$ A, $I_2 = -1$ A, $I_3 = 1.8$ A

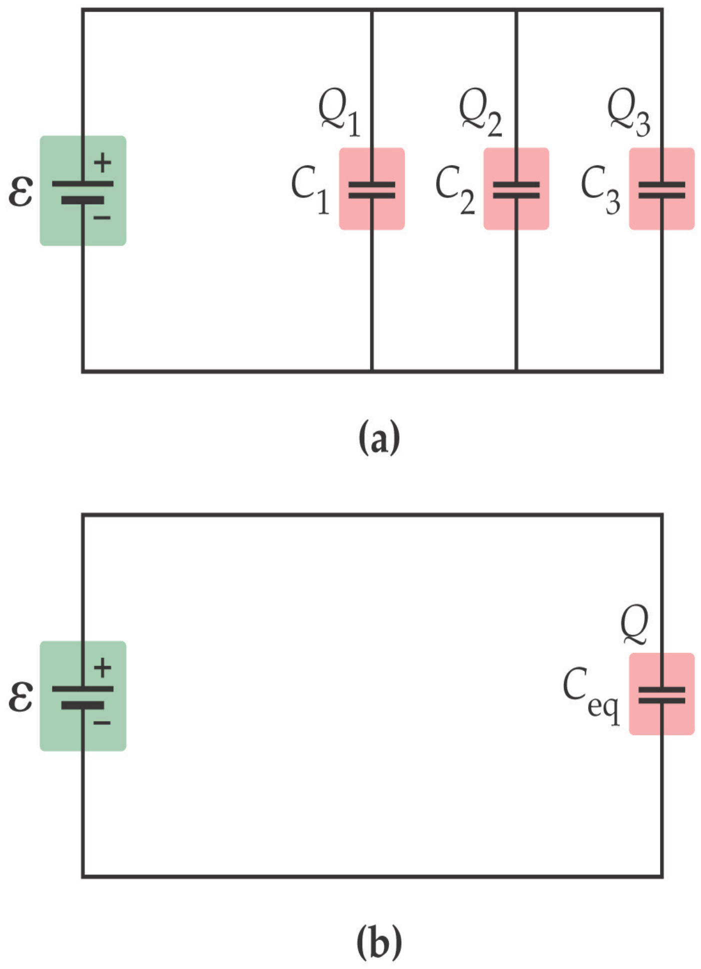
(b) $V = 2.2$ V



21.6 Circuits Containing Capacitors

Question: For an electrical circuit that consists of a ***battery*** and ***multiple capacitors***, how to find the ***equivalent capacitor***?
(look at the figure)

Answer: Capacitors can also be connected in series or in parallel.



(A) Capacitors in Parallel

When capacitors are connected in parallel:

1) The ***potential difference*** across each one ***is the same***

2) The ***charge stored*** by each capacitor ***is different***

$$Q_{total} = Q_1 + Q_2 + Q_3$$

$$C_{total}\,\varepsilon = C_1\,\varepsilon + C_2\,\varepsilon + C_3\,\varepsilon$$

Therefore, the ***equivalent capacitance*** is the sum of the individual capacitances:

Equivalent Capacitance for Capacitors in Parallel

$$C_{eq} = C_1 + C_2 + C_3 + \cdots = \sum C$$

SI unit: farad, F

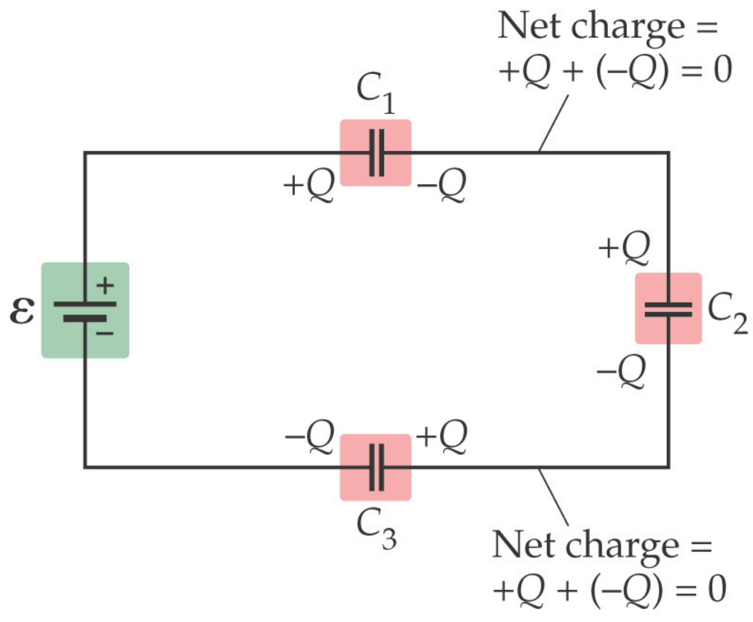
(B) Capacitors in Series:

Capacitors connected in series:

1) ***Do not have the same potential difference*** across them.

