

# Chapter 7

## Work and Kinetic Energy



## Units of Chapter 7

- Work Done by a Constant Force
- Kinetic Energy and the Work-Energy Theorem
- Work done by a variable forces
- Power

# 7-1 Work Done by a Constant Force

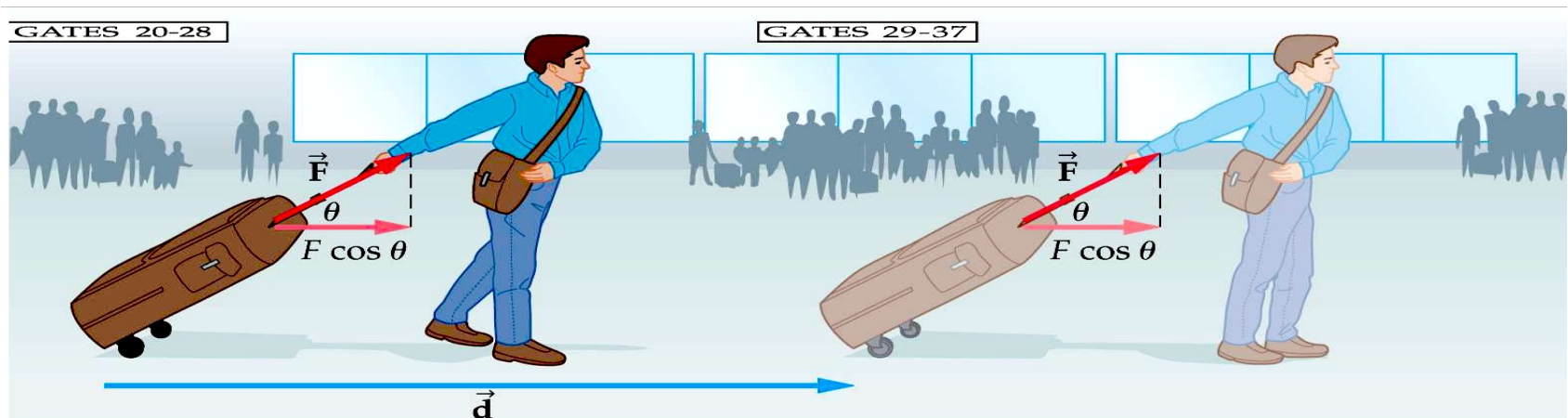
The work done by a constant force  $F$  is defined as:

$$W = Fd\cos\theta$$

$F$  is the applied force,  $d$  is the displacement of the object,  $\theta$  is the angle between the force and the displacement.

The SI unit of work is newton-meter (N.m) = joule, (J)

- Note: only by the component of the force that is along  $d$  does the work.**

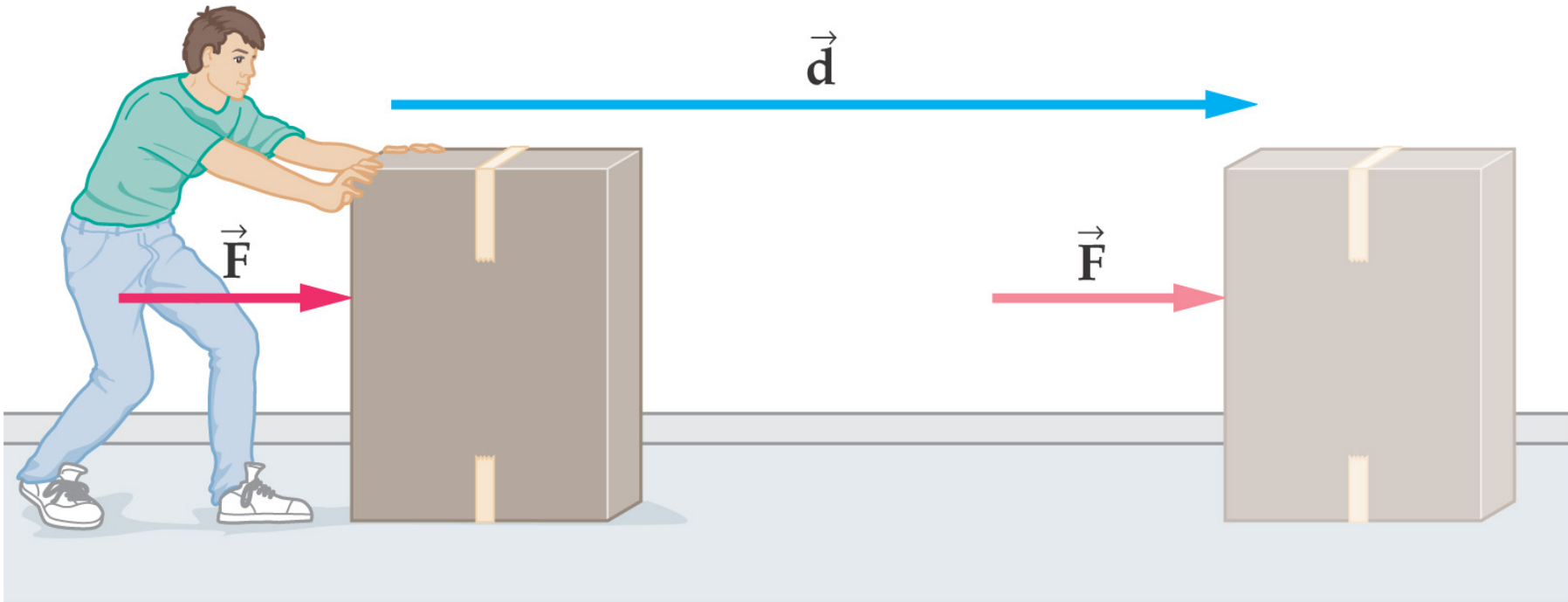


# 7-1 -Work Done by a Constant Force

The definition of work, when the force is parallel to the displacement:

$$W = Fd$$

Work is a scalar quantity (7-1)



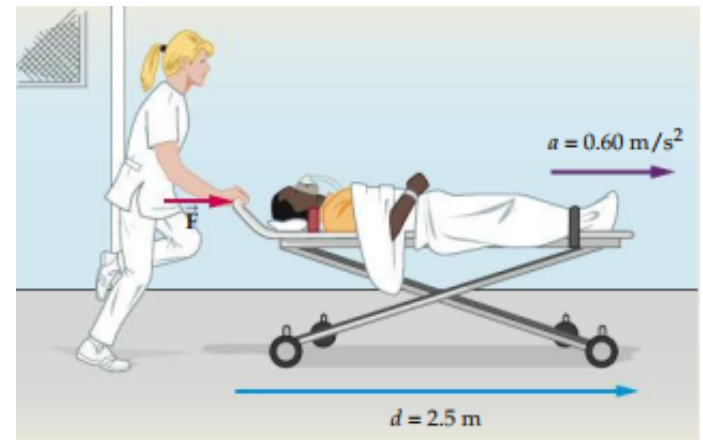
# Example:-7-1

- An intern pushes a 72kg patient on a 15kg gurney, producing an acceleration of  $0.60\text{m/s}^2$ .
- A) How much work does the intern do by pushing the patient and gurney through a distance of 2.5m? Assume the gurney moves without friction.-

Answer:  $W=130\text{J}$

- B) How far must the intern push the gurney to do 140J of work?

