Physics 102

Lecture 5

Motion i n Two Dimensions 2D

Text

- **Oscillatory Motion**
- **Free Fall**
- **Vectors and their components in 2D.**
- **Projectile Motion.**
- **Uniform Circular Motion.**

Periodic Motion

- Periodic motion is motion of an object that regularly returns to a given position after a fixed time interval

- A special kind of periodic motion occurs in mechanical systems when the force acting on the object is proportional to the position of the object relative to some equilibrium position

- If the force is always directed toward the equilibrium position, the motion is called simple harmonic motion

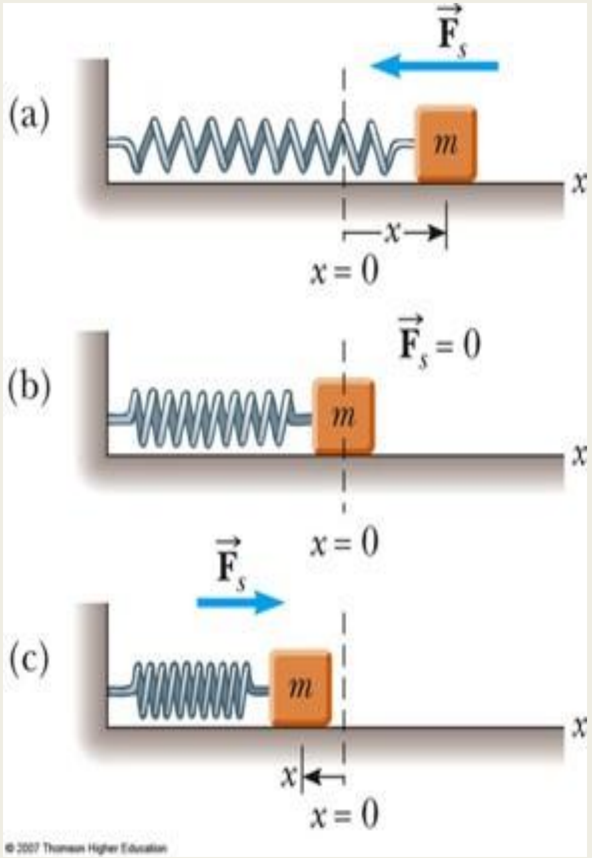
Motion of a Spring-Mass System

- A block of mass m is attached to a spring, the block is free to move on a frictionless horizontal surface
- Use the active figure to vary the initial conditions and observe the resultant motion
- When the spring is neither stretched nor compressed, the block is at the equilibrium position $x = 0$

Hooke's Law

Hooke's Law states $F_s = -kx$

- **F_s is the restoring force**
- It is always directed toward the equilibrium position
- Therefore, it is always opposite the displacement from equilibrium
- k is the force (spring) constant
- x is the displacement



Acceleration

The force described by Hooke's Law is the net force in Newton's Second Law

$$F_{Hooke} = F_{Newton}$$

$$-kx = ma_x$$

$$a_x = -\frac{k}{m}x$$

- The acceleration is proportional to the displacement of the block
- The direction of the acceleration is opposite the direction of the displacement from equilibrium
- An object moves with simple harmonic motion whenever its acceleration is proportional to its position and is oppositely directed to the displacement from equilibrium

Simple Harmonic Motion-Mathematical Representation

Model the block as a particle

- The representation will be particle in simple harmonic motion model
- Choose x as the axis along which the oscillation occurs

Acceleration

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$a = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

We let

$$\omega^2 = \frac{k}{m}$$

Then $a = -\omega^2 x$

The particle motion is represented by the second order differential equation

$$\frac{d^2x}{dt^2} = -w^2x$$

Simple Harmonic Motion – graphical Representation

A solution is $x(t) = A\cos(wt + \phi)$

A, w, ϕ are all constants

A cosine curve can be used to give physical significance to these constants

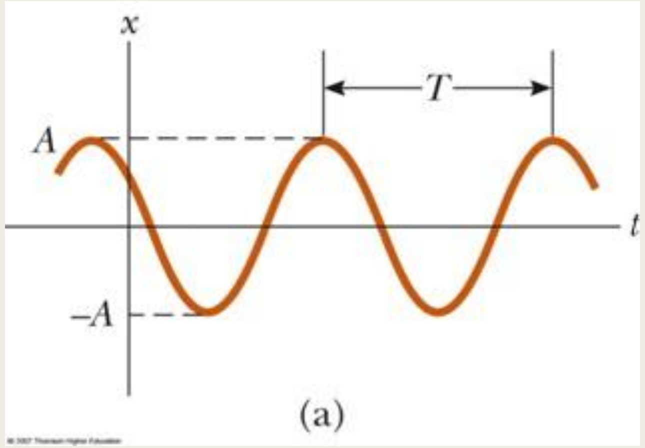
A is the amplitude of the motion

This is the maximum position of the particle in either the positive or negative direction

w is called the angular frequency

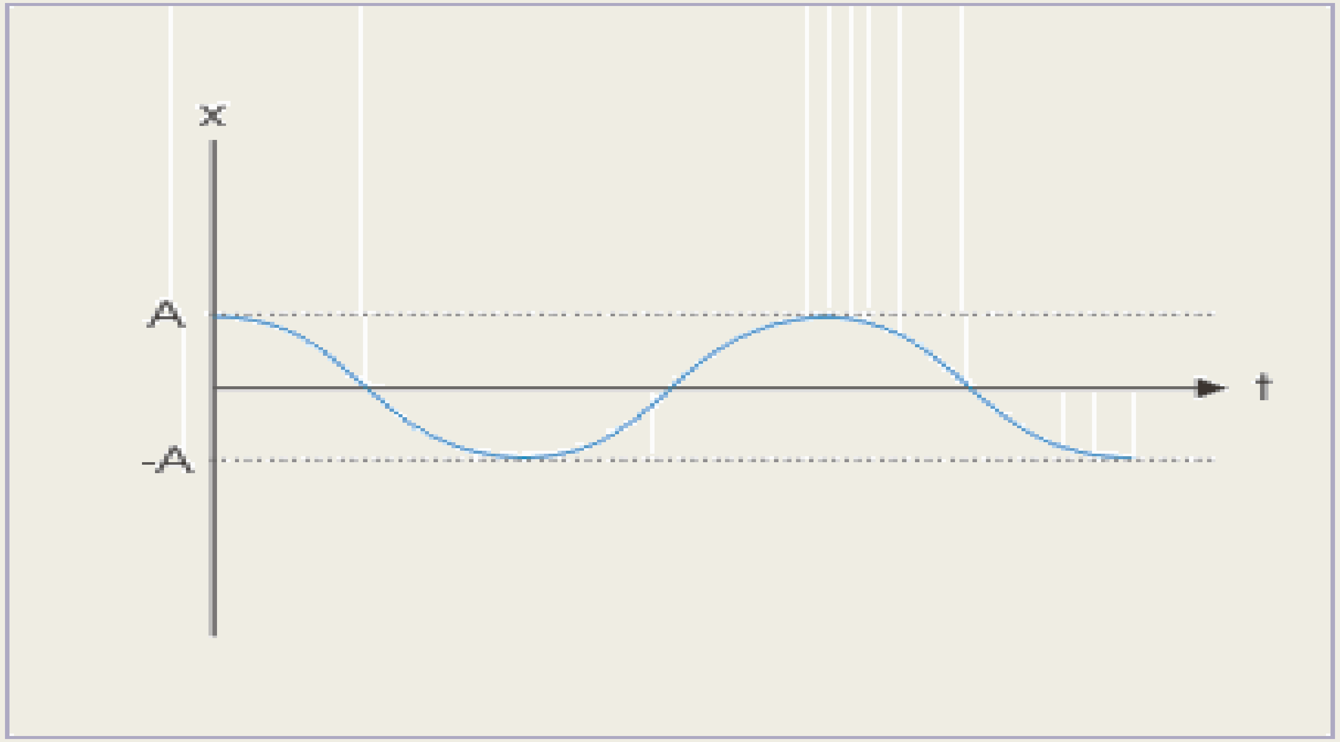
Units are rad/s

ϕ is the phase constant or the initial phase angle



2- Amplitude (A)

(A) amplitude of motion is : The maximum value of the position of the particle in either the (+x) or (–x) direction.