2) They do ***all carry the same charge***.

3) The total potential difference is the sum of the potential differences across each one.



(a)

(b)

Therefore, the equivalent capacitance is

Equivalent Capacitance for Capacitors in Series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots = \sum \frac{1}{C}$$

SI unit: farad, F

Note

1)This equation ***gives you the inverse of the capacitance***, not the capacitance itself!

2) ***Capacitors in series*** combine ***like*** ***resistors in parallel***, and vice versa.

Active Example 3, page 836

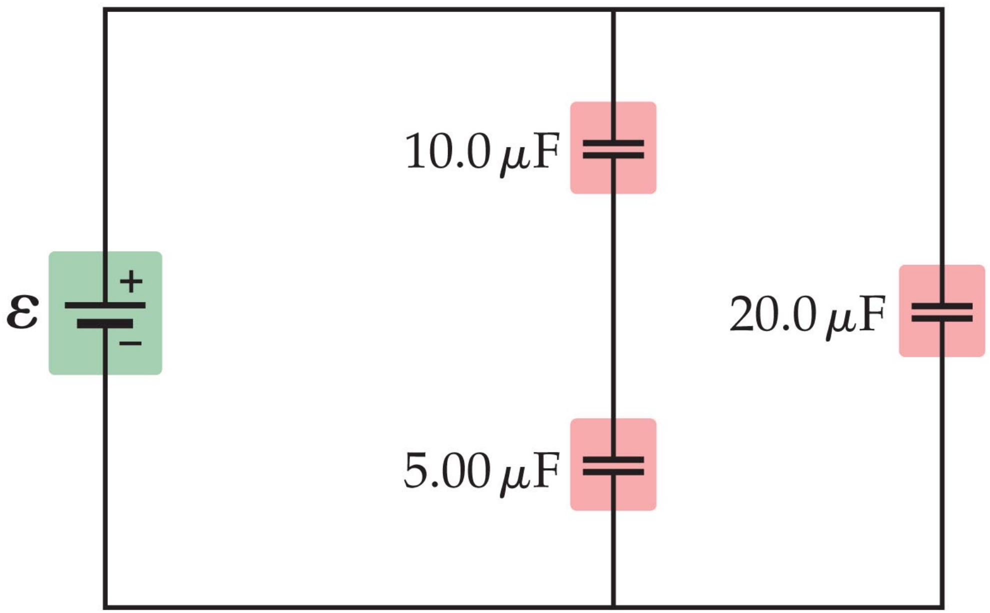
For the electric circuit shown, consisting of a (12 v) battery and three capacitors connected partly in series and partly in parallel. Find:

(a) The equivalent capacitance of this circuit

(b) The total energy stored in the capacitors

Answers: (a) $C_{eq} = 23.3\ \mu F$

(b) $E_{stored} = ½\ c_{eq}\ v^2 = 1.68 \times 10^{-3}$ J



Exercise 65, page 849

Two capacitors, $C_1 = C$, $C_2 = 2C$, are connected to a battery.

(a) Which capacitor will store more energy when they are connected to the battery in series? Explain

(b) Which capacitor stores more energy when they are connected in parallel? Explain.

Answers

(a) when capacitors are connected in series, then,

$Q_1 = Q_2 = Q$

$E = \frac{1}{2} Q V = \frac{1}{2} Q \left(\frac{Q}{C}\right) = \frac{1}{2} \frac{Q^2}{C}$

Which means that the capacitor with the ***smaller capacitance*** will store ***bigger energy***. And so, the capacitor ***C_1 will have bigger stored energy***

(b) When the capacitors are connected in parallel, then:

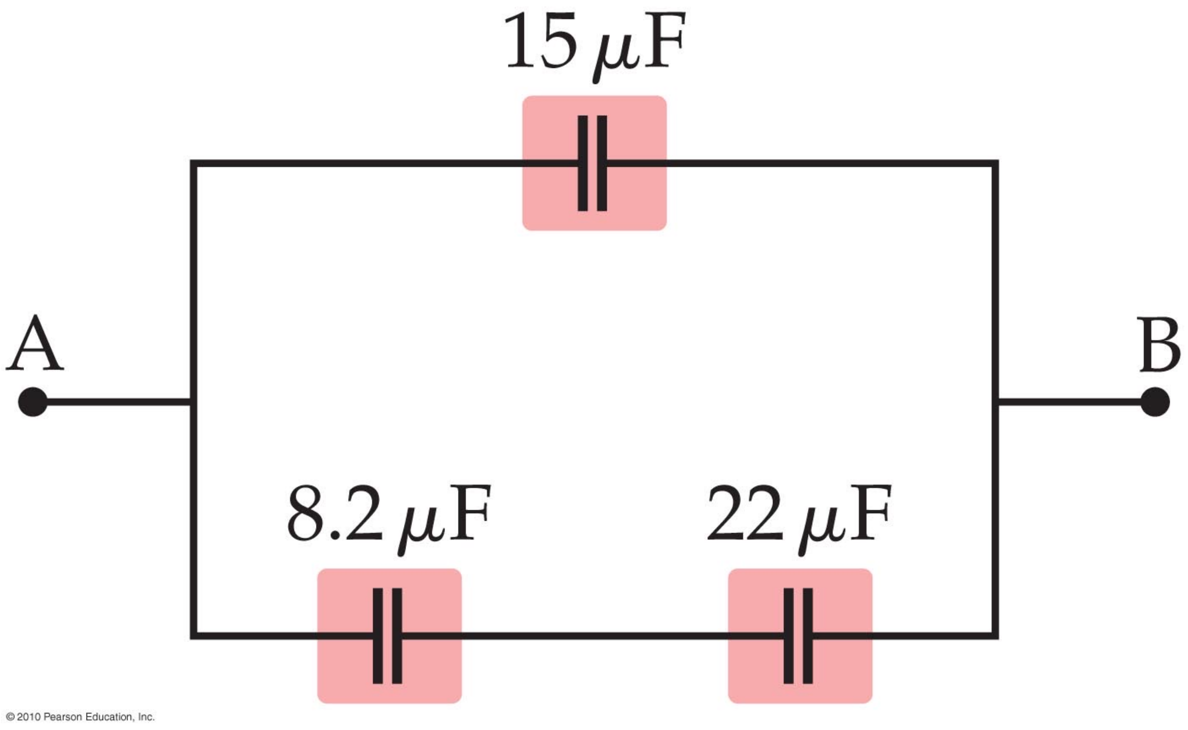
$V_1 = V_2 = V$

$E = \frac{1}{2} Q V = \frac{1}{2} C V^2$, Then, the ***capacitor with bigger capacitance*** will ***store more energy***. ***Which is C_2.***

Exercise 68, page 849

Find the ***equivalent capacitance*** between points ***a*** and ***b*** for the group of capacitors shown.