Answer:  $d= 2.7\text{m}$



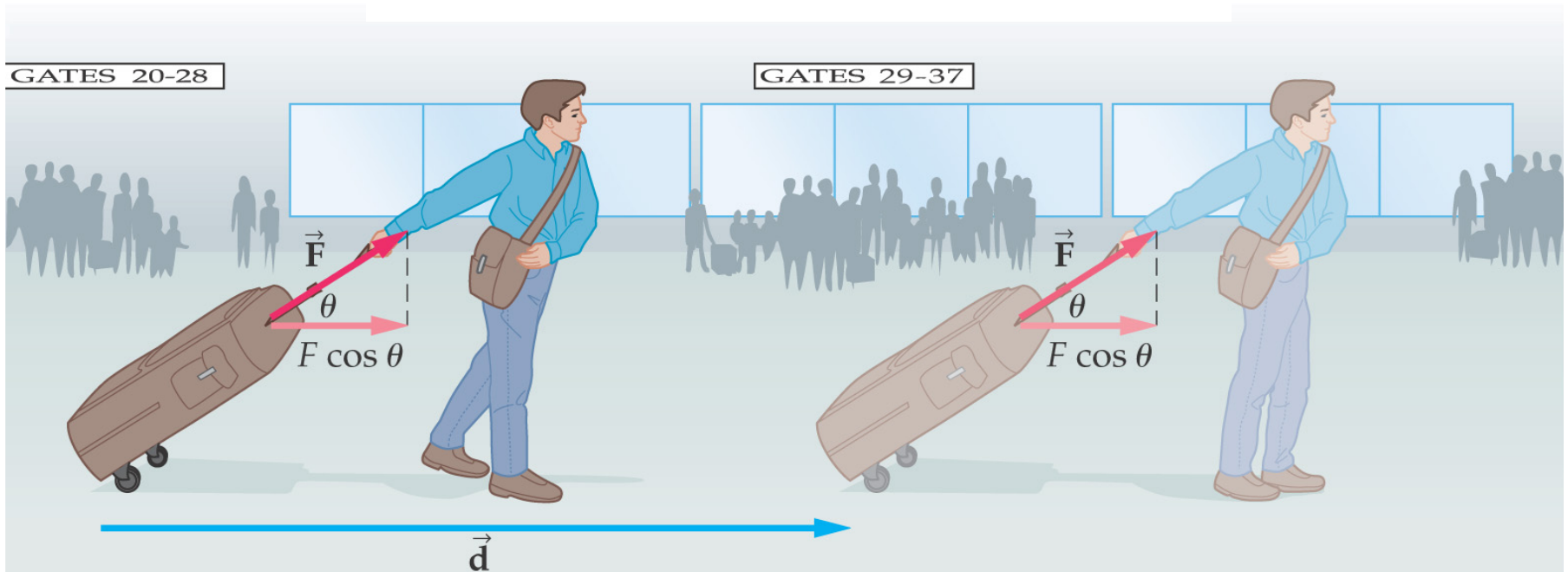
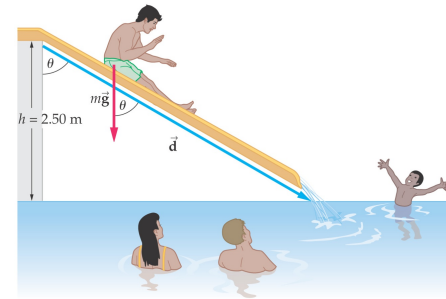
# 7-1 --Work Done by a Constant Force

If the force is at an angle to the displacement:

$$W = (F \cos \theta)d = Fd \cos \theta$$

The work can also be written as the dot product of the force and the displacement:

$$W = \vec{F} \cdot \vec{d} = Fd \cos \theta$$



# Example 7-2

## EXAMPLE 7-2 GRAVITY ESCAPE SYSTEM



### REAL-WORLD PHYSICS

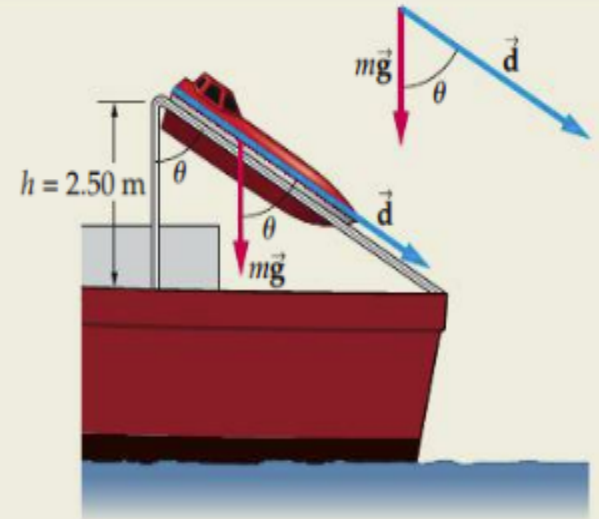
In a gravity escape system (GES), an enclosed lifeboat on a large ship is deployed by letting it slide down a ramp and then continuing in free fall to the water below. Suppose a 4970-kg lifeboat slides a distance of 5.00 m on a ramp, dropping through a vertical height of 2.50 m. How much work does gravity do on the boat?

### PICTURE THE PROBLEM

From our sketch, we see that the force of gravity  $m\vec{g}$  and the displacement  $\vec{d}$  are at an angle  $\theta$  relative to one another when placed tail-to-tail, and that  $\theta$  is also the angle the ramp makes with the vertical. In addition, we note that the vertical height of the ramp is  $h = 2.50$  m and the length of the ramp is  $d = 5.00$  m.

