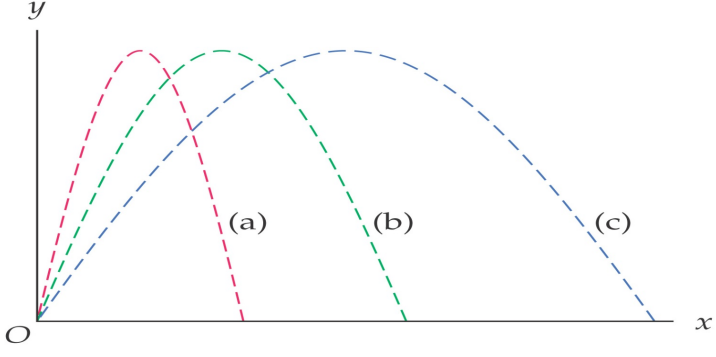
Chapter - 4

Two-Dimensional Kinematics

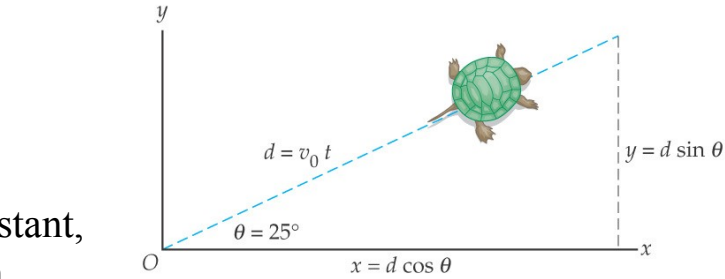


Units of Chapter 4

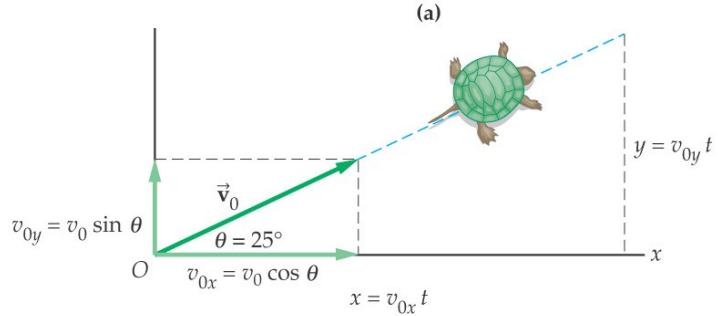
- Motion in Two Dimensions
- Projectile Motion: Basic Equations
- Zero Launch Angle
- General Launch Angle
- Projectile Motion: Key Characteristics

4-1- Motion in Two Dimensions

If velocity is constant, motion is along a straight line:



(a)



(b)

Example: 4-1

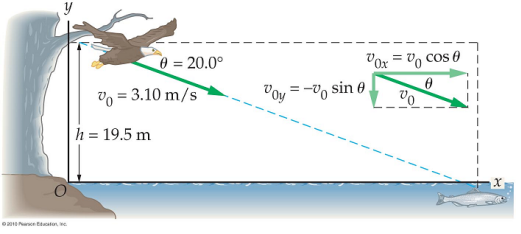
An eagle perched on a tree limb 19.5 m above the water spots at a fish swimming near the surface. The eagle pushes off from the branch and descends towards the water. By adjusting its body in flight, the eagle maintains a constant speed of 3.10 m/s at an angle of 20.0° below the horizontal.

A) How long does it take for the eagle to reach the water?

Ans. 18.4 s

B) How far has the eagle travelled in the horizontal direction when it reaches the water?

Ans. 53.5 m



4-1- Motion in Two Dimensions

Motion in the *x*- and *y*-directions should be solved separately:

TABLE 4–1 Constant-Acceleration Equations of Motion

Position as a function of time	Velocity as a function of time	Velocity as a function of position
$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	$v_x = v_{0x} + a_x t$	$v_x^2 = v_{0x}^2 + 2a_x \Delta x$
$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$	$v_y = v_{0y} + a_y t$	$v_y^2 = v_{0y}^2 + 2a_y \Delta y$

Problem 2

A sails boat runs before the wind with a constant speed of 4.2 m/s in the direction of 32° North / West.