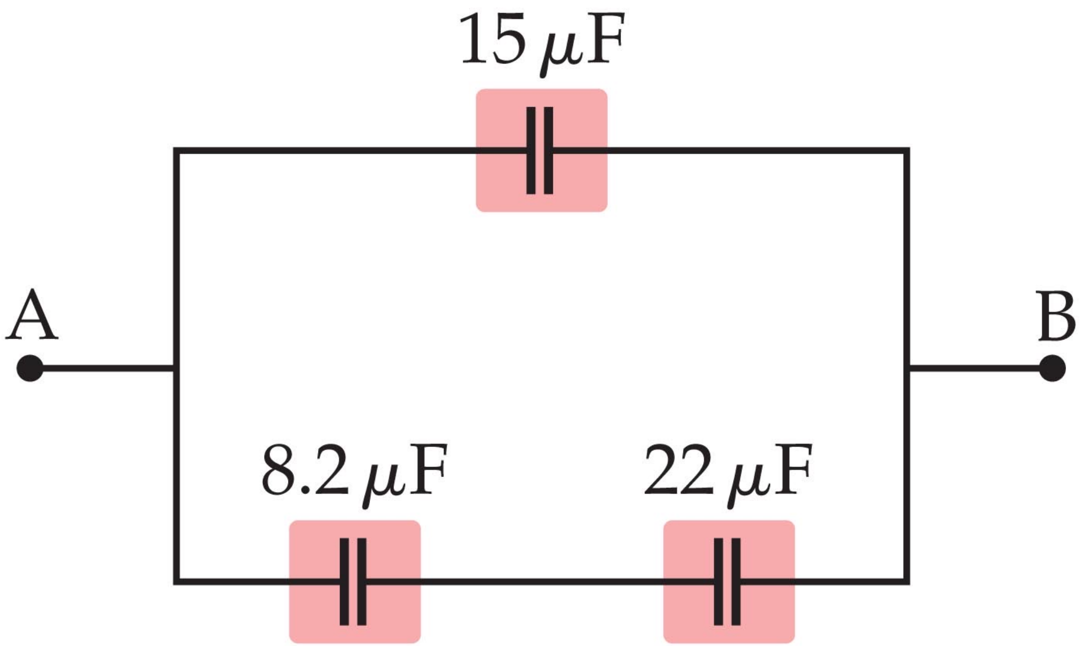
Answer: 20.97 μF



Exercise 72, page 849

For the same circuit in the previous exercise, if the terminals A and B are connected to a 9 volt.

Find the ***energy stored*** in ***each capacitor***.



Answer

Now the potential difference across the capacitors

V(15 μF) = V (8.2 μF, 22 μF) = 9 V

E (15 μF) = $\frac{1}{2}$ C V^2 = <u>***6.1 x 10 $^{-4}$ J***</u>

E(8.2 μF) = $\frac{1}{2}$ Q V = $\frac{1}{2}$ Q ($\frac{Q}{C}$) = $\frac{1}{2}\frac{Q^2}{C}$

E(22 μF) = $\frac{1}{2}$ Q V = $\frac{1}{2}$ Q ($\frac{Q}{C}$) = $\frac{1}{2}\frac{Q^2}{C}$

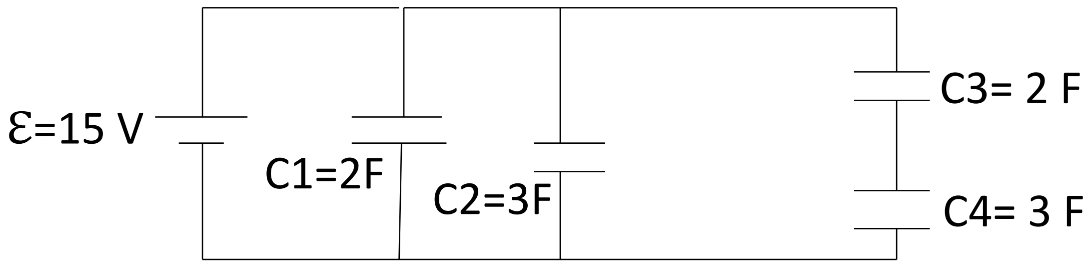
Now, since these two capacitors are connected on series, then, the charge passing through them is the same

Q_{tot}= V $C_{equivalent}$ = (9)(5.97 x 10^{-6}) = 5.4 x 10^{-5} C

E(8.2 μF)= (0.5)(5.4 x 10^{-5} $)^2$ / (8.2 X 10^{-6}) = <u>***1.8 x 10 $^{-4}$ J***</u>

E(22 μF)= (0.5)(5.4 x 10^{-5} $)^2$ / (22 X 10^{-6})= <u>***6.6 x 10^{-5} J***</u>

Extra Practice: Home Work



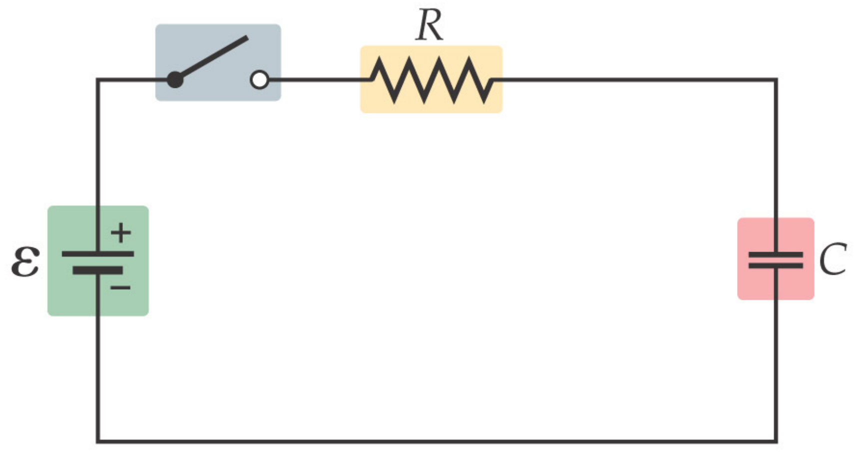
21.7 *RC* Circuits

1) When the circuit contains ***only*** ***batteries*** and ***capacitors***, charge appears almost ***instantaneously*** on the capacitors when the circuit is connected.