### STRATEGY

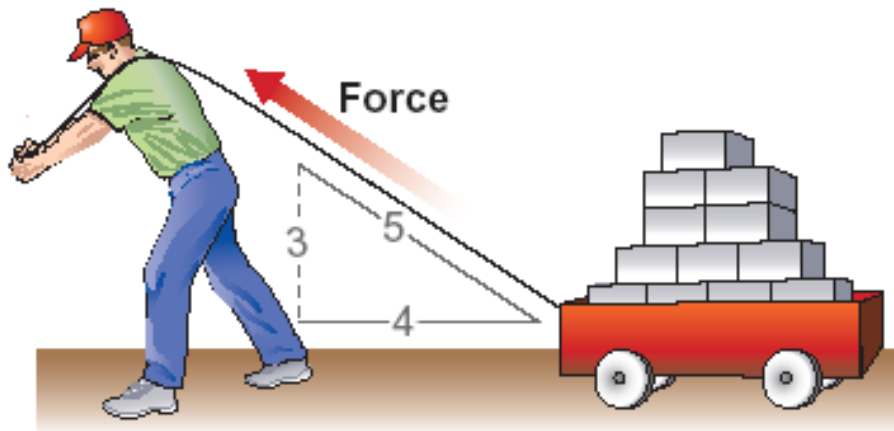
By definition, the work done on the lifeboat by gravity is  $W = Fd \cos \theta$ , where  $F = mg$ ,  $d = 5.00$  m, and  $\theta$  is the angle between  $m\vec{g}$  and  $\vec{d}$ . We are not given  $\theta$  in the problem statement, but from the right triangle that forms the ramp we see that  $\cos \theta = h/d$ . Once  $\theta$  is determined from the geometry of our sketch, it is straightforward to calculate  $W$ .



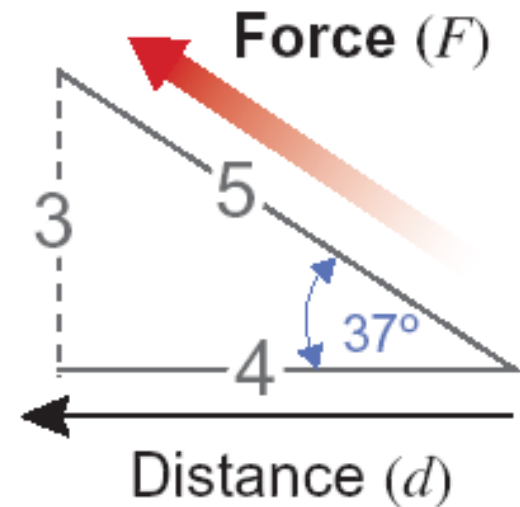
- *Answer*  $W = 122\text{kJ}$

# Force at an Angle to the Distance

## PROBLEM



## ANALYSIS



## SOLUTION

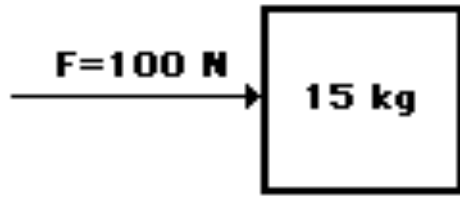
$$W = Fd \times \left( \frac{4}{5} \right) = Fd \cos 37^\circ$$

$$Fd \cos (\text{theta}) = 5 * 4 (4/5) = 15.97 = 16J$$

$$= 5 * 4 \text{ COS } 37 = 15.97J = 16J$$

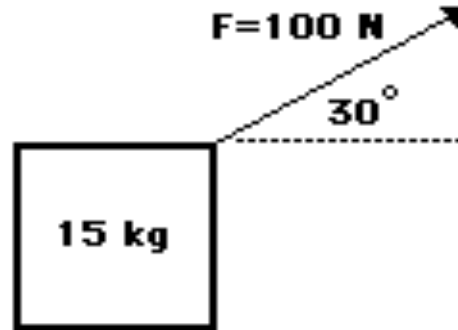
# Work Calculations

## Diagram A



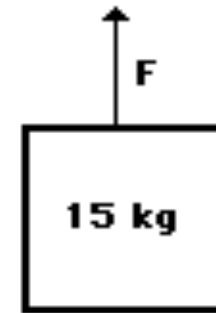
A 100 N force is applied to move a 15 kg object a horizontal distance of 5 meters at constant speed.

## Diagram B

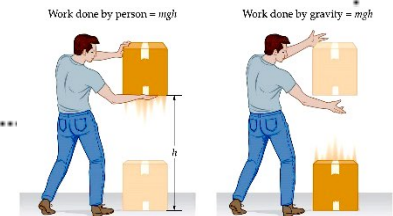


A 100 N force is applied at an angle of 30° to the horizontal to move a 15 kg object at a constant speed for a horizontal distance of 5 m.

## Diagram C



An upward force is applied to lift a 15 kg object to a height of 5 meters at constant speed.



$$W = F \times d$$

$$= 100 \times 5$$

$$= 500 \text{ J}$$

$$W = F \times d \cos 30^\circ$$

$$100 \times 5 \times .87$$

$$= 433 \text{ J}$$

$$W = mgd$$

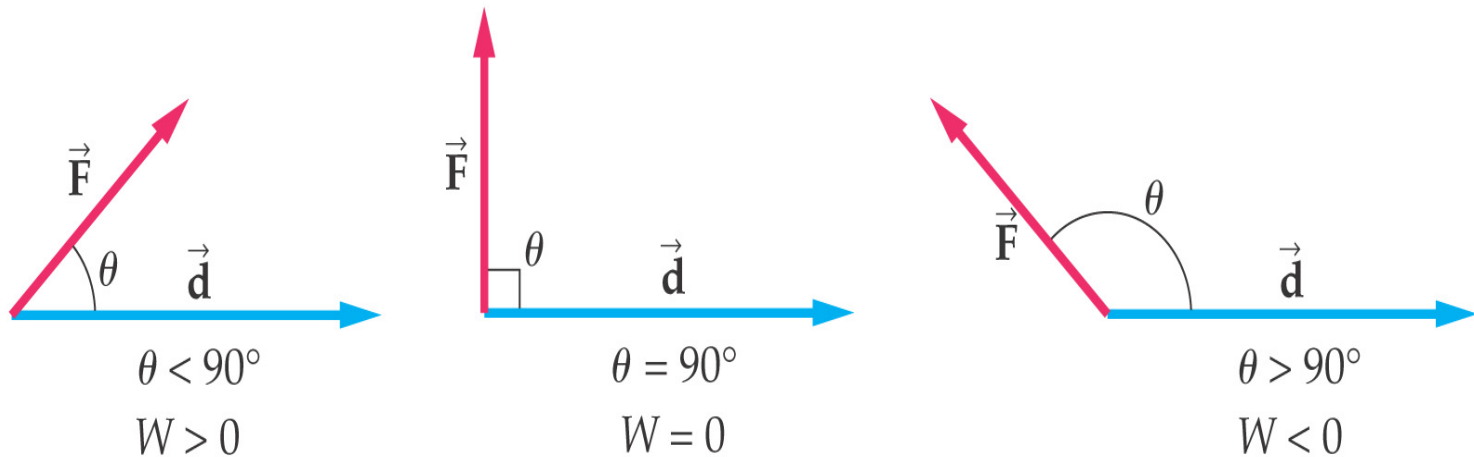
$$= 15 \times 9.8 \times 5$$

$$= 735 \text{ J}$$

# Negative work and positive work

The work done may be positive, zero, or negative, depending on the angle between the force and the displacement:

- (i) *Work is positive if the force has a component in the direction of motion*  
( $-90 < \theta < 90$ )
- (ii) *Work is zero if the force has no component in the direction of motion*  
( $\theta = \pm 90^\circ$ )
- (iii) *Work is negative if the force has a component opposite to the direction of motion*  
( $90^\circ < \theta < 270^\circ$ )



# - & + Work

- Positive work is performed when the direction of the force and the direction of motion are the same

- ascent phase of the bench press
- Throwing a ball
- push off (upward) phase of a jump

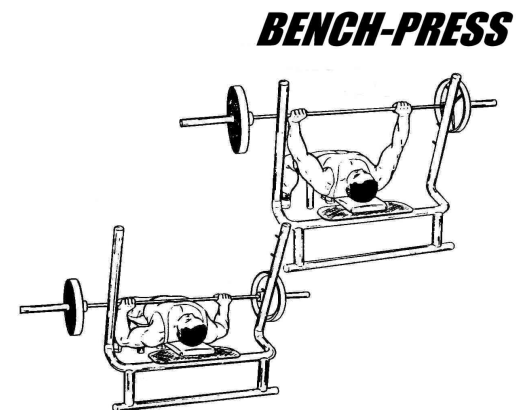
- Negative work is performed when the direction of the force and the direction of motion are the *opposite*

- descent phase of the bench press
- catching
- landing phase of a jump

# Calculate Work

- a) During the ascent phase of a rep of the bench press, the lifter exerts an *average* vertical force of 1000 N against a barbell while the barbell moves 0.8 m upward. How much work did the lifter do to the barbell?
- b) During the *descent* phase of a rep of the bench press, the lifter exerts an *average* vertical force of 1000 N against a barbell while the barbell moves 0.8 m *downward*. How much work did the lifter do to the barbell?

Answer : a)  $W = 800J$   
b)  $W = -800J$



## Problems ( Page 211 )

Problem 5: Children in a tree house lift a small dog in a basket 4.7 m up to their house. If it takes 201 J of work to do this, what is the combined mass of the dog and basket?

(Answer: 4.36 kg)



Problem 6: Early one October, you go to a pumpkin patch to select your Halloween pumpkin. You lift the 3.2 kg pumpkin to a height of 1.2 m, then carry it 50.0 m ( on level ground ) to check-out stand. (a) Calculate the work you do on the pumpkin as you lift it from the ground. (b) How much work do you do on the pumpkin as you carry it from the field?

(Ans. 37.4

Answer : a) Work = 37.7J , b) Work = zero



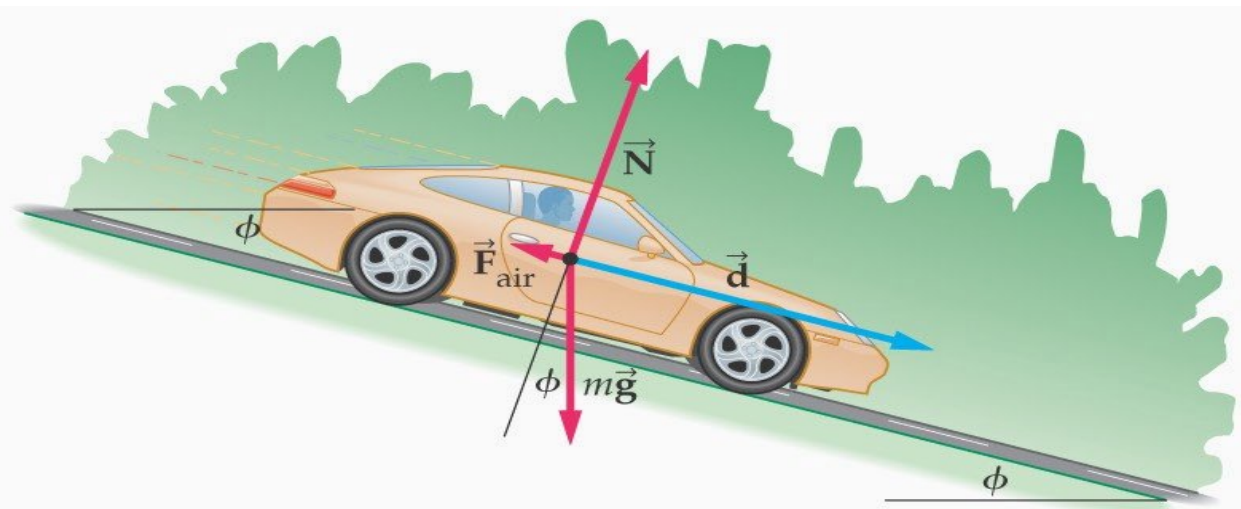
## 7-1-- Work Done by a Constant Force

If there is more than one force acting on an object, we can find the work done by each force, and also the work done by the net force:

$$W_{\text{total}} = (F_{\text{total}} \cos \theta) d = F_{\text{total}} d \cos \theta$$

If  $\vec{F}_1$  does work  $W_1$ ,  $\vec{F}_2$  does work  $W_2$ ,  $\vec{F}_3$  does work  $W_3$ , and so on, the total work is (7-5)

$$W_{\text{total}} = W_1 + W_2 + W_3 + \dots = \sum W$$



## Example 7-3

A car of mass 1550 kg coasts down a hill at an angle of  $5.0^\circ$  below the horizontal. The car is acted on by three forces :