How far: **(a) W**est and **(b)** North has the sailboat travelled in 25 min.

- Answers : **(a)** - 5343 m , **(b)** 3338 m

Problem 3

As you walk to class with a constant speed of 1.75 m/s, you are moving in a direction that is 18° north of east.

How much time does it take to change your displacement by **(a)** 20.0 m East or **(b)** 30.0 m North.

Answers : **(a)** 12 s , **(b)** 55 s

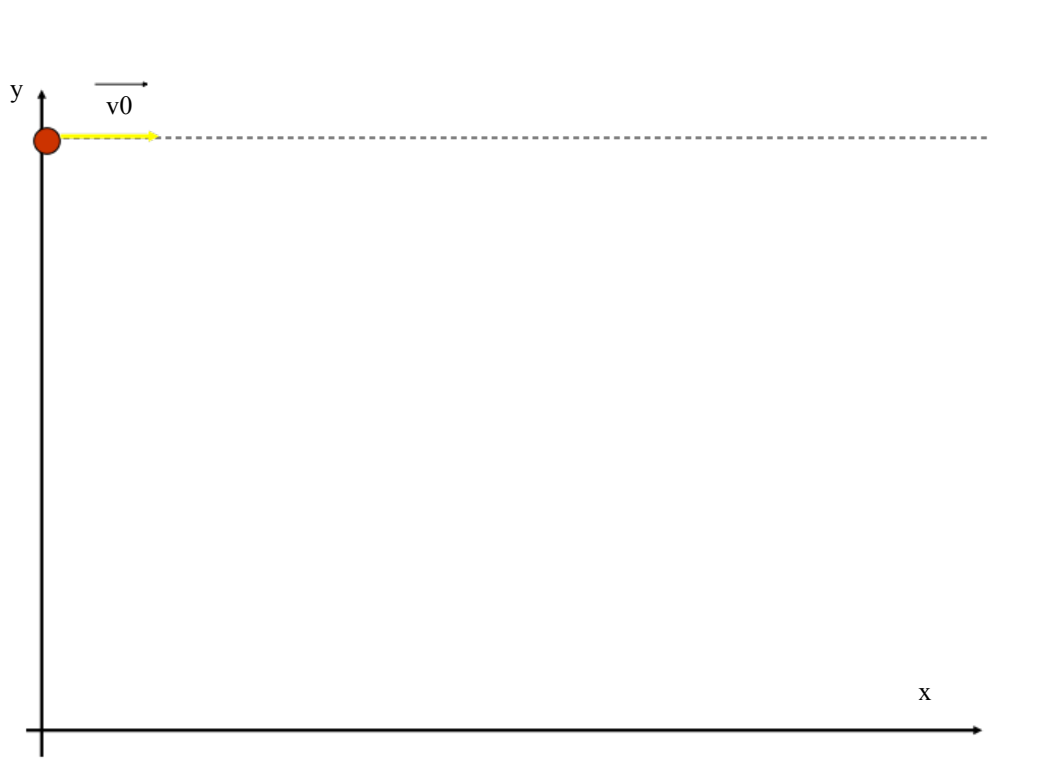
Part 1.

