Chapter 3

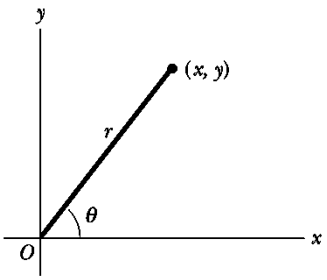
Vectors

CHAPTER OUTLINE

▲ *These controls in the cockpit of a commercial aircraft assist the pilot in maintaining control over the* velocity *of the aircraft—how fast it is traveling and in what direction it is traveling—allowing it to land safely. Quantities that are defined by both a magnitude and a direction, such as velocity, are called* vector quantities. *(Mark Wagner/Getty Images)*

3.1 Coordinate Systems

Cartesian coordinates are also called rectangular coordinates.



plane polar coordinates (r, θ)

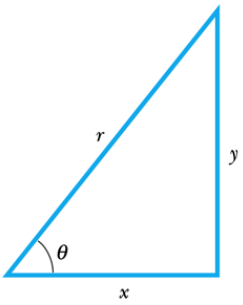
r is the distance from the origin to the point having Cartesian coordinates (x, y)

θ is the angle between a line drawn from the origin to the point and a fixed axis

$$\cos\theta = \frac{x}{r}$$

$$\sin\theta = \frac{y}{r}$$

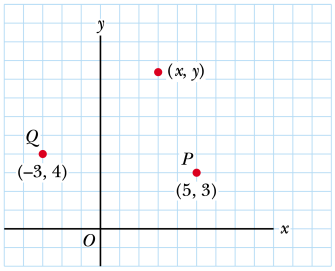
$$\tan\theta = \frac{y}{x}$$



$$x = r\cos\theta \qquad r = \sqrt{x^2 + y^2}$$

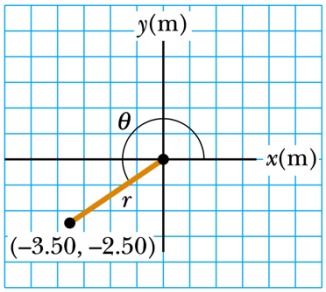
$$y = r\sin\theta \qquad \theta = \left(\tan\frac{y}{x}\right)^{-1}$$

In this practice determine the x , y , r and θ for each point



Example 3.1: Polar Coordinates

The Cartesian coordinates of a point in the xy plane are $(x, y) = (-3.50, -2.50)\ m$, *as shown in Figure 3.3. Find the polar coordinates of this point.*



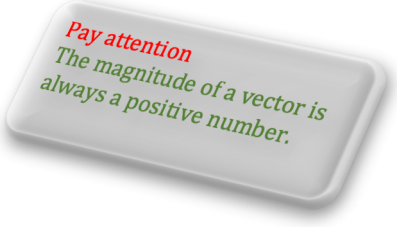
1- *Two points in the xy plane have cartesian coordinates (2.00 , -4.00) m and (-3.00 , 3.00) m , determine the distance between them and their polar coordinates*

a) $25m\ , (2m\ , -37.9^o\ )$ & $(9\ , -45^o)$
b) $8.6m\ , (4.47m\ , -63.4^o\ )$ & $(4.24\ , -45^o)$
c) $4.3m\ , (3.32m\ , -26.6^o\ )$ & $(18.9\ , -26.6^o)$
d) $49m\ , (9m\ , -36.9^o\ )$ & $(9\ , -37.9^o)$
e) $54.5m\ , (2.8m\ , -38.1^o\ )$ & $(32\ , -23^o)$