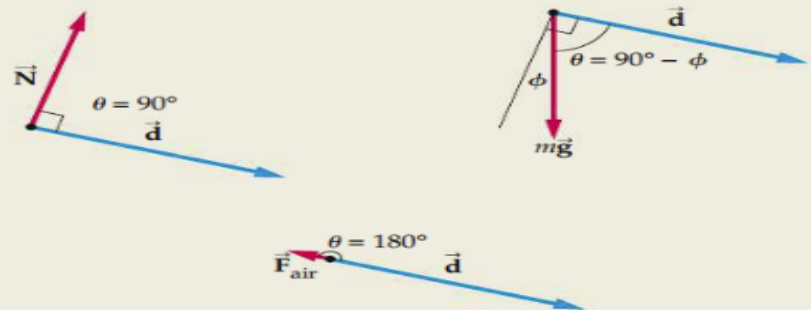
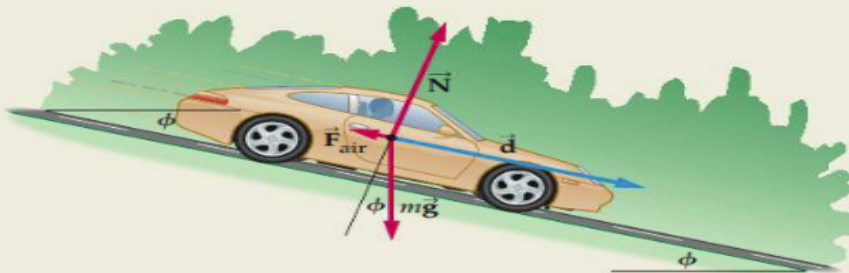
- (i) the normal force  $\mathbf{N}$  exerted by the road,
  - (ii) a force due to air resistance ,  $F_{\text{air}}$ , and
  - (iii) the force of gravity,  $mg$ .
- Find the work done by each of the forces and the total work done on the car as it travels a distance 20.4 m down the hill. The force due to air resistance is 15.0 N.

### EXAMPLE 7-3 A COASTING CAR I

A car of mass  $m$  coasts down a hill inclined at an angle  $\phi$  below the horizontal. The car is acted on by three forces: (i) the normal force  $\vec{N}$  exerted by the road, (ii) a force due to air resistance,  $\vec{F}_{\text{air}}$ , and (iii) the force of gravity,  $m\vec{g}$ . Find the total work done on the car as it travels a distance  $d$  along the road.

#### PICTURE THE PROBLEM

Because  $\phi$  is the angle the slope makes with the horizontal, it is also the angle between  $m\vec{g}$  and the downward normal direction, as was shown in Figure 5-15. It follows that the angle between  $m\vec{g}$  and the displacement  $\vec{d}$  is  $\theta = 90^\circ - \phi$ . Our sketch also shows that the angle between  $\vec{N}$  and  $\vec{d}$  is  $\theta = 90^\circ$ , and the angle between  $\vec{F}_{\text{air}}$  and  $\vec{d}$  is  $\theta = 180^\circ$ .



#### STRATEGY

For each force we calculate the work using  $W = Fd \cos \theta$ , where  $\theta$  is the angle between that particular force and the displacement  $\vec{d}$ . The total work is the sum of the work done by each of the three forces.

# Different solution for example 7-3

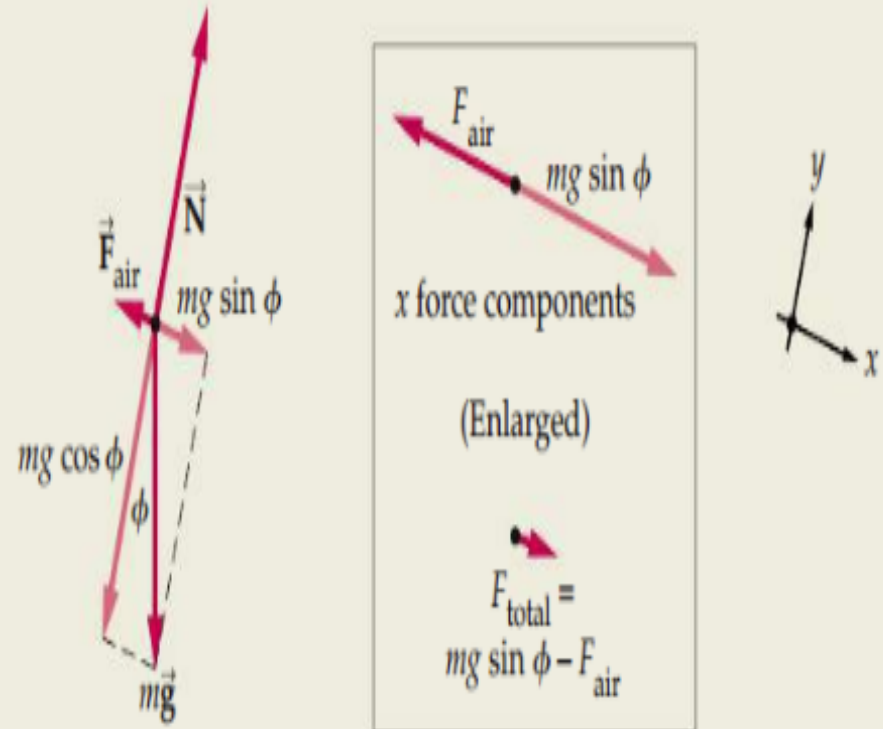
Consider the car described in **Example 7-3**. Calculate the total work done on the car using  $W_{\text{total}} = F_{\text{total}}d \cos \theta$ .

## PICTURE THE PROBLEM

First, we choose the  $x$  axis to point down the slope, and the  $y$  axis to be at right angles to the slope. With this choice, there is no acceleration in the  $y$  direction, which means that the total force in that direction must be zero. As a result, the total force acting on the car is in the  $x$  direction. The magnitude of the total force is  $mg \sin \phi - F_{\text{air}}$ , as can be seen in our sketch.