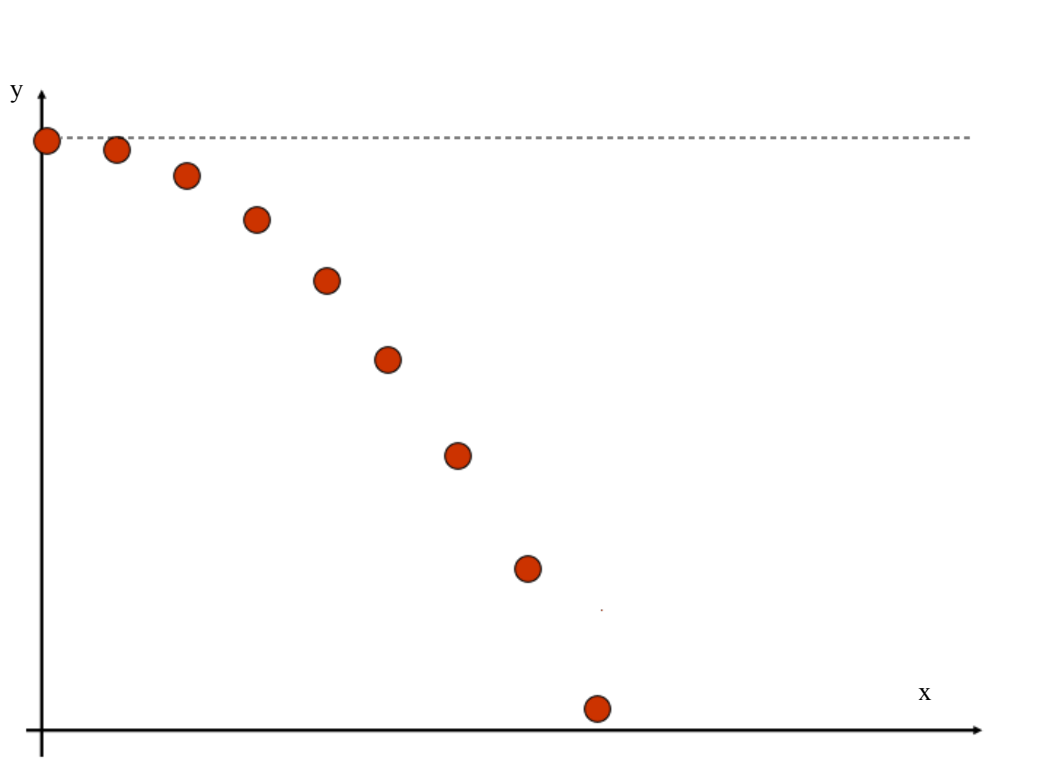
Motion of Objects Projected Horizontally

PROJECTILE is a body which is thrown horizontally or at an angle relative to the horizontal which follows a curved path called **trajectory**

Examples:

- Ball being thrown,
- Water coming out of the hose,
- A bullet fired from a gun,
- Arrow shot from a bow,
- Cannonballs,
- Fountains.