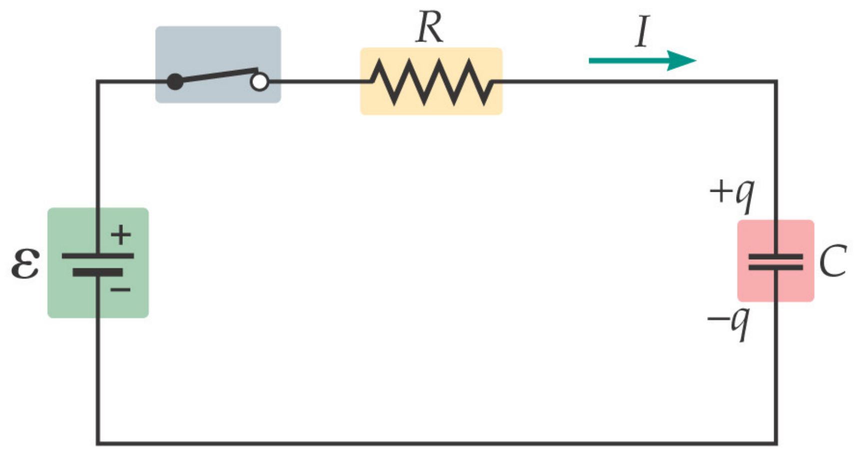
2) If the circuit contains a ***resistance*** , ***battery*** and a ***capacitor***, this is ***NOT*** the case . This circuit is called an ***RC circuit***.

(look at the figure)

1) ***Resistors limit the rate at which charge can flow***, and an amount of time may be required for the capacitor to become charged.



(a) $t < 0$



(b) $t > 0$

(A) Charging a Capacitor

Using calculus, it can be shown that ***the charge on the capacitor increases as:***

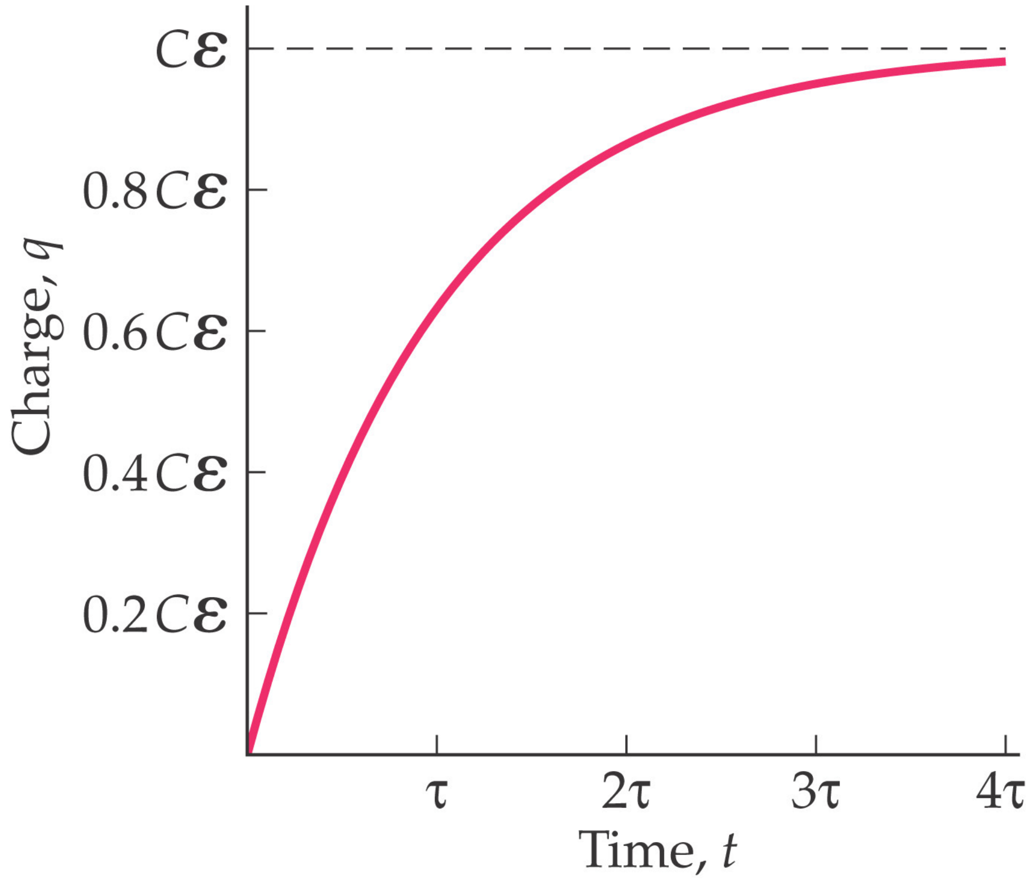
$$q(t) = C\mathcal{E}(1 - e^{-t/\tau})$$