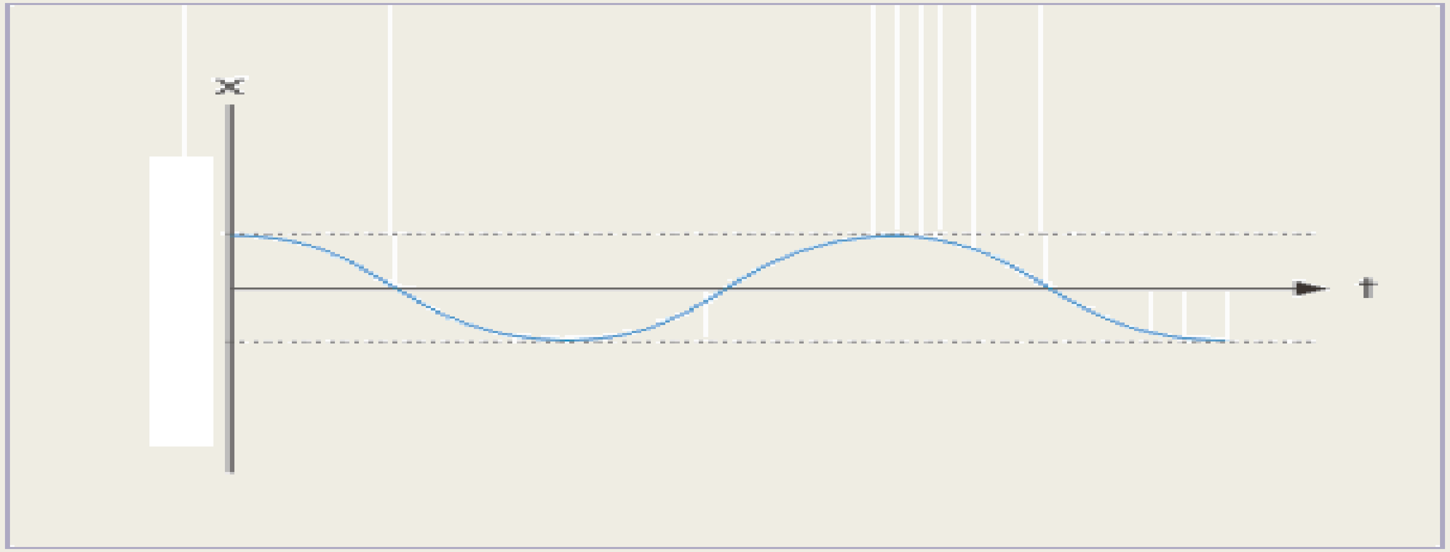


1- Position (x)

Position versus time for an object in simple harmonic motion as following:

$$x = A\cos(\omega t + \phi)$$

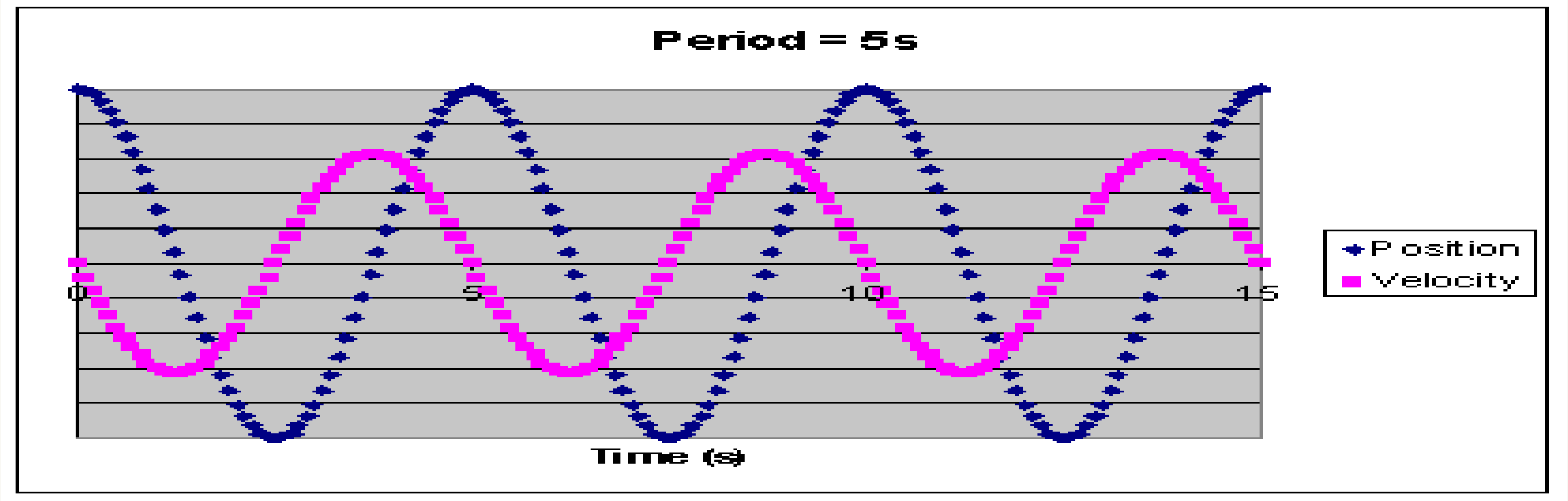
where A , ω , and ϕ are constants.



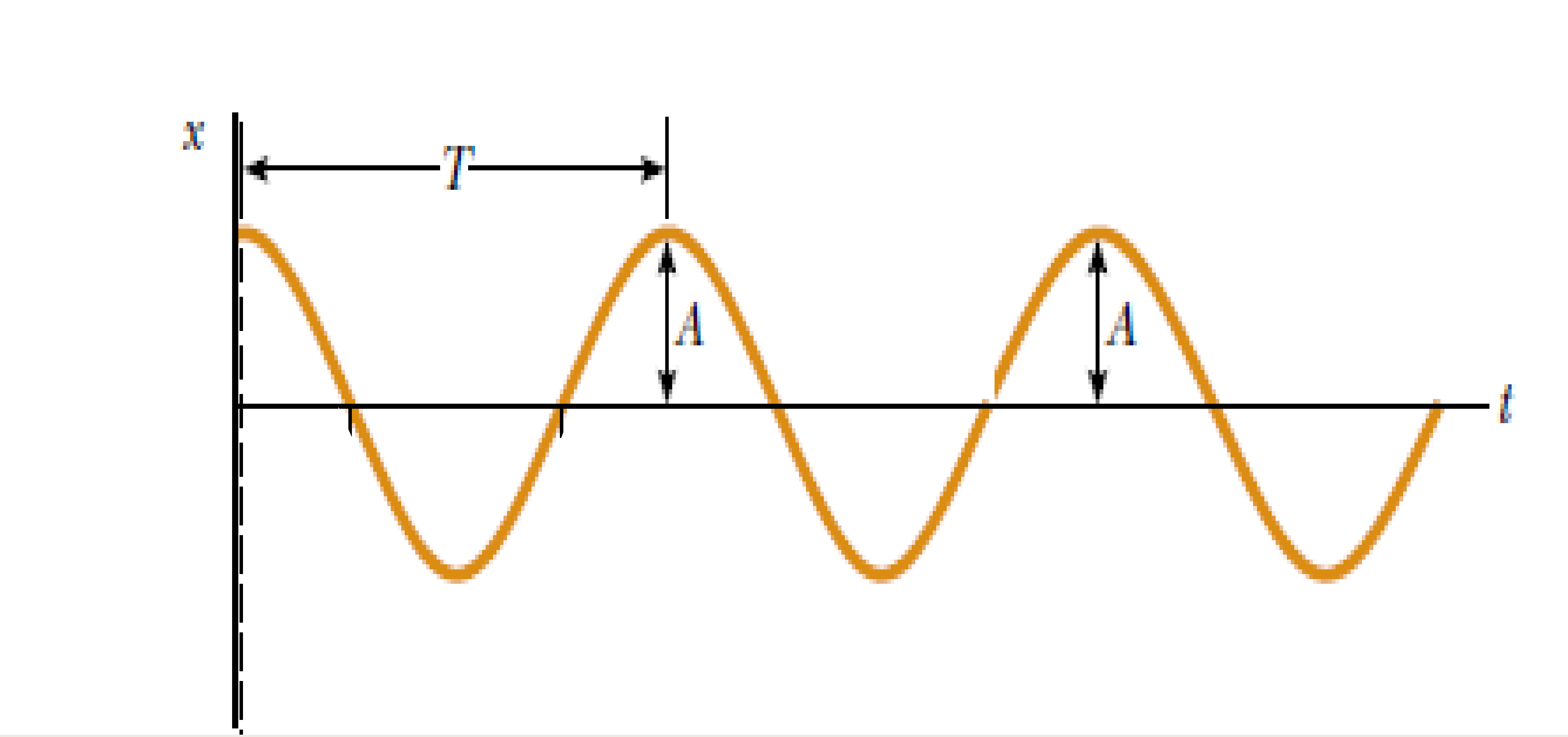
3- Phase constant (∅)