y
v0
x


y
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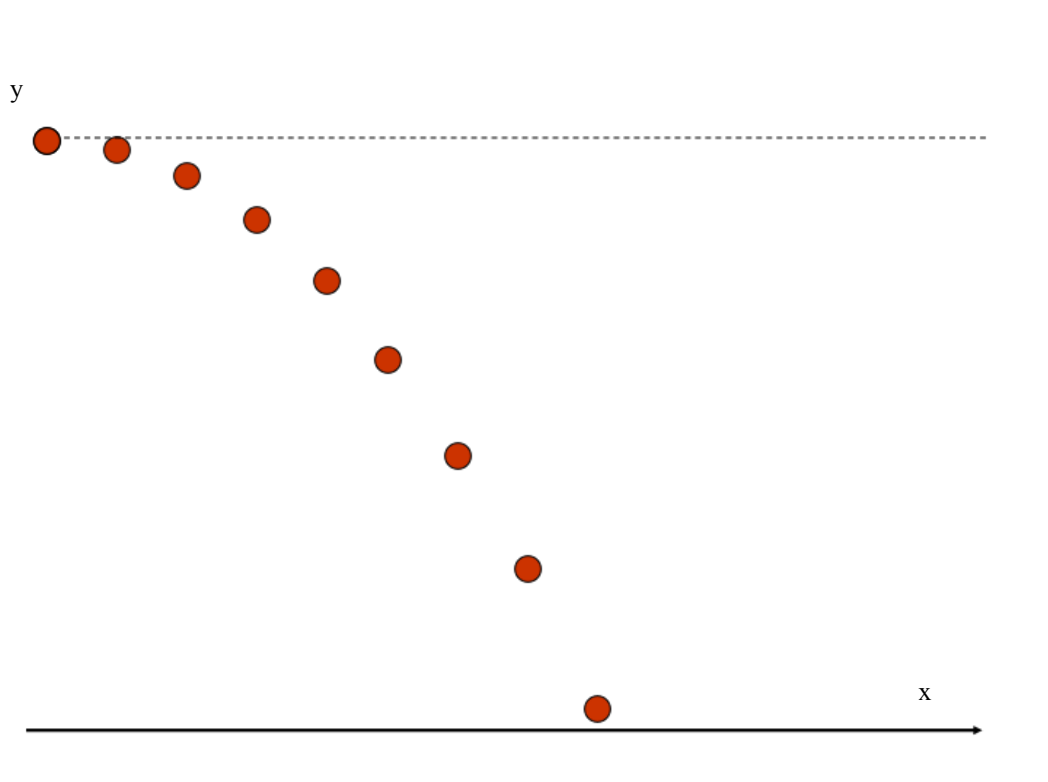
y
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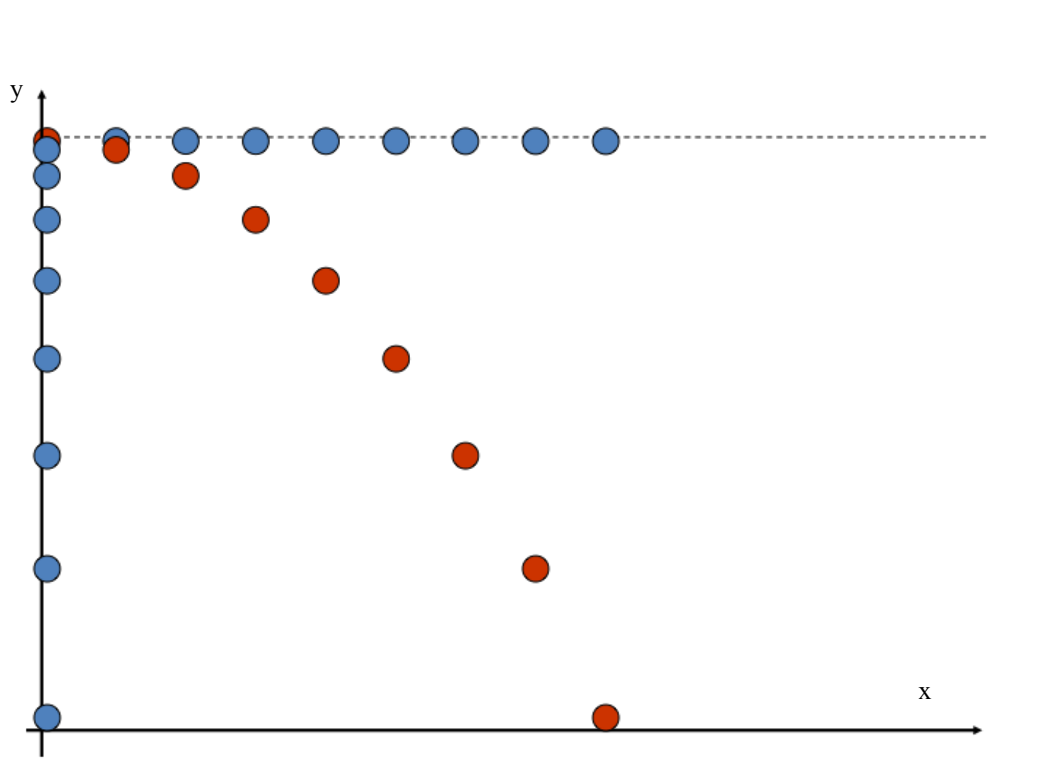
y
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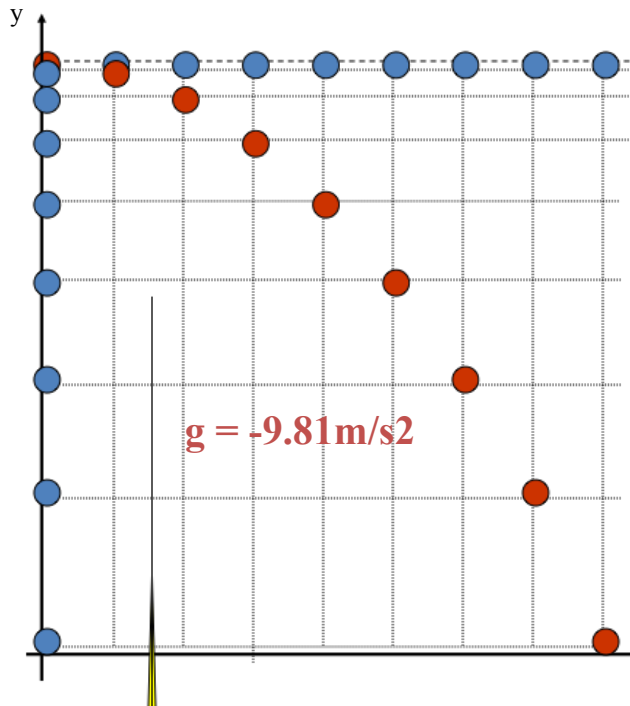