3.2 Vector and Scalar Quantities

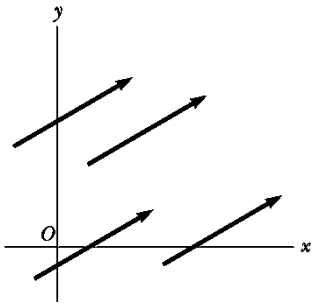
A scalar quantity	*A vector quantity*
is completely specified by a single value with an appropriate unit and has no direction.	*is completely specified by a number and appropriate units plus a direction.*
distance	*displacement*
speed	*Velocity*
time intervals	*Acceleration*
mass	*Force*
temperature	*Momentum*
volume	*torque*

to represent a vector quantity
a ***boldface*** *letter, such as* ***A***
an arrow is written over the symbol for the vector : $\vec{A}$
The magnitude of the vector A is written either A or $|A|$



Quick Quiz 3.1

Which of the following are vector quantities and which are scalar quantities?
(a) your age (b) acceleration (c) velocity (d) speed (e) mass



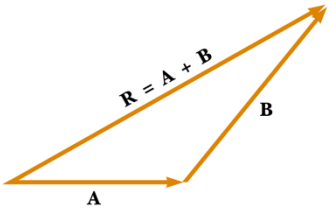
3.3 Some Properties of Vectors

Equality of Two Vectors

two vectors A and B are equal if they have the same magnitude & point in the same direction.

Adding Vectors

The resultant vector $\vec{R} = \vec{A} + \vec{B}$ *is the vector drawn from the tail of* $\vec{A}$ *to the tip of* $\vec{B}$.
$\vec{R}$ *is the vector drawn from the tail of the first vector to the tip of the last vector.*



When two vectors are added, the sum is independent of the order of the addition.

the commutative law of addition: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

the associative law of addition: $\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$

a vector quantity has both magnitude and direction and also obeys the laws of vector addition

Negative of a Vector

The vectors $\vec{A}$ *and* $-\vec{A}$ *have the same magnitude but point in opposite directions.*

$\vec{A} + (-\vec{A}) = 0$

Subtracting Vectors

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

Quick Quiz 3.2

The magnitudes of two vectors $\vec{A}$ and $\vec{B}$ are $\vec{A} = 12\ units$ and $\vec{B} = 8\ units$, Which of the following pairs of numbers represents the largest and smallest possible values for the magnitude of the resultant vector $\vec{R} = \vec{A} + \vec{B}$?

(a) 14.4 units, 4 units
(b) 12 units, 8 units
(c) 20 units, 4 units
(d) none of these answers,

Quick Quiz 3.3

If vector $\vec{B}$ is added to vector $\vec{A}$, under what condition does the resultant vector $\vec{A} + \vec{B}$ have magnitude $\vec{A} + \vec{B}$

(a) $\vec{A}$ and $\vec{B}$ are parallel and in the same direction
(b) $\vec{A}$ and $\vec{B}$ are parallel and in opposite directions,
(c) $\vec{A}$ and $\vec{B}$ are perpendicular.

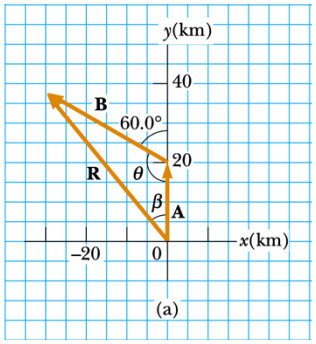
Quick Quiz 3.4.

If vector B is added to vector A, which two of the following choices must he true in order for the resultant vector to be equal to zero?

(a) $\vec{A}$ and $\vec{B}$ are parallel and in. the same direction.
(b) $\vec{A}$ and $\vec{B}$ are parallel and in opposite directions.
(c) $\vec{A}$ and $\vec{B}$ have the same magnitude.
(d) $\vec{A}$ and $\vec{B}$ are perpendicular.

Example 3.2 A Vacation Trip

A car travels 20.0 km due north and then 35.0 km in a direction 60^o west of north, as shown in Figure 3.12a. Find the magnitude and direction of the car's resultant displacement.