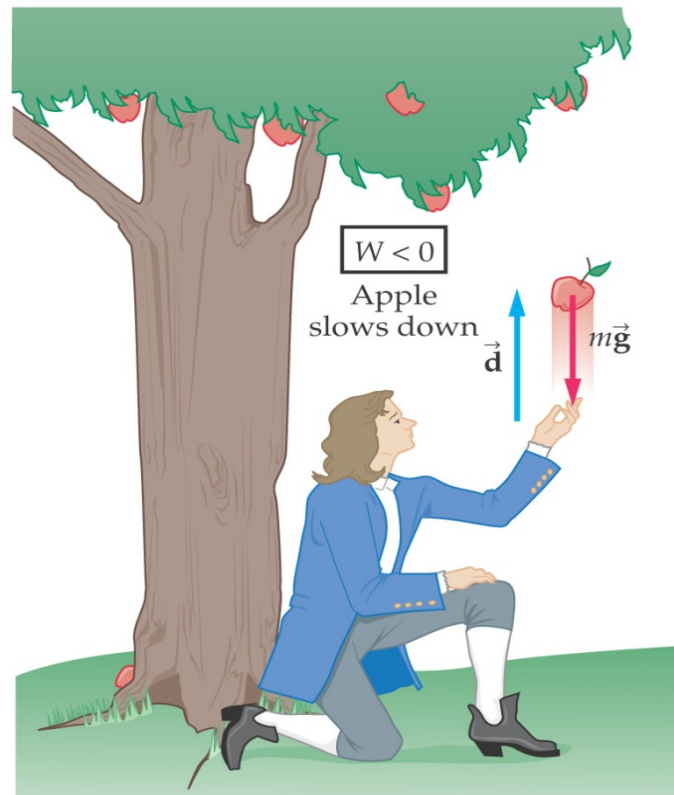
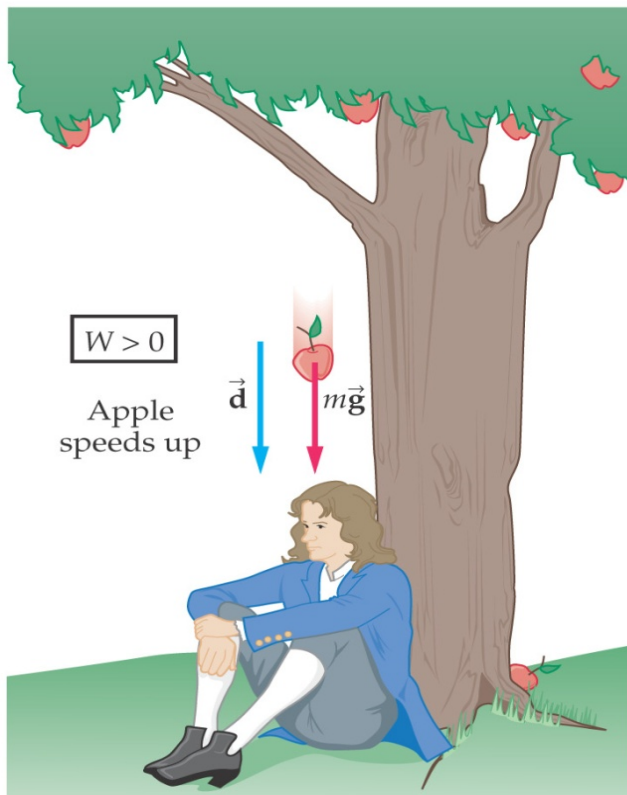
## STRATEGY

We begin by finding the  $x$  component of each force vector and then summing them to find the total force acting on the car. As can be seen from the figure, the total force points in the positive  $x$  direction; that is, in the same direction as the displacement. Therefore, the angle  $\theta$  in  $W = F_{\text{total}}d \cos \theta$  is zero.



# 7-2 --Kinetic Energy and the Work-Energy Theorem

When positive work is done on an object, its speed increases; when negative work is done, its speed decreases.



# 7-2-- Kinetic Energy and the Work-Energy Theorem

After algebraic manipulations of the equations of motion, we find:

Work-Energy Theorem: The total work done on an object is equal to its change in kinetic energy.

$$W_{\text{total}} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Therefore, we define the kinetic energy:

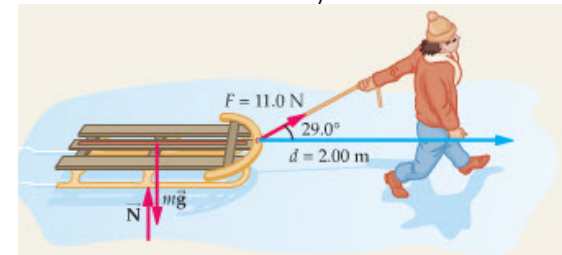
(7-6)

$$K = \frac{1}{2}mv^2$$

# Problems

**Exercise 7-1 :** A boy exerts a force of 11 N at 29 degrees above the horizontal on a 6.4 kg sled. Find (a) the work done by the boy and (b) the final speed of the sled after it moves 2 m., assuming the sled starts with an initial speed of 0.5 m/s and slides horizontally without friction.

**Answer:**  $W = 19.2\text{J}$  ,  $V_f = 2.50\text{m/s}$



**Exercise 7-2 :** A truck moving at 15 m/s has kinetic energy of  $4.2 \times 10^5\text{J}$ .

(a) What is the mass of the truck?

(b) By what multiplicative factor does the kinetic energy increase if its speed is doubled?

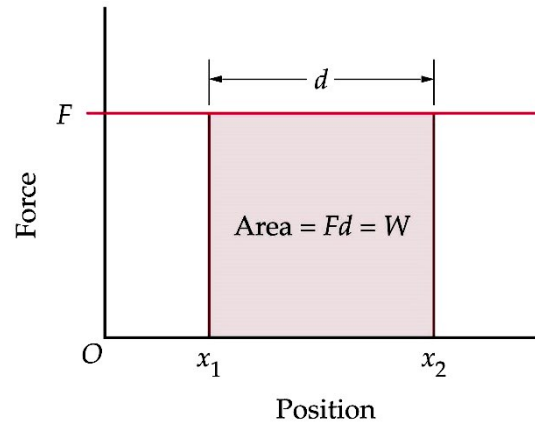
**Answer:** a)  $m = 3733\text{ Kg}$  . b) K.E increases by four

**Exercise 7-3 :** How much work is required for 74 kg sprinter to accelerate from rest to 2.2 m/s?

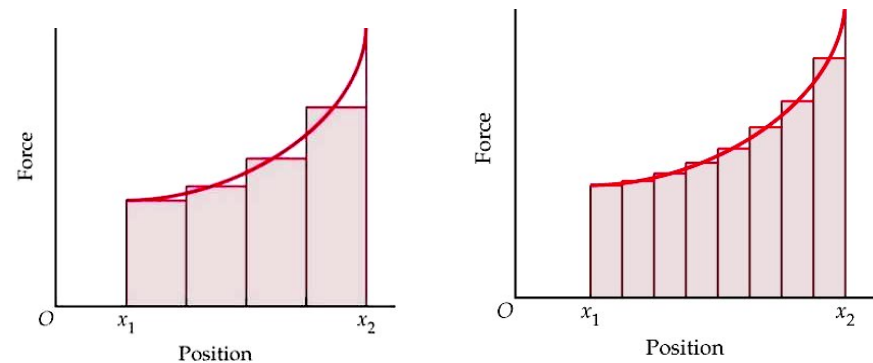
**Answer:**  $W = 180\text{J}$

# 7-3 Work Done by a Variable Force

- If the force is constant, we can graphically interpret the work done ( $W = F d$ ) as the area of a rectangle  $F$  tall and  $d$  wide