- Motion is accelerated
- Acceleration is constant, and downward a = g = -9.81m/s2
- The horizontal (x) component of velocity is constant
- The horizontal and vertical motions are independent of each other, but they have a common time

ANALYSIS OF MOTION

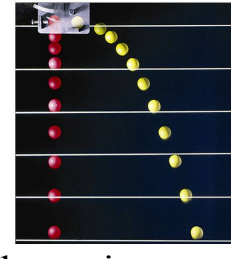
- x-direction (horizontal): uniform motion
- y-direction (vertical): accelerated motion

QUESTIONS:

- What is the trajectory?
- What is the horizontal range?
- What is the final velocity of the projectile?

4-2- Projectile Motion: Basic Equations

Projectile is an object that is launched into motion and then allowed to follow a path determined solely by the influence of gravity.



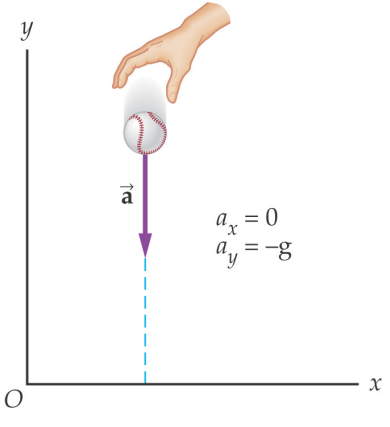
Assumptions:

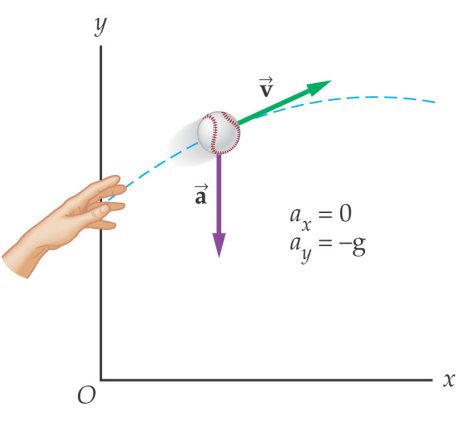
- Ignore air resistance
- g = 9.81 m/s2, downward
- Ignore Earth's rotation
- With these assumptions, an object in projectile motion will follow a parabolic path

If y-axis points upward, acceleration in x-direction is zero and acceleration in y-direction is -9.81 m/s2

4-2- Projectile Motion: Basic Equations

The acceleration is independent of the direction of the velocity:





4-2- Projectile Motion: Basic Equations

The basic equations of projectile motion:

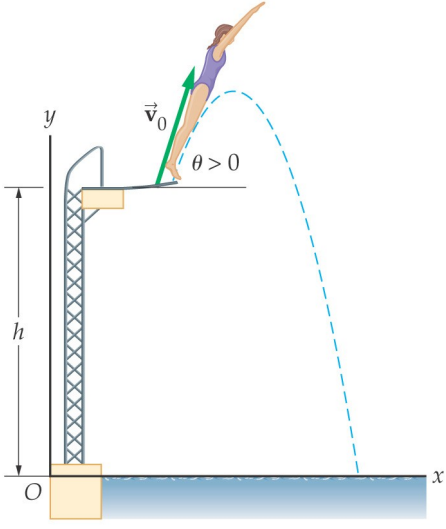
$$x = x_0 + v_{0x}t$$
$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