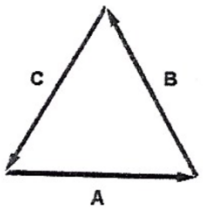


Multiplying a vector by a scalar

If vector $\vec{A}$ is multiplied by a positive scalar quantity m
If m is positive (same direction)
If m is negative (opposite direction)
If $|m| > 1$ (the magnitude increase)
If $|m| < 1$ (the magnitude decrease)

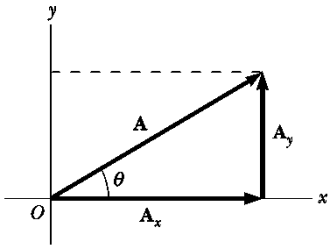
2- *The diagram below shows 3 vectors which sum to zero, all of equal length. Which statement below is true?*

A) $\vec{A} + \vec{B} = \vec{B} - \vec{C}$
B) $\vec{A} - \vec{B} = 2\vec{A} - \vec{C}$
C) $\vec{A} - \vec{B} = 2\vec{A} + \vec{C}$
D) $2\vec{A} + 2\vec{B} = 2\vec{C}$



3.4 Components of a Vector and Unit Vectors

the components of the vector are the projections of vectors along coordinate axes



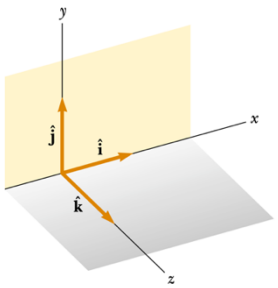
$$A_x = A\cos\theta \qquad A = \sqrt{{A_x}^2 + {A_y}^2}$$

$$A_y = A\sin\theta \qquad \theta = \left(\tan\frac{A_y}{A_x}\right)^{-1}$$

the signs of the components A_x and A_y depend on the angle θ.

Quick Quiz 3.5
Choose the correct response to make the sentence true: A component of a vector is (a) always, (b) never, or (c) sometimes larger than the magnitude of the vector

Unit Vectors



A unit vector is a dimensionless vector having a magnitude of exactly 1

$\hat{\imath}$ ⟶ X

$\hat{\jmath}$ ⟶ Y

$\hat{k}$ ⟶ Z

$$|\hat{\imath}| = |\hat{\jmath}| = |\hat{k}| = 1$$

Quick Quiz 3.6

If at least one component of a vector is a positive number, the vector cannot
(a) have any component that is negative
(b) be zero
(c) have three dimensions.

Quick Quiz 3.7

If $\vec{A} + \vec{B} = 0$*, the corresponding components of the two vectors* $\vec{A}$ *and* $\vec{B}$ *must be*
(a) equal (b) positive (c) negative (d) of opposite sign.

Quick Quiz 3.8

For which of the following vectors is the magnitude of the vector equal to one of the components of the vector?
(a) $A = 2\hat{\imath} + 5\hat{\jmath}$
(b) $B = -3\hat{\jmath}$
(c) $C = +5\hat{k}$

Example 3.3 The Sum of Two Vector

Find the sum of two vectors ***A*** *and* ***B*** *lying in the xy plane and given by*
$A = (2.0\hat{\imath} + 2.0\hat{\jmath})\ m$ *and* $B = (2.0\hat{\imath} - 4\hat{\jmath})\ m$

Example 3.4 The Resultant Displacement

A particle undergoes A three consecutive displacements:

$d_1 = (15\hat{\imath} + 30\hat{\jmath} + 12\hat{k})\ cm,$

$d_2 = (23\hat{\imath} - 14\hat{\jmath} - 5.0\hat{k})\ cm$

$d_3 = (-13\hat{\imath} + 15\hat{\jmath})\ cm$

Find the components of the resultant displacement and its magnitude.

Example 3.5 Taking a Hike

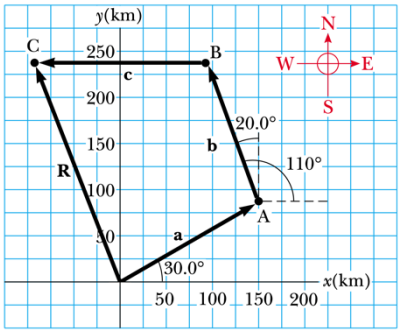
A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

(A) Determine the components of the hiker's displacement for each day.