The phase constant ∅ (or initial phase angle) is determined uniquely by the position and velocity of the particle. If the particle is at its maximum position $x = A$ at t=0, the phase constant is ∅= 0 .

The quantity ($\boldsymbol{\omega t} +$ ∅) is called the phase of the motion. Note that the function $x(t)$ is periodic and its value is the same each time ωt increases by 2π radians.



Exercice



Period

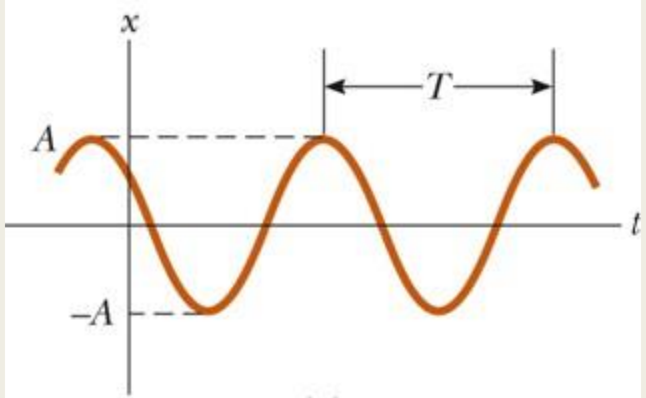
- The period, T, is the time interval required for the particle to go through one full cycle of its motion
- The values of x and v for the particle at time t equal the values of x and v at t + T because the phase increase by $2\pi\ rad$ in time interval of T
- $w(t+T) - wt = 2\pi$ then

$$T = \frac{2\pi}{\omega}$$

Frequency

- The inverse of the period is called the frequency
- The frequency represents the number of oscillations that the particle undergoes per unit time interval
- Units are cycles per second = hertz (Hz)

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$



(a)

Summary Equations – Period and Frequency

- The frequency and period equations can be rewritten to solve for w

$$\omega = 2\pi f = \frac{2\pi}{T}$$

- The period and frequency can also be expressed as:

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\omega^2 = \frac{k}{m}$$

Motion Equations for Simple Harmonic Motion

Simple harmonic motion is one-dimensional and so directions can be denoted by + or – sign

$$x(t) = A\cos(\omega t + \phi)$$

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

$$a = \frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \phi)$$

Maximum Values of v and a

- Because the sine and cosine functions oscillate between ±1, we can easily find the maximum values of velocity and acceleration for an object in SHM

$$v_{max} = \omega A = \sqrt{\frac{k}{m}} A$$

$$a_{max} = \omega^2 A = \frac{k}{m} A$$

Energy of the SHM Oscillator

- Assume a spring-mass system is moving on a frictionless surface
- the total energy is constant
- The kinetic energy can be found by
- $K = \frac{1}{2} mv^2 = \frac{1}{2} mw^2 A^2 \sin^2 (wt + \phi)$
- The elastic potential energy can be found by
- $U = \frac{1}{2} kx^2 = \frac{1}{2} kA^2 \cos^2 (wt + \phi)$

$$x(t) = A \cos(\omega t + \phi)$$

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

$$a = \frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \phi)$$

Energy of the SHM Oscillator

The total energy is $E = K + U = \frac{1}{2} kx^2 + \frac{1}{2} mv^2 = \frac{1}{2} kA^2$

- The total mechanical energy is constant
- The total mechanical energy is proportional to the square of the amplitude
- Energy is continuously being transferred between potential energy stored in the spring and the kinetic energy of the block
- Use the active figure to investigate the relationship between the motion and the energy

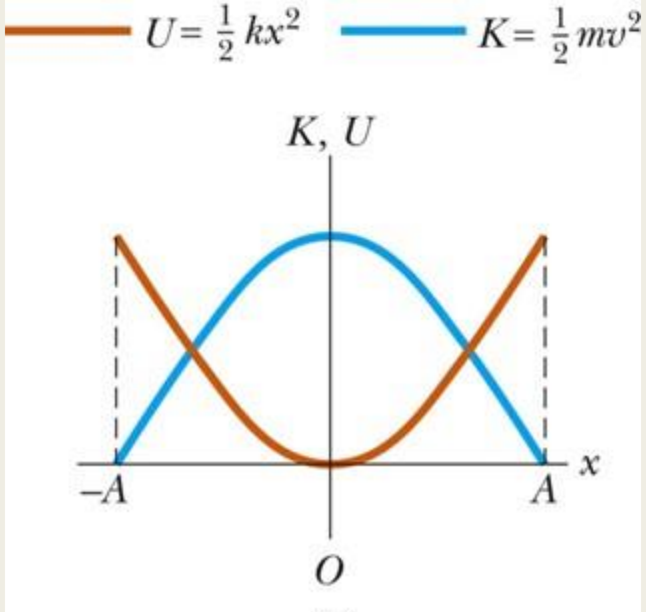
  - Eenergy of the SHM Oscillator,

As the motion continues, the exchange of energy also continues

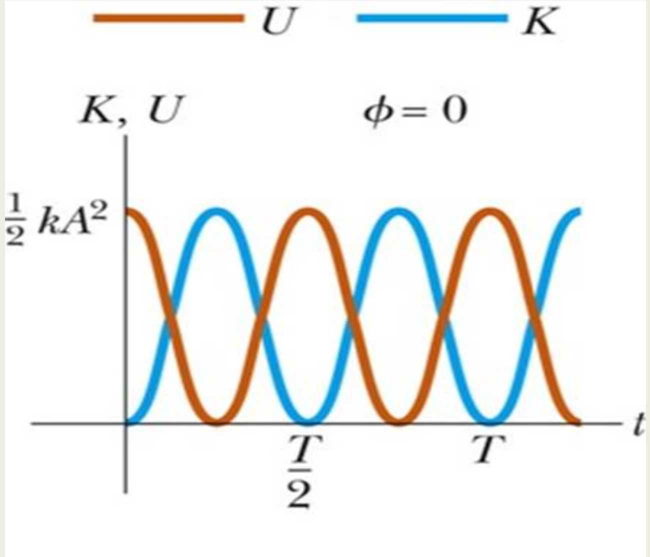
Energy can be used to find the velocity

$$v = \pm\sqrt{\frac{k}{m}\left(A^2 - x^2\right)}$$

$$= \pm\omega^2\sqrt{A^2 - x^2}$$



(b)