Here, ***τ*** is the ***time constant*** (***FIND ITS UNIT***) of the circuit (***which is the time required for the capacitor to become almost 63.2% charged***) and it equals:

$$\tau = RC$$

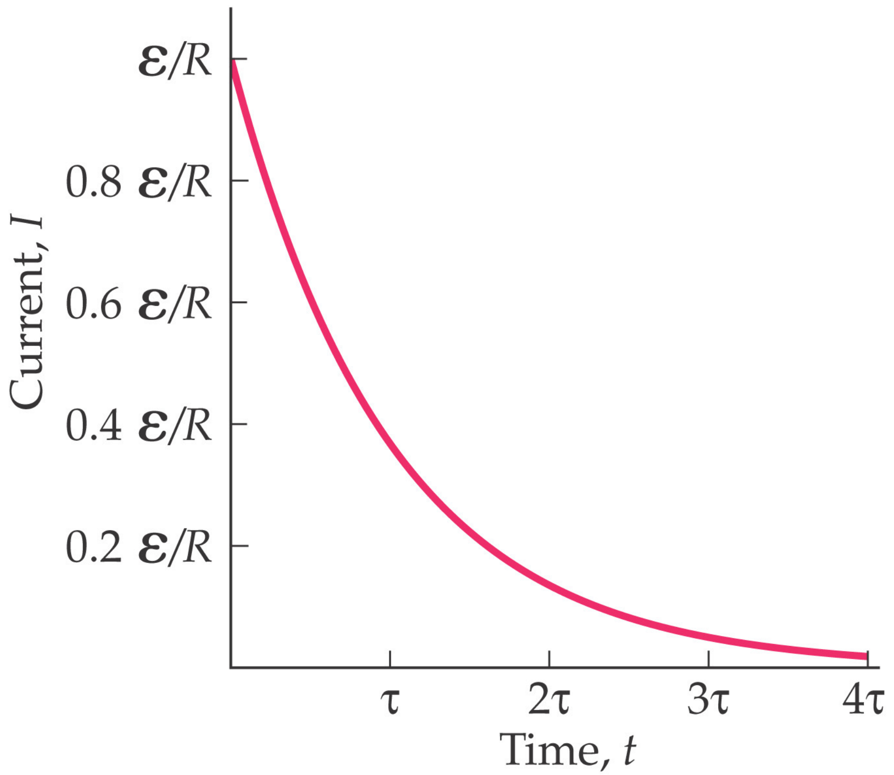
And $C\mathcal{E}$ is the ***final charge*** on the capacitor, Q.

Here is the ***charge*** vs. ***time*** for an RC circuit:



It can be shown that ***the current*** in the circuit has a related behavior:

$$I(t) = \left(\frac{\mathcal{E}}{R}\right)e^{-t/\tau}$$



NOTE

After ***along time*** of the current flowing in the circuit, the ***capacitor behaves like an open switch***