(B) Determine the components of the hiker's resultant displacement R for the trip. Find an expression for R in terms of unit vectors.

Example 3.6 Let's Fly Away!

A commuter airplane takes the route shown in Figure 3.20. First, it flies from the origin of the coordinate system shown to city A, located 175 km in a direction 30.0° north of east. Next, it flies 153 km 20.0° west of north to city B. Finally, it flies 195 km due west to city C. Find the location of city C relative to the origin.



3- *Vector A has y-component $A_y = +9.6\ m$. The vector A makes an angle of 32° counterclockwise from the + y-axis. The x-component A_x and the magnitude of A are:*

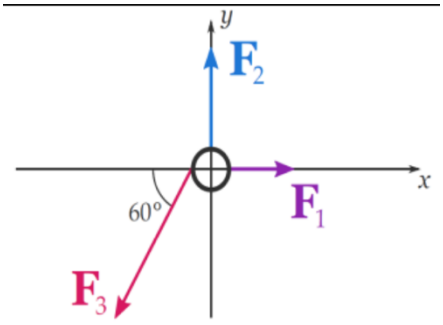
A) 4i m, 14.3m
B) - 7i m, 13.7m
C) -10i m, 18.4m
D) 2i m, 7.3 m
E) - 6i m, 11.3m

4- *Starting from one point, a man walks 25 km in a direction 30° south of west and then walks 30 km toward the north to a terminal point. His resultant direction is:*

A) 71°N of W
B) 69° N of E
C) 51° W of N
D) 29° E of N
E) 21°W of N

5- *Find the magnitude of the resultant F_R of the three vectors shown in the figure. The vectors have the following magnitudes: $F_1 = 10\ N$, $F_2 = 20\ N$ and $F_3 = 40\ N$.*

A) 17.73 N
B) 11.02 N
C) 16.45 N
D) 21.87 N
E) 13.88 N



6- *The magnitude and direction of a vector having an x component of 6.75 units and a y component of -4.3 units can be approximated by:*

A) 5 units, 323°
B) 5 units, 37°
C) 3 units, 44.5°
D) 8 units, 327.5°
E) 8 units, 32.5°

7- *Vector $\vec{A}$ has x and y components of -8.7 cm and 15 cm respectively; vector $\vec{B}$ has x and y componenets of 13.5 cm and -6.6 cm, respectively. If $\vec{A} - \vec{B} + 2\vec{C} = 0$. The components of vector C are:*

A) 5.5i, -5.4j
B) -3.6i, 5.1j
C) -7.3i, 7.2j
D) 3.2i, -4.1j
E) 10.95i,-10.8j

8- *A vector starts at x,y coordinates (3,4) and ends at x,y coordinates (-2,16). What are the magnitude and direction of this vector ?*

A) $13m, 113^o$
B) $17m, 120^o$
C) $13m, 220^o$
D) $19m, 137^o$
E) $19m, 173^o$

9- *A car travels 20 km due north and then 35 km in a direction 60° west of north. The magnitude of displacement of the car is:*

A) 48.2 km
B) 40.9 km
C) 56.3 km
D) 36.7 km
E) 11.8 km

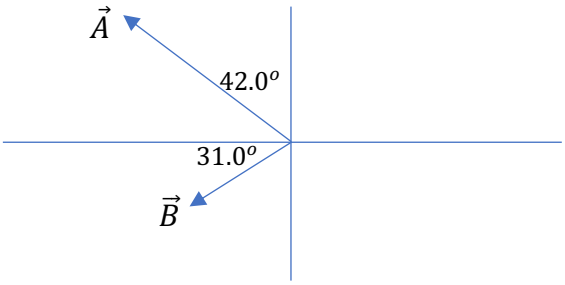
10- *If vector $\vec{A} = 2\hat{\imath} + 6\hat{\jmath}$ and vector $\vec{B} = 4\hat{\imath} - 2\hat{\jmath}$, then the magnitude of vector $2\vec{A} + \vec{B}$ is:*