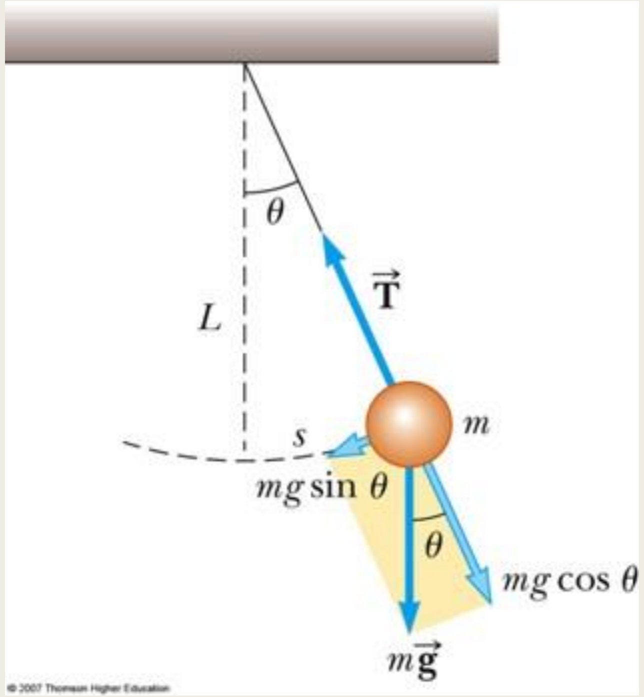
(a)

Simple Pendulum

- A simple pendulum also exhibits periodic motion
- The motion occurs in the vertical plane and is driven by gravitational force
- The motion is very close to that of the SHM oscillator
- If the angle is <10o

ϕ

- $\vec{T}$ The forces acting on the bob are the tension and the weight
- $m\vec{g}$ is the force exerted on the bob by the string
- is the gravitational force
- The tangential component of the gravitational force is a restoring force



- **Simple Pendulum, In the tangential direction,**

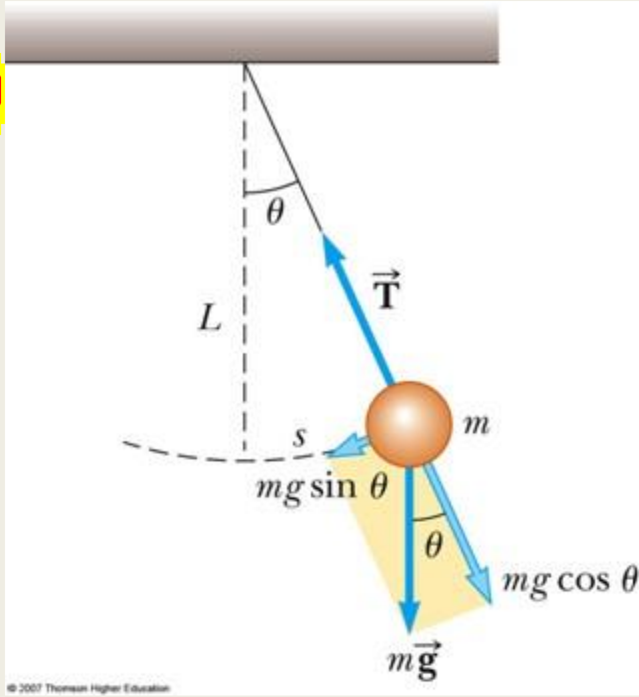
$$F_t = -mg\sin\theta = m\frac{d^2s}{dt^2}$$

- S= θ L
- The length, L, of the pendulum is constant, and for small values of θ , sin θ = θ
- $$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta = -\frac{g}{L}\theta$$
- **Simple Pendulum, The function θ can be written as** $x(t) = A\cos(wt + \phi)$
- $\theta = \theta_{max}\cos(wt + \theta)$
- The angular frequency is

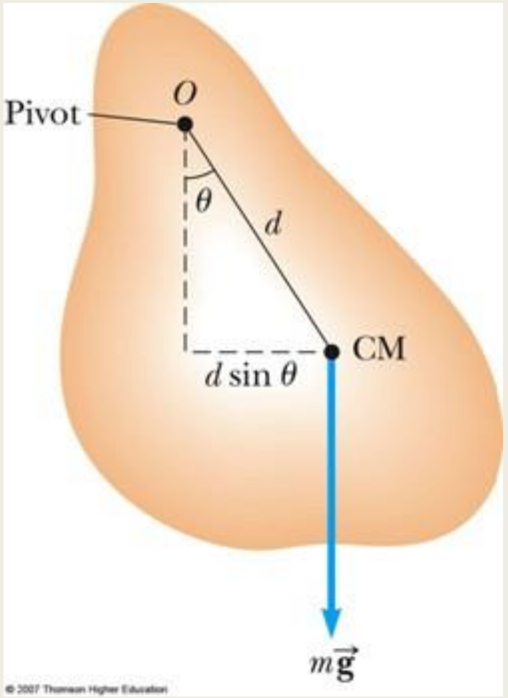
$$\omega = \sqrt{\frac{g}{L}}$$

- The period is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$



- Physical Pendulum
- If a hanging object oscillates about a fixed axis that does not pass through the center of mass and the object cannot be approximated as a particle, the system is called a physical pendulum
- It cannot be treated as a simple pendulum



Physical Pendulum,

The gravitational force provides a torque about an axis through O

The magnitude of the torque is *mgd sin θ*

I is the moment of inertia about the axis through O

From Newton second law,

$$-mgd\sin\theta = I\frac{d^2\theta}{dt^2}$$

Assuming *θ is small this become*

$$\frac{d^2\theta}{dt^2} = -\left(\frac{mgd}{I}\right)\theta = -\omega^2\theta$$

$$\omega = \sqrt{\frac{mgd}{I}}$$

EXAMPLE 15.1 A Block-Spring System

A 200-g block connected to a light spring for which the force constant is 5.00 N/m is free to oscillate on a horizontal, frictionless surface. The block is displaced 5.00 cm from equilibrium and released from rest as in Active Figure 15.6.

(A) Find the period of its motion.

SOLUTION

Conceptualize Study Figure 15.6 and imagine the block moving back and forth in simple harmonic motion once it is released. Set up an experimental model in the vertical direction by hanging a heavy object such as a stapler from a strong rubber band.

Categorize The block is modeled as a *particle in simple harmonic motion.*

Analyze

Use Equation 15.9 to find the angular frequency of the block–spring system:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{5.00 \text{ N/m}}{200 \times 10^{-3} \text{ kg}}} = 5.00 \text{ rad/s}$$

Use Equation 15.13 to find the period of the system:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{5.00 \text{ rad/s}} = 1.26 \text{ s}$$

EXAMPLE 15.3 **Oscillations on a Horizontal Surface**

A 0.500-kg cart connected to a light spring for which the force constant is 20.0 N/m oscillates on a horizontal, frictionless air track.

(A) Calculate the total energy of the system and the maximum speed of the cart if the amplitude of the motion is 3.00 cm.

SOLUTION

Conceptualize The system oscillates in exactly the same way as the block in Active Figure 15.10

Analyze Use Equation 15.21 to express the total energy of the oscillator system and equate it to the kinetic energy of the system when the cart is at $x = 0$:

$$E = \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$$

Solve for the maximum speed and substitute numerical values:

$$v_{max} = \sqrt{\frac{k}{m}}A = \sqrt{\frac{20.0 \text{ N/m}}{0.500 \text{ kg}}}(0.030\,0 \text{ m}) = 0.190 \text{ m/s}$$

INTRODUCTION
Freely Falling Objects

When air resistance is negligible, all objects dropped under the influence of gravity near Earth's surface fall toward Earth with the same constant acceleration.

We denote the magnitude of the **free-fall acceleration** by the symbol ***g***. The value of ***g*** decreases with increasing altitude.

At Earth's surface, the value of ***g*** is approximately 9.80 m/s^2. Unless stated otherwise, we will use this value for ***g*** in doing calculations.