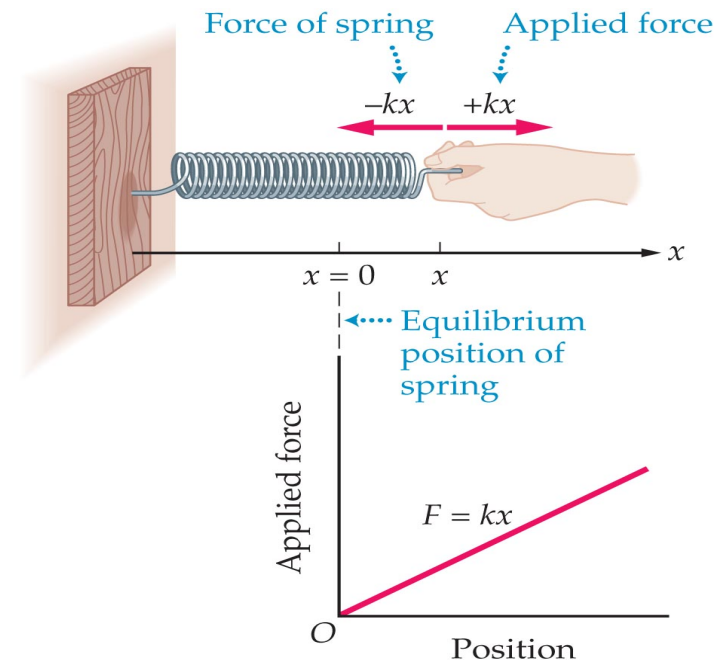
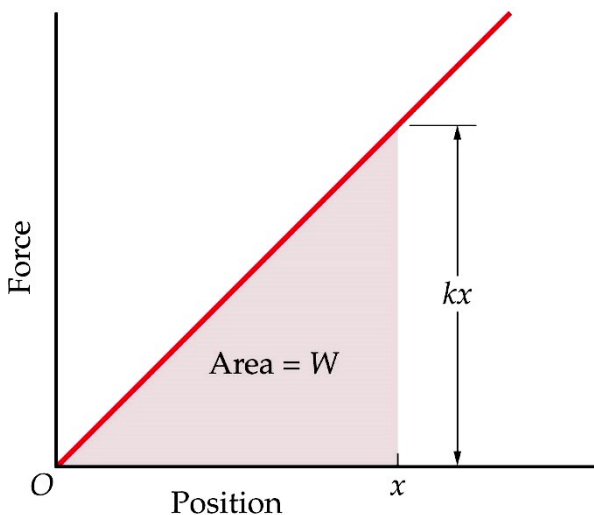
- We can then approximate a continuously varying force by a succession of constant values:
- work = area under the curve**



# 7-3 Work Done by a Variable Force

- The force needed to stretch a spring an amount  $x$  is  $F = kx$ .
- Therefore, the work done in stretching the spring is

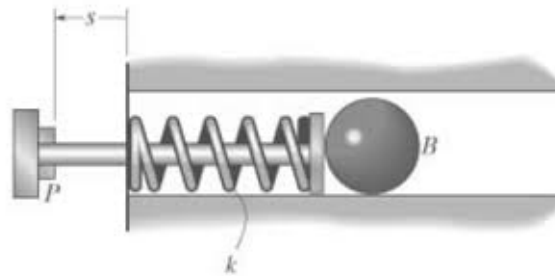
$$W = \frac{1}{2}kx^2$$



# Problems

- Exercise 4
- The spring in a pinball launcher has a force constant of 405 N/m. How Much work is required to compress the spring a distance of 3.00 cm?

- *Answer :  $W = 0.18 \text{ J}$*



# 7-4 ---Power

**Power is a measure of the rate at which work is done:**

$$P = \frac{W}{t}$$

SI unit:

- J/s = watt, W
- Kg m<sup>2</sup> / s<sup>2</sup> /s
- N m / s

1 horsepower = 1 hp = 746 W

**Power is the rate that we use energy.**

If an object is moving at a constant speed in the face of friction, gravity, air resistance, and so forth, the power exerted by the driving force can be written:

Power = Work or Energy / Time

$$P = \frac{Fd}{t} = F\left(\frac{d}{t}\right) = Fv$$

**Power is a scalar quantity**



# Power Calculation

- A 5 Kg Cart is pushed by a 30 N force against friction for a distance of 10m in 5 seconds. Determine the Power needed to move the cart.

Answer :

- $P = 60 \text{ N.m /s}$
- $P = 60 \text{ watts}$

# Examples of power

## Power



Light  
bulb

Electrical energy  $\longrightarrow$  light  
and heat at 100 J/s.

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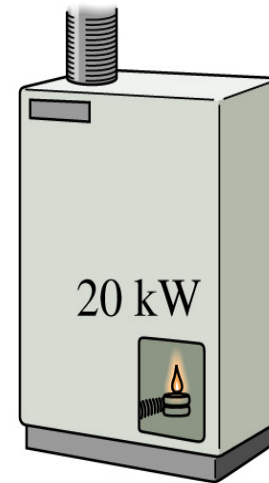


Athlete

$\frac{1}{2}$  hp

Chemical energy of glucose  
and fat  $\longrightarrow$  mechanical  
energy at  $\approx 350$  J/s  $\approx \frac{1}{2}$  hp.

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Gas  
furnace

Chemical energy of gas  $\longrightarrow$   
thermal energy at 20,000 J/s.

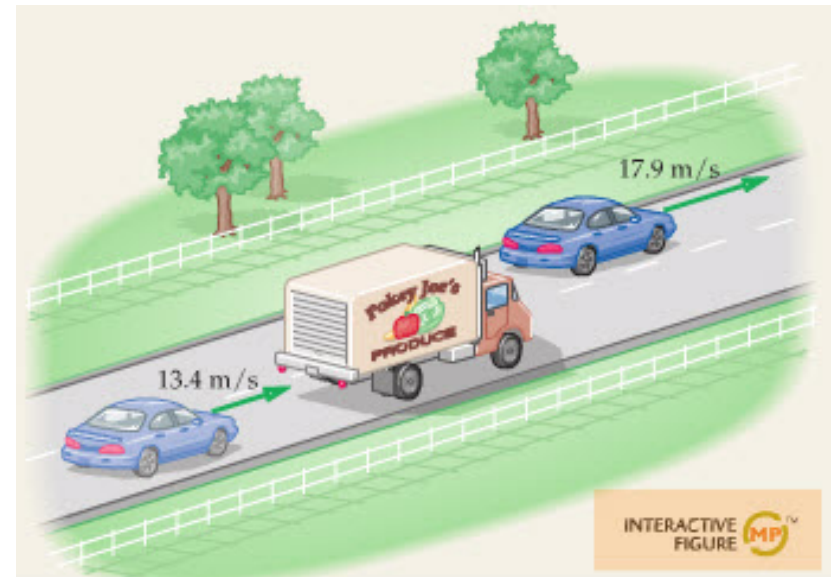
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# 7-4 Power

- **Example 8:** To pass a slow moving truck you want your  $1.3 \times 10^3$  kg car to accelerate from 12 m/s to 18 m/s in 3 seconds. What is the minimum power required for this pass.