Example 9, charging a capacitor, page 837

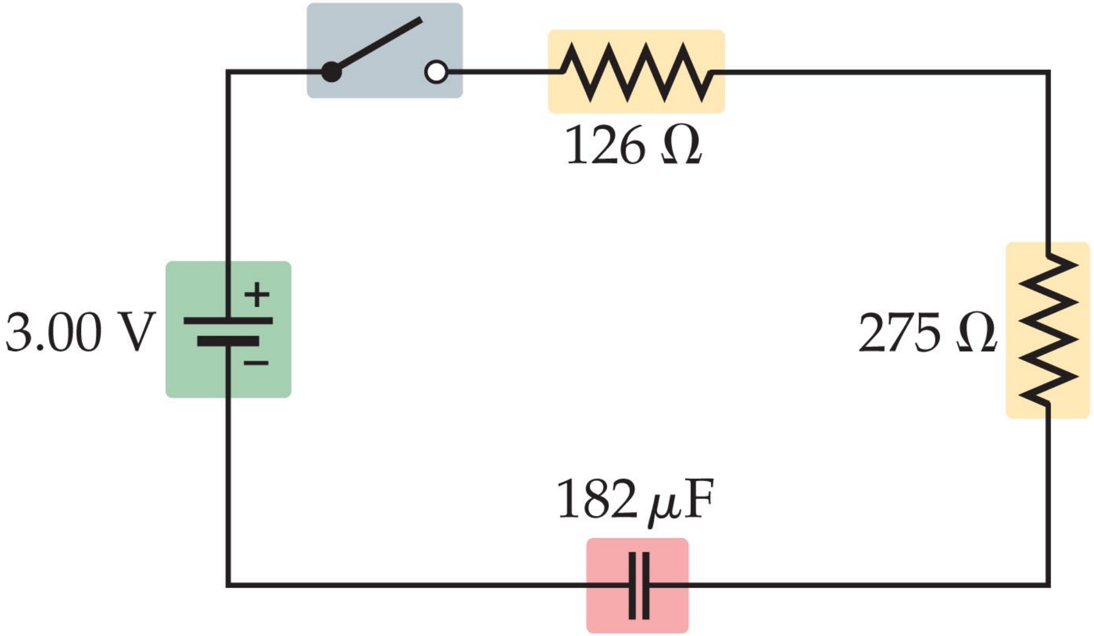
For the RC circuit shown, the capacitor is initially uncharged

a) What ***charge*** will the capacitor have ***along time*** after the switch is closed.

b) At what time will the ***charge*** on the capacitor ***be 80%*** of the value found in part a?

Answers: (a) C= 5.46X10^{-4} C

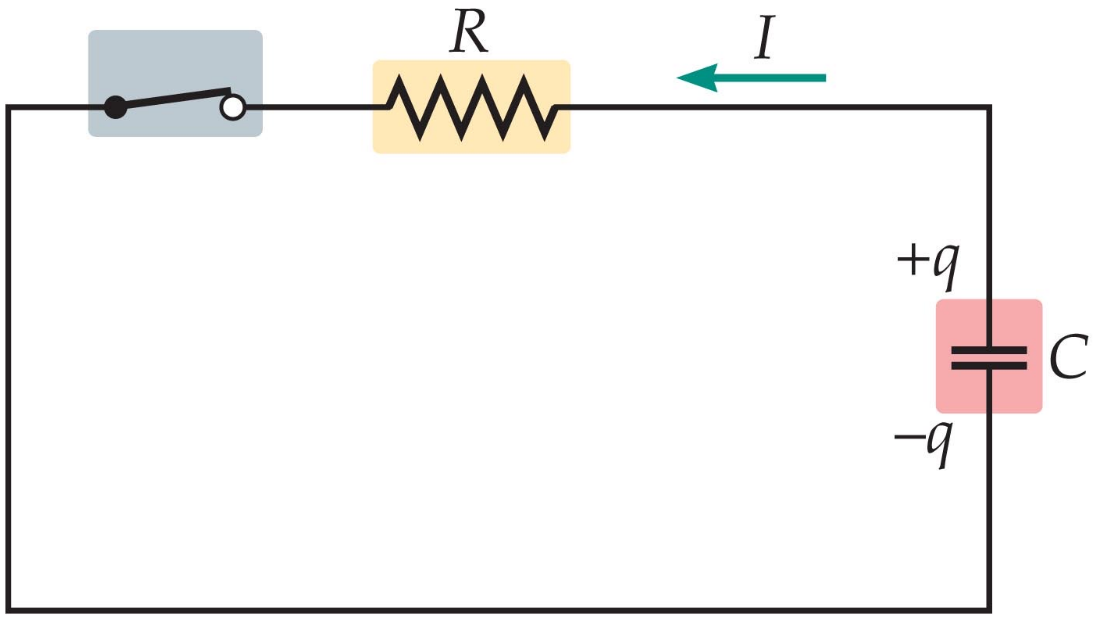
(b) t= 118 ms



(B) Discharging a Capacitor

The shown circuit will cause the ***capacitor to discharge*** according to the following equation:

$$\mathbf{q(t) = Q}\ e^{-t/\tau}$$



(b) $t > 0$

Conceptual Checkpoint 4. page 839

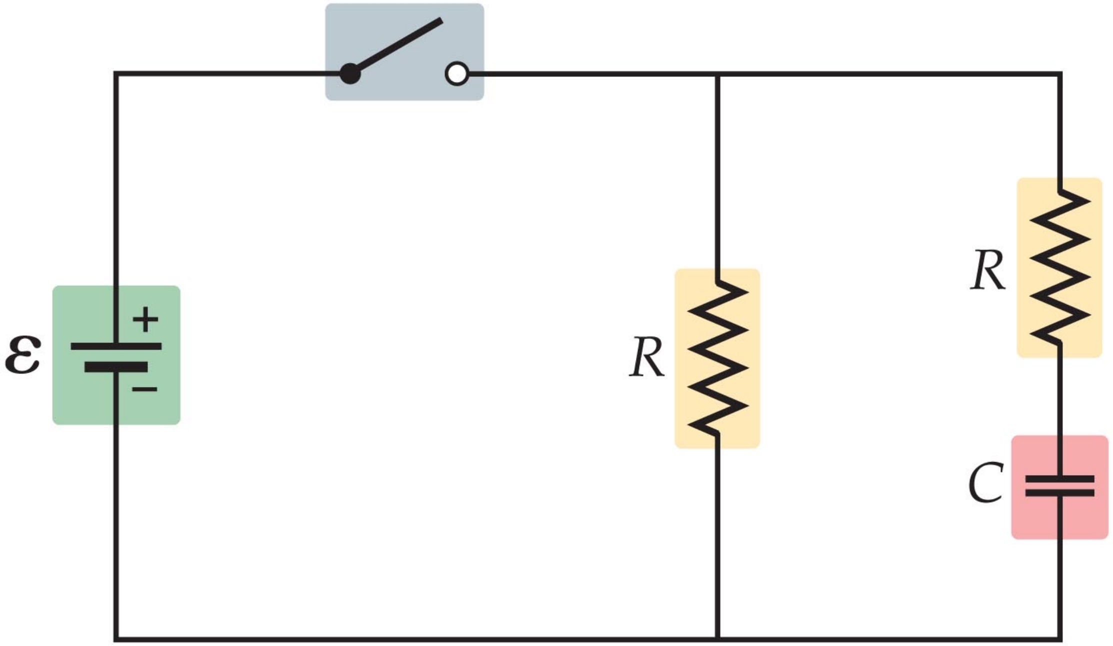
For the circuit shown, what current flows ***through the battery***

(a) Immediately after the switch is closed

(b) Along time after the switch is closed

Answers: (a) 2Ɛ/R (short circuit)

(b) Ɛ/R (open switch)



Exercise 78, page 850

The switch on an RC circuit is closed at t = 0 sec

Given that: $\varepsilon = 9$ V; R = 150 Ω; C = 23×10^{-6} F

How much charge is on the capacitor at time t = 4.2 ms?

Answer: 150×10^{-6} C

Exercise 85, page 850

Consider the RC circuit shown in the figure. Find:

(a) The time constant

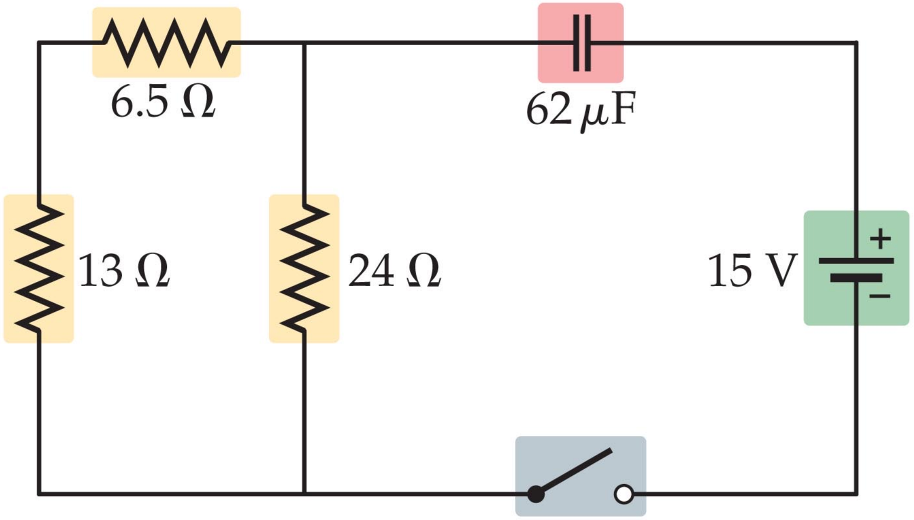
(b) The initial current for this circuit

(c) It is desired to increase the time constant of this circuit by adjusting the value of the (6.5 Ω) resistor. Should this resistance of this resistor be increased or decreased to have the desired effect?

Answers: (a) 0.67 s

(b) 1.4 A

(c) increase



Extra Practice