The acceleration is independent of the object's characteristic such as mass, density, or shape; it is the same for all objects

Freely Falling Objects

The free-fall acceleration near Earth's surface is $a = -g = -9.8\ \mathrm{m/s^2}$, and the *magnitude* of the acceleration is $g = 9.8\ \mathrm{m/s^2}$. Do not substitute $-9.8\ \mathrm{m/s^2}$ for g.

The equations of motion for object moving along straight line and under constant acceleration g are given (free-falling) as:

$$v = v_o - gt$$

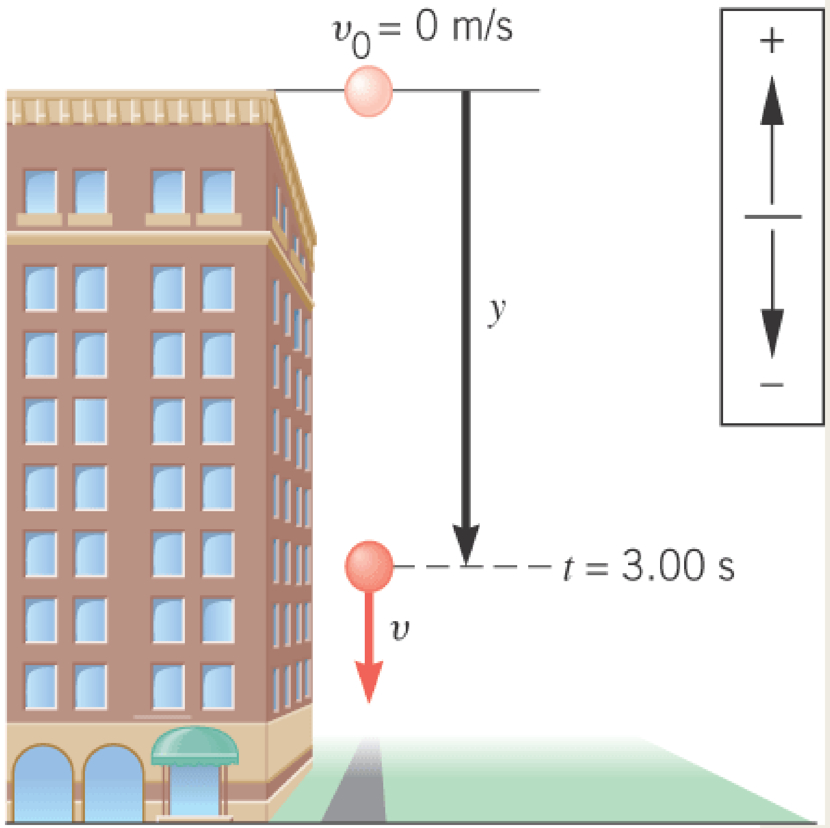
$$\Delta y = y - y_o = v_o t - \frac{1}{2} g t^2$$

$$v^2 = v_o^2 - 2g\Delta y$$

Freely Falling Objects

Example 2.10

A stone is dropped from rest from the top of a building, as shown in Figure 2.4. After 3s of free fall, what is the displacement *y* of the stone?



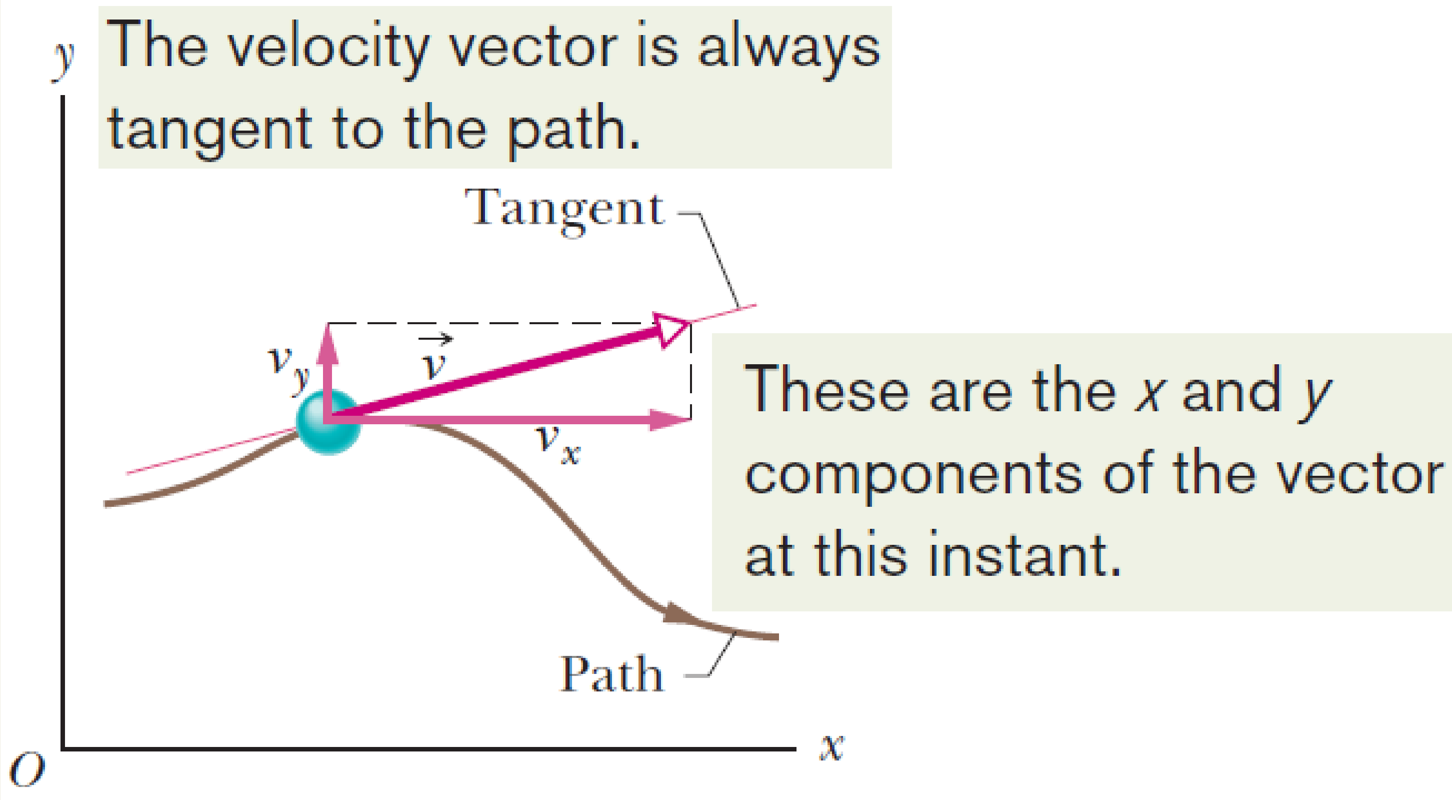
Solution

$$y = y_o + v_o t - \frac{1}{2} g t^2$$

$$y = 0 + 0 - \frac{1}{2}(9.8) \times (3)^2 = -44.1\text{m}$$

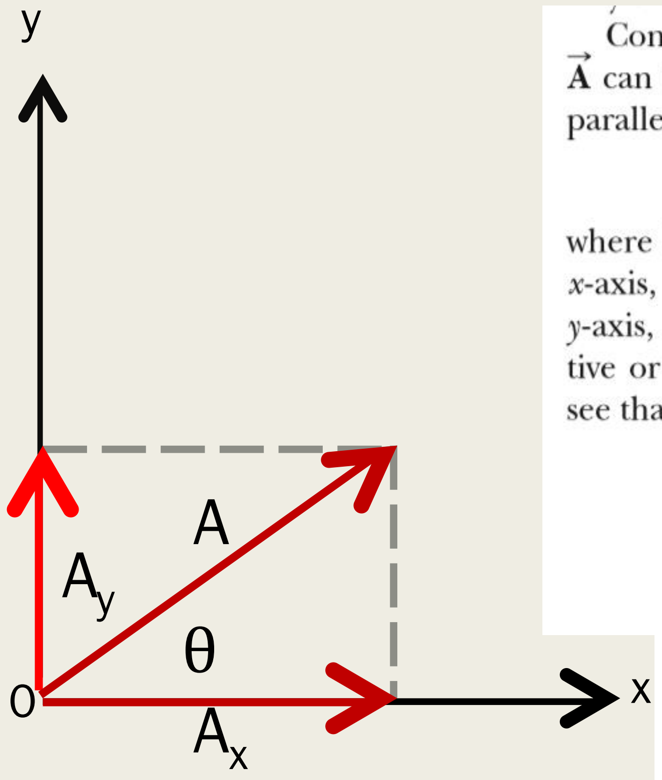
Average Velocity and Instantaneous Velocity

Figure shows a velocity vector $\vec{v}$ and its scalar x and y components. Note that $\vec{v}$ is tangent to the particle's path at the particle's position.



vector components

A **component** of a vector is the projection of the vector on an axis.