A) 32.1
B) 12.8
C) 14
D) 7.2
E) 22.2

11- If vector $\vec{C}$ is added to vector $\vec{B}$, the result is $-9\hat{\imath} - 8\hat{\jmath}$. is $\vec{B}$ is subtracted from $\vec{C}$, the result is $5\hat{\imath} + 4\hat{\jmath}$. what is the direction of $\vec{B}$?

A) 131^o
B) 221^o
C) 139^o
D) 236^o

12- Vectors $\vec{A}$ and $\vec{B}$ are shown in the figure. Vector $\vec{C}$ is given by $\vec{C} = \vec{B} - \vec{A}$. the magnitude of vector $\vec{A}$ is 16.0 units , and the magnitude of vector $\vec{B}$ is 7.00 units what is the magnitude of vector C ?



A) 9.53
B) 15.5
C) 16.2
D) 17.5

13- under what condition does the resultant vector A+B have magnitude A+B ?

A) Vectors A+B are parallel and in opposite directions.
B) Vector A and B are parallel and in the same direction
C) Vector A and B are perpendicular
D) Non of the above

14- The magnitude of the sum of two vectors A and B, $|A + B|$ is equal to $|A| + |B|$

A) Vectors A and B are perpendicular
B) Vectors A and B are in opposite direction
C) Angle between Vectors A and B is 45
D) Vectors A and B are in the same direction
E) Non of these

15- A hiker begins a trip by first walking 4.0 km to the east then walks 3.0 km in north direction, what is the magnitude and direction of her resultant displacement?

A) 5km, 53.1 from the east to the north
B) 7km, 53.1 from the east to the north
C) 25km, 63.8 from the east to the north
D) 7km, 36.8 from the east to the north
E) 5km, 36.8 from east to north

16- If $A + B = 0$, the corresponding components of the two vectors A and B must be:

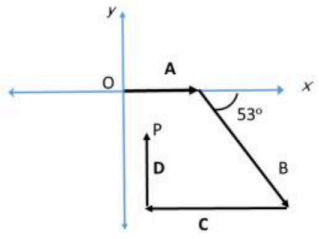
A) Equal
B) negative
C) of opposite sign
D) positive
E) none of those

17- For which of the following vectors is the magnitude of the vector equal to one of the components of the vector?

A) $2\hat{\imath} + 5\hat{\jmath}$
B) $-\hat{\imath}$
C) $-3\hat{\jmath}$
D) $5\hat{\imath} + \hat{\jmath}$
E) $3\hat{\imath}$

18- A boy walks froms O to P in four straight lines as shwon in the diagram. Let A=60m, B=C=100m and D=70m. D is parallel to the y-axis. The magnitude of the boy's displacement as he travels from O to P is :

A) 16.4m
B) 14.17m
C) 22.5m
D) 12.8m
E) 18.1m



Exercise

Section 3.1 Coordinate Systems

1. The polar coordinates of a point are $r = 5.50\ m$ and $\theta = 240^o$. What are the Cartesian coordinates of this point?

2. Two points in the xy plane have Cartesian coordinates (2.00, - 4.00) m and (- 3.00, 3.00) m. Determine (a) the distance between these points and (b) their polar coordinates.

3. A vector has an x component of $-25.0\ units$ and a y component of $40.0\ units$. Find the magnitude and direction of this vector.

4. Obtain expressions in component form for the position vectors having the following polar coordinates: (a) $12.8\ m, 150^o$ (b) $3.30\ cm, 60.0°$ (c) $22.0\ in., 215°$.