Answers :

- $\Delta K = 9.16 \times 10^4 \text{ J}$
- $P = 3.05 \times 10^4 \text{ watt}$



# Summary of Chapter 7

- If the force is constant and parallel to the displacement, work is force times distance
- If the force is not parallel to the displacement,

$$W = (F \cos \theta)d = Fd \cos \theta$$

- The total work is the work done by the net force:

$$W_{\text{total}} = (F_{\text{total}} \cos \theta)d = F_{\text{total}}d \cos \theta$$

# Summary of Chapter 7

- SI unit of work: the joule, J
- Total work is equal to the change in kinetic energy:

$$W_{\text{total}} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

where

$$K = \frac{1}{2}mv^2$$

# Summary of Chapter 7

- Work done by a spring force:

$$W = \frac{1}{2}kx^2$$

- Power is the rate at which work is done:

$$P = \frac{W}{t}$$

- SI unit of power: the watt, W

# Exersises

- EXERCISE:7-1 (Assignment)

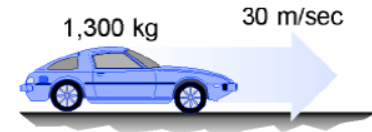
**Problem 4:** A farmhand pushes a 26 kg bale of hay 3.9 m across the floor of a barn. If she exerts a horizontal force of 88 N on the bay, how much work has she done? (Answer 343.2)

Answer : Work = 343.2J

A car with a mass of 1,300 kg is going straight ahead at a speed of 30 m/sec (67 mph). The brakes can supply a force of 9,500 N.

- Calculate:

- a) The kinetic energy of the car. ( KE: 585000 J )
- b) The distance it takes to stop. ( 61.58 m)



**Problem 19:** How much work is needed for a 73 kg runner to accelerate from rest to 7.7 m/s?

Answer : (Work=2164.08 J)

**Problem 21:** A 9.5 g bullet has a speed of 1.3 km/s. (a) What is its kinetic energy in Joules? (b) What is the bullet's kinetic energy if its speed is halved? (c) If its speed is doubled?

Answers: ( 8027.5 J, 2006.87 J, 32110 J )

# Exercises

- **Exercise :** At what rate is work being done when a 26 N horizontal force moves a book 2.5 m horizontally in 13 s?

(Answer:  $W = 65 \text{ J} \Rightarrow \text{Power} = 5 \text{ Watts}$ )

**Exercise :** What is the power of a machine that can produce 3600 J of work in 2 minutes?

(Answer:  $\text{Power} = \quad$  )

**Exercise :** How much work is done by a 25000 watt car engine in 5 minutes? **Example:** A spring is hanged vertically from the ceiling of the room. When a block of mass 7 kg is hanged from the lower end, the spring is stretched by 14 cm. calculate the spring constant of this spring

(Answer:  $k = \quad$  )

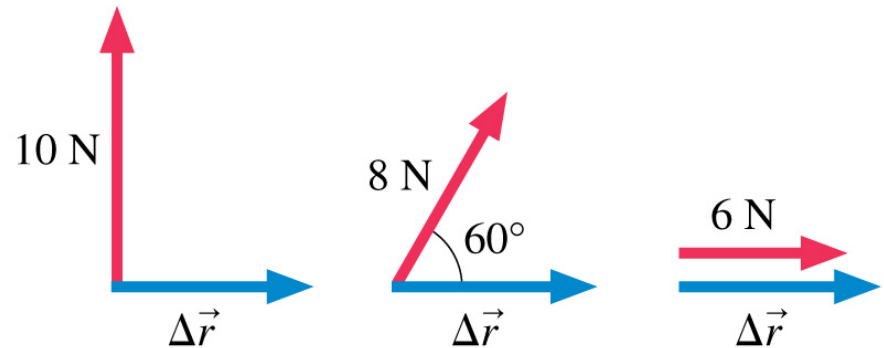
**Example:** A 4.1 kg box of books is lifted vertically from rest a distance of 1.6 m with a constant upward applied force of 52.7 N. Find the work done by the applied force, the work done by gravity, and the final speed of the box.

(Answer:  $\text{Work} = \quad , v_f \quad$  )

# Class Question

Which force does the most work?

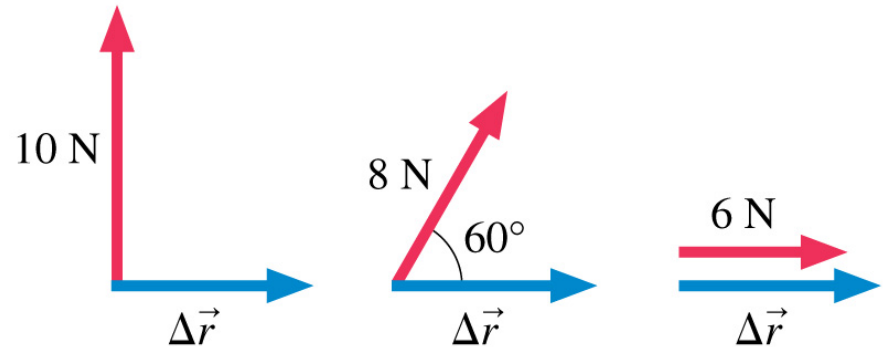
1. The 6 N force.
2. The 8 N force.
3. The 10 N force.
4. They all do the same amount of work.



# Class Question

Which force does the most work?

- ✓ 1. The 6 N force.
- 2. The 8 N force.
- 3. The 10 N force.
- 4. They all do the same amount of work.




# *Class Question*

A crane lowers a steel girder into place at a construction site. The girder moves with constant speed. Consider the work  $W_g$  done by gravity and the work  $W_T$  done by the tension in the cable. Which of the following is correct?

1.  $W_g$  is positive and  $W_T$  is positive.
2.  $W_g$  is negative and  $W_T$  is negative.
3.  $W_g$  is positive and  $W_T$  is negative.
4.  $W_g$  and  $W_T$  are both zero.
5.  $W_g$  is negative and  $W_T$  is positive.

# *Class Question*

A crane lowers a steel girder into place at a construction site. The girder moves with constant speed. Consider the work  $W_g$  done by gravity and the work  $W_T$  done by the tension in the cable. Which of the following is correct?

1.  $W_g$  is positive and  $W_T$  is positive.
2.  $W_g$  is negative and  $W_T$  is negative.
-  3.  **$W_g$  is positive and  $W_T$  is negative.**
4.  $W_g$  and  $W_T$  are both zero.
5.  $W_g$  is negative and  $W_T$  is positive.