Consider a vector $\vec{\mathbf{A}}$ in a rectangular coordinate system, as shown in Figure $\vec{\mathbf{A}}$ can be expressed as the sum of two vectors: $\vec{\mathbf{A}}_x$, parallel to the x-axis; and $\vec{\mathbf{A}}_y$, parallel to the y-axis. Mathematically,

$$\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y$$

where $\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$ are the component vectors of $\vec{\mathbf{A}}$. The projection of $\vec{\mathbf{A}}$ along the x-axis, A_x, is called the x-component of $\vec{\mathbf{A}}$, and the projection of $\vec{\mathbf{A}}$ along the y-axis, A_y, is called the y-component of $\vec{\mathbf{A}}$. These components can be either positive or negative numbers with units. From the definitions of sine and cosine, we see that $\cos\theta = A_x/A$ and $\sin\theta = A_y/A$, so the components of $\vec{\mathbf{A}}$ are

$$A_x = A\cos\theta$$

$$A_y = A\sin\theta$$

These components form two sides of a right triangle having a hypotenuse with magnitude A. It follows that $\vec{\mathbf{A}}$'s magnitude and direction are related to its components through the Pythagorean theorem and the definition of the tangent:

$$A = \sqrt{A_x^2 + A_y^2}$$

$$\tan\theta = \frac{A_y}{A_x}$$

To solve for the angle θ, which is measured from the positive x-axis by convention, we can write Equation 3.4 in the form

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

Projectile Motion

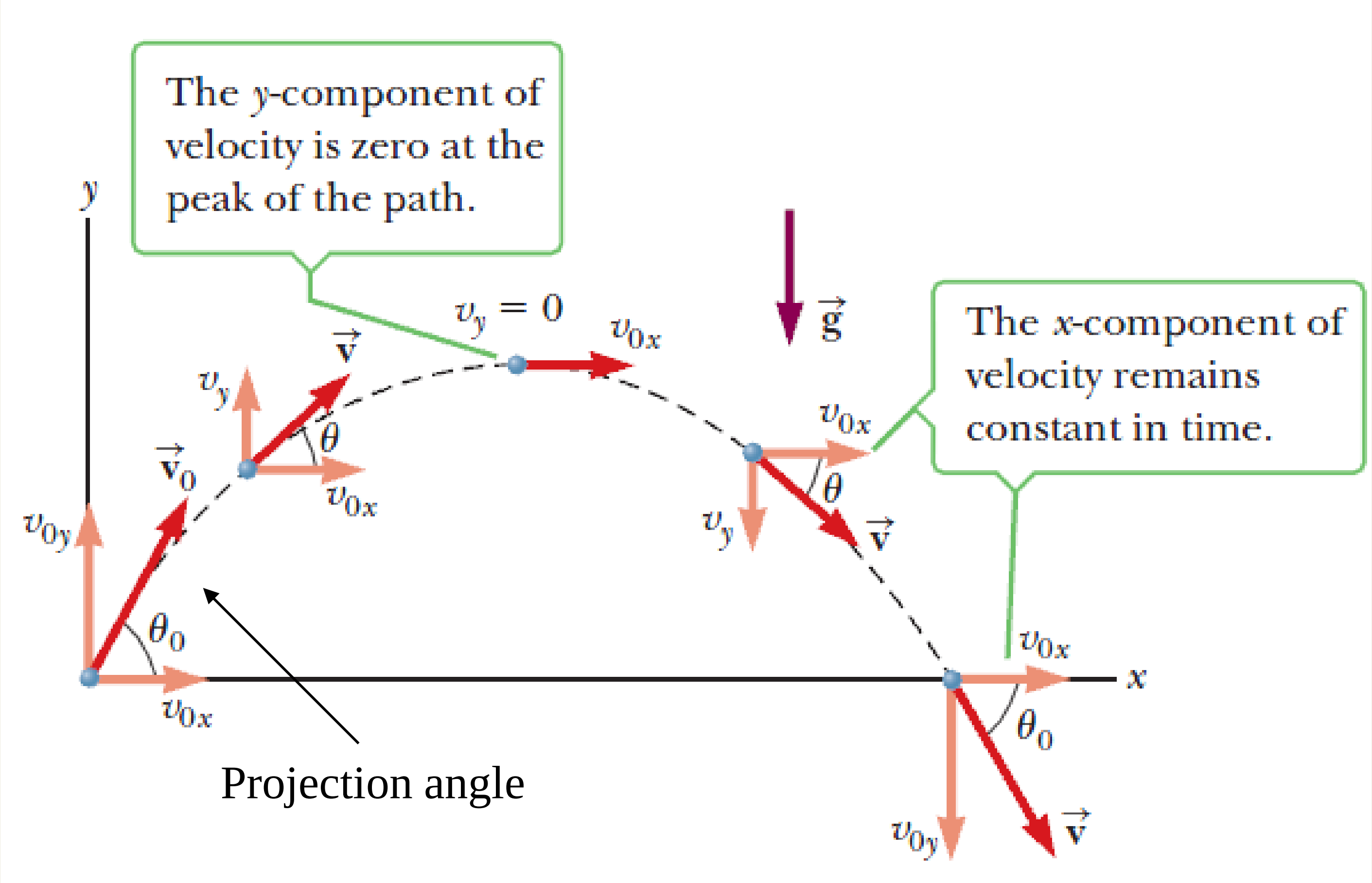
We next consider a special case of two-dimensional motion: A particle moves in a vertical plane with some initial velocity $\vec{v}_0$ but its acceleration is always the freefall acceleration $\vec{g}$, which is downward. Such a particle is called a **projectile** and its motion is called **projectile motion.**

the horizontal and vertical motions are completely independent of each other. This means that motion in one direction has no effect on motion in the other direction.

Here we assume that has no effect on the projectile motion.

Projectile Motion

The positive *x*-direction is horizontal and to the right, and the *y*-direction is vertical and positive upward.



Projectile Motion

The projectile with an initial velocity $\vec{v}_0$ that can be written as

$$\vec{v}_0 = v_{0x}\hat{\mathrm{i}} + v_{0y}\hat{\mathrm{j}}.$$

The components v_{0x} and v_{0y} can then be found if we know the angle θ_0 between $\vec{v}_0$ and the positive x direction:

$$v_{0x} = v_0 \cos\theta_0 \quad \text{and} \quad v_{0y} = v_0 \sin\theta_0.$$

During its two-dimensional motion, the projectile's position vector $\vec{r}$ and velocity vector $\vec{v}$ change continuously, but its acceleration vector $\vec{a}$ is constant and *always* directed vertically downward. The projectile has *no* horizontal acceleration.

Projectile Motion

Projectile motion, involving two-dimensional motion into two separate and easier one-dimensional motion

one for the horizontal (with *zero acceleration*) and

one for the vertical motion (with *constant downward acceleration*)

Projectile Motion Analyzed

The Horizontal Motions

Because there is *no acceleration* in the horizontal direction, the horizontal component v_x of the projectile's velocity remains unchanged from its initial value v_{0x} throughout the motion.