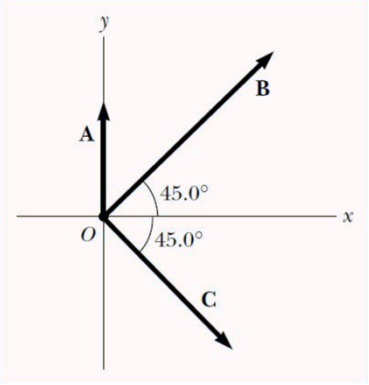
5. Given the vectors A = 2.00i + 6.00j and B = 3.00i - 2.00j,
   (a) draw the vector sum C = A + B and the vector difference D = A — B.
   (b) Calculate C and D, first in terms of unit vectors and then in terms of polar coordinates, with angles measured with respect to the + x axis.

6. Vector A has x and y components of - 8.70 cm and 15.0 cm, respectively; vector B has x and y components of 13.2 cm and - 6.60 cm, respectively. If A - B + 3C = 0, what are the components of C?

7. *Consider the two vectors* $A = 3\hat{\imath} - 2\hat{\jmath}$ *and* $B = -\hat{\imath} - 4\hat{\jmath}$.
Calculate
(a) $\boldsymbol{A + B}$,
(b) $\boldsymbol{A - B}$,
(c) $|\boldsymbol{A + B}|$
(d)$|\boldsymbol{A - B}|$,
(e) the directions of $\boldsymbol{A + B}$ *and* $\boldsymbol{A - B}$.

8. *Vector B has x, y, and z components of 4.00, 6.00, and 3.00 units, respectively. Calculate the magnitude of B and the angles that B makes with the coordinate axes.*

9. *Three displacement vectors of a croquet ball are shown in Figure , where* $|A| = 20.0\ units$, $|B| = 40.0\ units$, *and* $|C| = 30.0\ units$. *Find (a) the resultant in unit—vector notation and (b) the magnitude and direction of the resultant displacement.*



10. *If* $\overrightarrow{A} = (6.00\hat{\imath} - 8.00\hat{\jmath})$ *units,* $B = (-8.00\hat{\imath} + 3\hat{\jmath})$*units, and* $C = (26.0\hat{\imath} - 19.0\hat{\jmath})$ *units, determine* a *and* b *such that* $aA + bB + C = 0$.

Test your self — Chapter 3 — Phys 103 KSU

1- If $\vec{A} = (6m\hat{i} - 8m\hat{j})$ then $4\vec{A}$ has magnitude:

a) 10 m b) 20 m

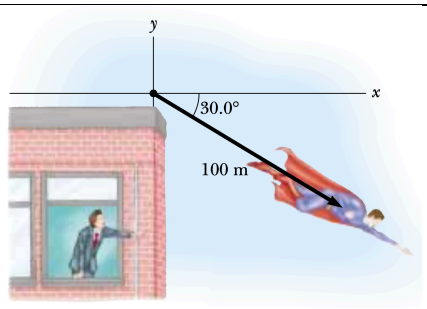
c) 30 m d) 40 m

2- Two points in a plane have polar coordinates (2.50 m, 30.0°) and (3.80 m, 120.0°). Determine the distance between them.

a) 4.5m b) 1.3 m

c) 6.3m d) 90m

3- Find the horizontal and vertical components of the 100-m displacement of a superhero who flies from the top of a tall building following the path shown in Fig. P3.18.



a) $50m\ , 50\sqrt{3}m$ b) $100m\ , 100\sqrt{3}m$

c) $30m\ , 30\sqrt{3}m$ d) $50m\ , 50m$

4- If the rectangular coordinates of a point are given by (2, y) and its polar coordinates are (r, 30°), determine y and r.

a) r=2.3 , y=1.15 b) r=3.3 , y=2.6

c) r=1.7 , y=1 d) r=4 , y=2

5- Find the magnitude and direction of the resultant of three displacements having rectangular components (3.00, 2.00) m, (- 5.00, 3.00) m, and (6.00, 1.00) m.