At any time t, the projectile's horizontal displacement $x - x_0$ from an initial position x_0 with $a = 0$, is given by

$$x - x_0 = v_{0x}t.$$

second motion law

Because $v_{0x} = v_0 \cos \theta_0$, this becomes

$$x - x_0 = (v_0 \cos \theta_0)t. \qquad (1)$$

The Vertical Motion

The vertical motion is the motion for a particle in free fall. Most important is that the acceleration is constant.

$$y - y_0 = v_{0y}t - \tfrac{1}{2}gt^2$$

second motion law

$$= (v_0 \sin \theta_0)t - \tfrac{1}{2}gt^2, \quad (2)$$

where the initial vertical velocity component v_{0y} is replaced with the equivalent $v_0 \sin \theta_0$.

$$v_y = v_0 \sin \theta_0 - gt$$

First motion law

$$v_y^2 = (v_0 \sin \theta_0)^2 - 2g(y - y_0)$$

third motion law

The Horizontal Range

The *horizontal range* R of the projectile is the *horizontal* distance the projectile has traveled when it returns to its initial height

To find range R, let us put $x - x_0 = R$

and $y - y_0 = 0$ obtaining

$$R = (v_0 \cos \theta_0)t$$

$$0 = (v_0 \sin \theta_0)t - \tfrac{1}{2}gt^2$$

Eliminating t between these two equations yields

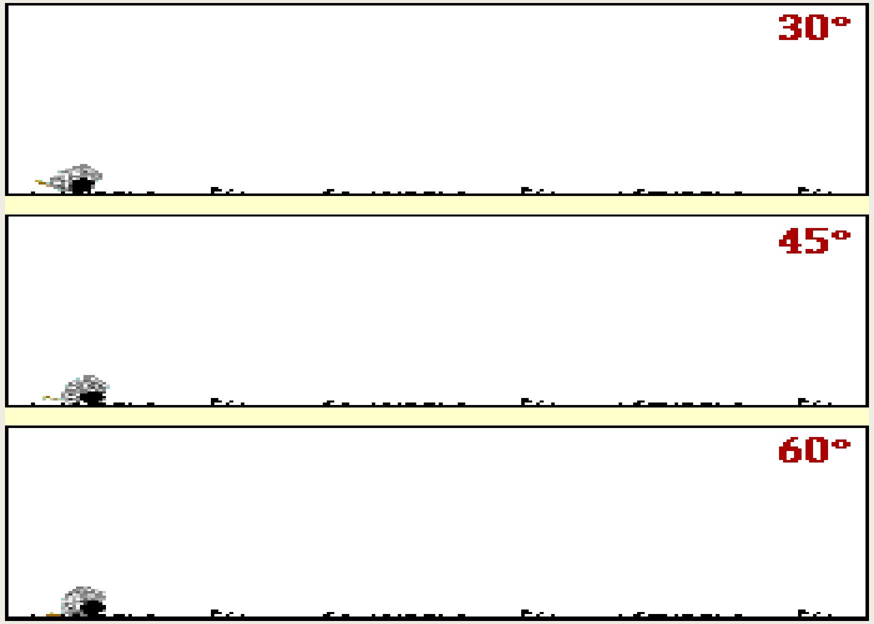
$$R = \frac{2v_0^2}{g} \sin \theta_0 \cos \theta_0.$$

The Horizontal Range

Using the identity $\sin 2\theta_0 = 2 \sin \theta_0 \cos \theta_0$, we obtain

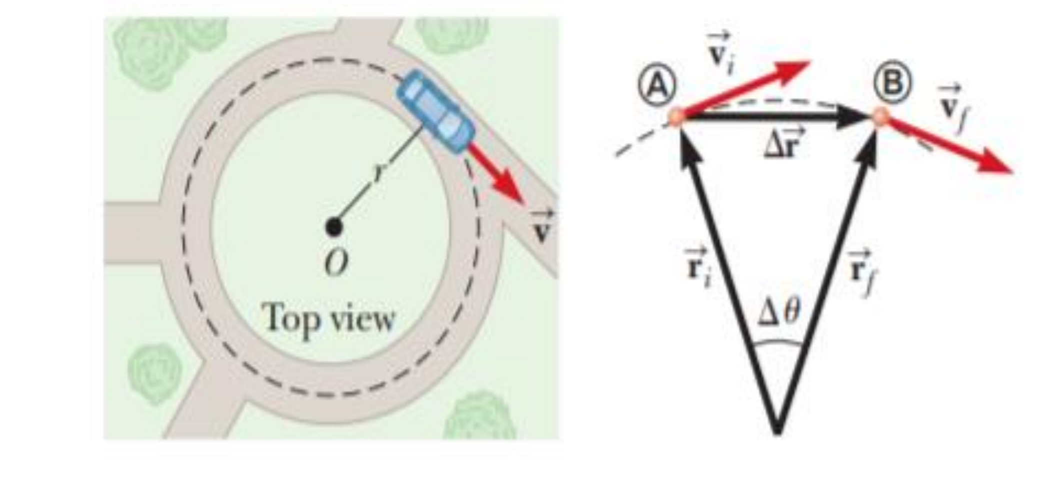
$$R = \frac{v_0^2}{g} \sin 2\theta_0$$

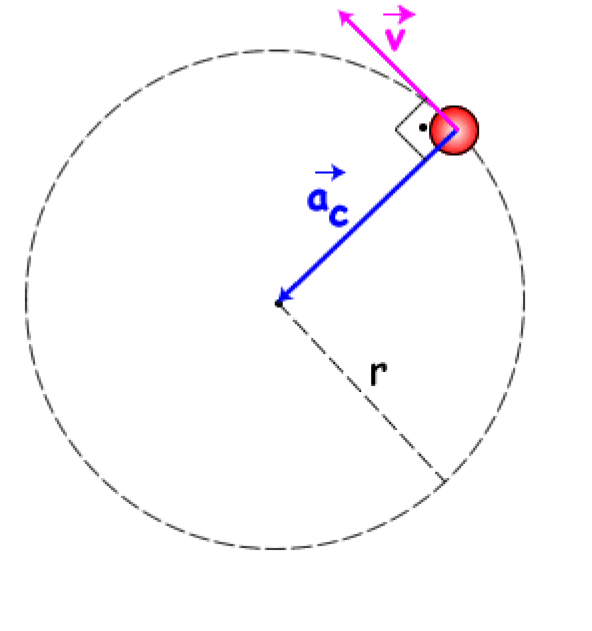
- 1- This equation does not give the horizontal distance traveled by a projectile when the final height is not the launch height (like basket ball).
- 2- the horizontal range R is maximum for a launch angle of 45º



Uniform Circular Motion

A particle is in **uniform circular motion** if it travels around a circle or a circular arc at constant (*uniform*) speed. Although the speed does not vary, *the particle is accelerating* because the velocity changes in direction.

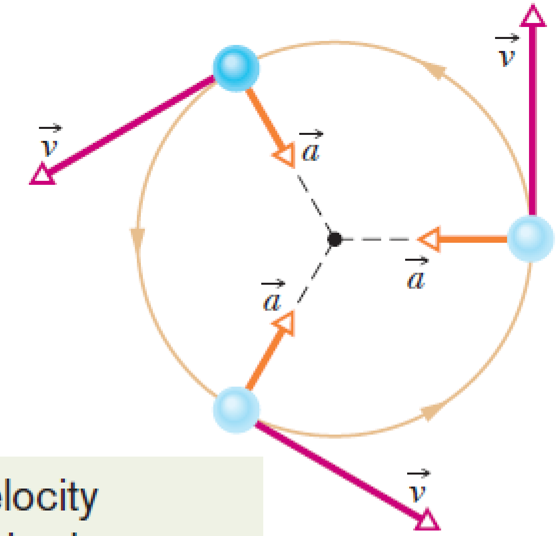




Uniform Circular Motion

Figure shows the relationship between the velocity and acceleration vectors at various stages during uniform circular motion. Both vectors have constant magnitude, but their directions change continuously. The velocity is always directed tangent to the circle in the direction of motion. The acceleration is always directed *radially inward*.

The acceleration vector always points toward the center.



The velocity vector is always tangent to the path.

Uniform Circular Motion

the magnitude of this acceleration $\vec{a}$ is

$$a = \frac{v^2}{r} \qquad \text{(acceleration)},$$

where r is the radius of the circle and v is the speed of the particle.

In addition, during this acceleration at constant speed, the particle travels the circumference of the circle (a distance of $2\pi r$) in time

$$T = \frac{2\pi r}{v} \qquad \text{(period)}.$$

T is called the *period of revolution*, or simply the *period*, of the motion. It is, in general, the time for a particle to go around a closed path exactly once.

Uniform Circular Motion

The period of a particle in uniform circular motion is a measure of the number of seconds for one revolution of the particle around the circle. The inverse of the period is the *rotation rate* and is measured in revolutions per second. Because one full revolution of the particle around the circle corresponds to an angle of 2π radians, the product of 2π and the rotation rate gives the **angular speed** ω of the particle, measured in radians/s or s^{-1}:

$$\omega = \frac{2\pi}{T}$$

$$\omega = 2\pi\left(\frac{v}{2\pi r}\right) = \frac{v}{r} \quad \rightarrow \quad v = r\omega$$

Example 1

A ball launched in 25 m/s with an angle 35^o above x axis, find the horizontal range, maximum height, the time that the ball remain in the air.

Example 2

56. An Earth satellite moves in a circular orbit 640 km above Earth's surface with a period of 98.0 min. What are the (a) speed and (b) magnitude of the centripetal acceleration of the satellite?

Givens:

The earth satellite is moving in a uniform circular motion.
The distance between the satellite orbit and the earth's surface, $d = 640$ km.
The satellite moves around the earth's surface for a period, $T = 98$ min.

The radius of the satellite orbit is given by:

$$R_{\text{sat}} = R_{\text{earth}} + d = 6371000 + 640 \times 10^3 = 7011000 \text{ m}.$$

Where R_{earth} is the earth's radius.

Part a:

The speed of the satellite:

$$v = \frac{2\pi R_{\text{sat}}}{T} = \frac{2\pi \times 7011000}{98 \times 60} = 7491.74 \text{ m/s}.$$

Where R_{sat} is the radius of the satellite orbit, and T is the period of the motion.

$$\boxed{v = 7491.74 \text{ m/s}}$$

Part b:

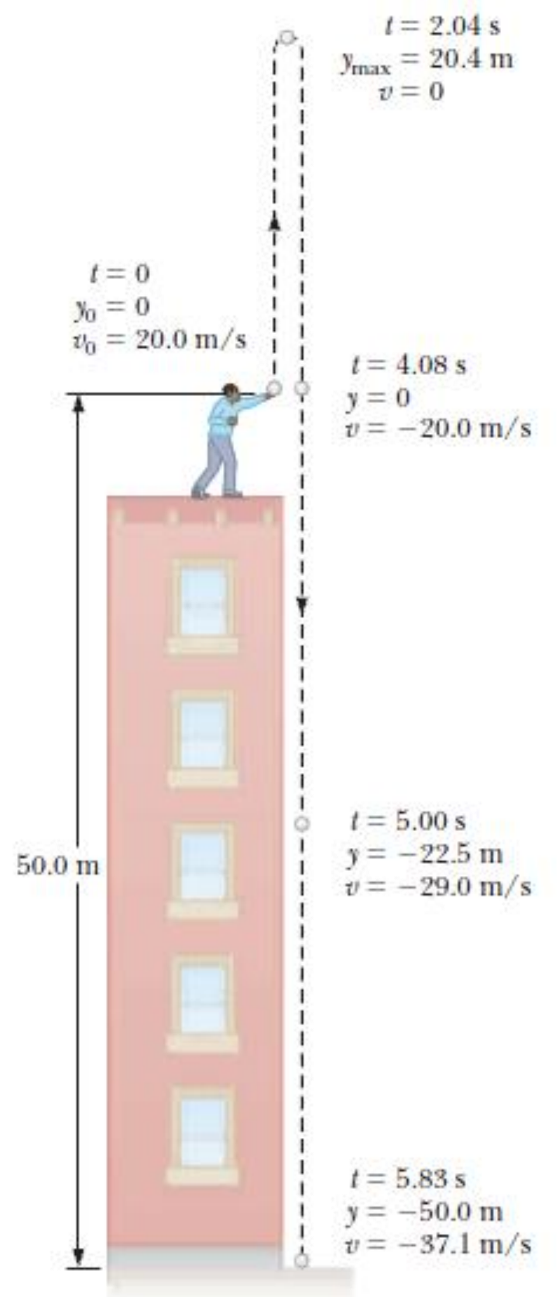
The magnitude of the centripetal acceleration of the satellite:

$$a_c = \frac{v^2}{R_{\text{sat}}} = \frac{7491.74^2}{7011000} = 8.005 \text{ m/s}^2.$$

$$\boxed{a_c = 8.005 \text{ m/s}^2}$$

EXERCISES

A ball is thrown from the top of a building with an initial velocity of 20.0 m/s straight upward, at an initial height of 50.0 m above the ground. The ball just misses the edge of the roof on its way down, as shown in Figure . Determine **(a)** the time needed for the ball to reach its maximum height, **(b)** the maximum height, **(c)** the time needed for the ball to return to the height from which it was thrown and the velocity of the ball at that instant, **(d)** the time needed for the ball to reach the ground, and **(e)** the velocity and position of the ball at t = 5.00 s. Neglect air drag.



EXERCISES:

••32 GO You throw a ball toward a wall at speed 25.0 m/s and at angle $\theta_0 = 40.0°$ above the horizontal (Fig. 4-35). The wall is distance $d = 22.0$ m from the release point of the ball. (a) How far above the release point does the ball hit the wall? What are the (b) horizontal and (c) vertical components of its velocity as it hits the wall? (d) When it hits, has it passed the highest point on its trajectory?

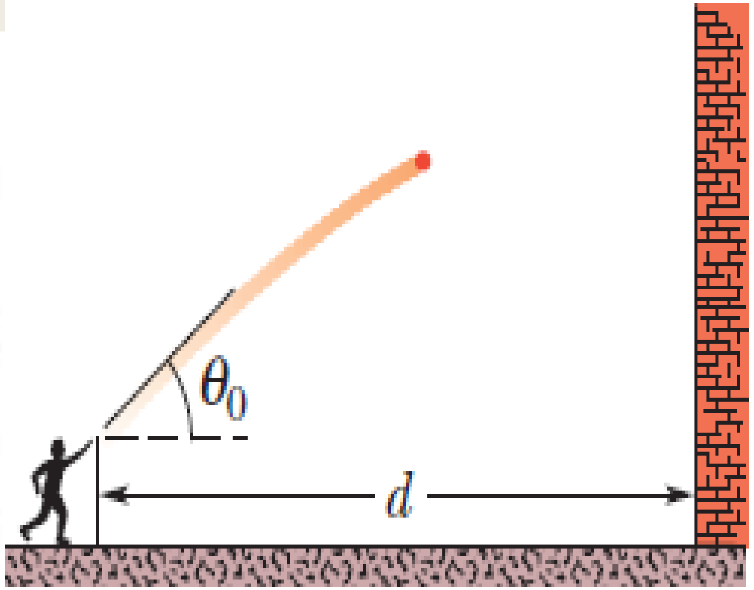


Fig. 4-35 Problem 32.

EXERCISES:

- A ball is thrown with initial velocity v_o at 45^o of x axis, and then hit the ground at 350m after a period of time 4sec, calculate its initial velocity?

HOME WORK

- A fireman, 50 m away from a burning building, directs a stream of water from a hose at an angle of 30° above the horizontal. If the velocity of the stream of water is 40 m/s, at what height will the stream of water strike the building?