a) $7.21m\ , 56.3^o$ b) $5.08m\ , 89.1^o$

c) $2.14m\ , 33.6^o$ d) $12.0m\ , 13.9^o$

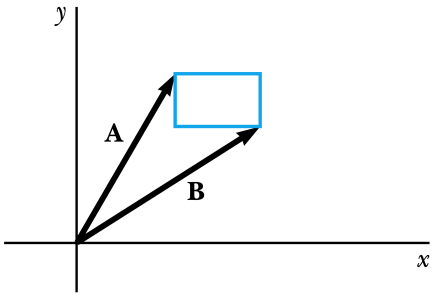
6- A person walks 25.0° north of east for 3.10 km. How far would she have to walk due north and due east to arrive at the same location?

a) 2.8km north , 1.31km east
b) 2km north , 3km east
c) 1.6km north , 1.5km east
d) 3km north , 0.1km east

7- A girl delivering newspapers covers her route by traveling 3.00 blocks west, 4.00 blocks north, and then 6.00 blocks east. (a) What is her resultant displacement? (b) What is the total distance she travels?

a) Displacement=5 blocks
Distance= 13 blocks
b) Displacement=5 blocks
Distance= 5 blocks
c) Displacement=13 blocks
Distance= 5 blocks
d) Displacement=13 blocks
Distance= 13 blocks

8- The rectangle shown in Figure P3.57 has sides parallel to the x and y axes. The position vectors of two corners are A = 10.0 m at 50.0° and B = 12.0 m at 30.0°. (a) Find the perimeter of the rectangle.



a) 10.4
b) 11.26
c) 7.66
d) 6

9- Given the displacement vectors $\vec{A} = (3\hat{i} - 4\hat{j} + 4\hat{k})$ m and $\vec{B} = (2\hat{i} + 3\hat{j} - 7\hat{k})$ m, find the magnitudes of the vectors (a) $\vec{C} = 2\vec{A} - \vec{B}$

a) 16.3
b) 6.48
c) 4.93
d) 2.12

11- *Consider the displacement vector **A**=(3i+3j) m, **B**=(i−5j) m, and **C**=(−i+4j) m. If **D** = **A** + **B** − **C**, Determine the magnitude and direction of **D***

a) $7.81m$, 310^o b) $1.41m$, 110^o

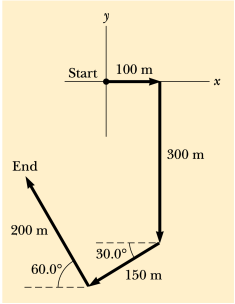
c) $5.15m$, 150^o d) $12.3m$, 220^o

12- *Consider the three displacement vectors* $\vec{A} = (3\hat{\imath} - 3\hat{\jmath})\ m$, $\vec{B} = (\hat{\imath} - 4\hat{\jmath})m$, *and* $\vec{C} = (-2\hat{\imath} + 5\hat{\jmath})m$. *determine (a) the magnitude and direction of* $\vec{E} = -\vec{A} - \vec{B} + \vec{C}$.

a) $13.4m$, 63.4^o b) $9.41m$, 34.8^o

c) $40m$, 40^o d) $14.9m$, 76.1^o

13- *A person going for a walk follows the path shown in Fig. P3.59. The total trip consists of four straight-line paths. At the end of the walk, what is the person's resultant displacement measured from the starting point?*



a) *240* b) *300*

c) *350* d) *550*

14- *Velocity vectors **A** and **B** have equal magnitudes of 10 m/s. If the sum of **A** and **B** is the vector (12j)m/s, determine the components of **A***

a) $A_x = 8\,,\ A_y = 8$ b) $A_x = 8\,,\ A_y = 6$

c) $A_x = 6\,,\ A_y = 6$ $A_x = 4\,,\ A_y = 8$

15- *A vector has an x component of -25.0 units and a y component of 45.0 units. Find the direction of this vector.*

a) *119* b) *219*

c) *75* d) *31*

انتظروا

Chapter 2

مع تحيات

Mr. Abdullah Mohamed

دعواتي للجميع بالتفوق والنجاح