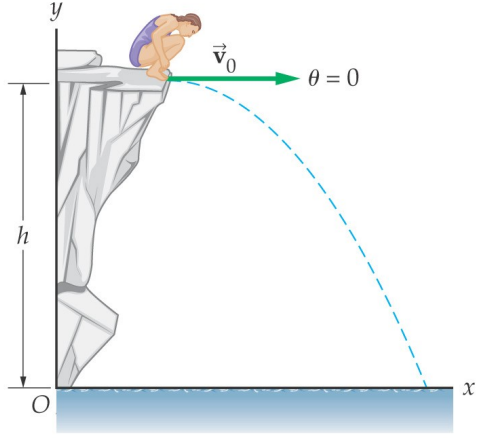
$$v_x = v_{0x}$$
$$v_y = v_{0y} - gt$$

$$v_x^2 = v_{0x}^2$$
$$v_y^2 = v_{0y}^2 - 2g\Delta y$$

4-3- Zero Launch Angle

Launch angle: direction of initial velocity with respect to horizontal





4-3- Zero Launch Angle

In this case, the initial velocity in the *y*-direction is zero. Here are the equations of motion, with $x0 = 0$ and $y0 = h$:

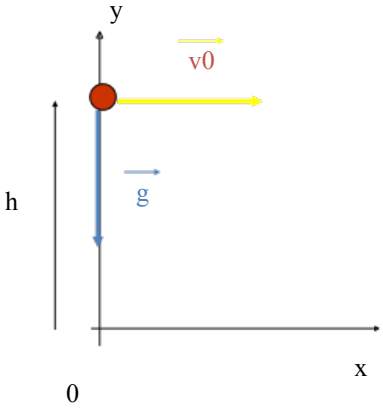
$$x = v_0 t$$

$$y = h - \tfrac{1}{2} g t^2$$

$$v_x = v_0 = \text{constant} \qquad v_x^2 = v_0^2 = \text{constant}$$

$$v_y = -gt \qquad v_y^2 = -2g\Delta y$$

Frame of reference:



Equations of motion:

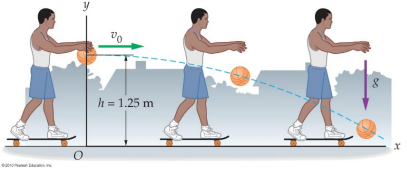
	X Uniform m.	Y Accel. m.
acceleration	ax = 0	ay = -g = -9.81 m/s2
velocity	vx = v0	vy = -g t
displacement	x = v0 t	y = h - ½ g t2

Example 4-3

A person skateboarding with a constant speed of 1.30 m/s releases a ball from a height of 1.25 m above the ground. Given that x0 = 0 and y0 = h = 1.25 m.

Find x and y for

- (a) t = 0.250 s
- (b) t= 0.500s



Find the velocity, speed and direction of motion of ball at t=0.500 s

- Answers : (a) x= 0.325 m , y= 0.943 m
  (b) x= 0.650 m , y= 0.0238 m

vx= 1.30 m/s, vy= -4.91 m/s, Speed=5.08 m/s, Direction= -75.2o

Problem: 12

A diver runs horizontally off the end of diving board with an initial speed of 1.85 m/s. If the diving board id 3.00 m above the water, what is the divers speed just before she enters the water?

Answers :

- Vy2 = 58.9 m**2**/s2
- Speed = 7.89 m/s

4-4- General Launch Angle

In general, $v0x = v0 \cos\theta$ and $v0y = v0 \sin\theta$, xo=0 , yo=0

This gives the equations of motion:

$$x = (v_0 \cos\theta)t$$

$$y = (v_0 \sin\theta)t - \tfrac{1}{2}gt^2$$

$$v_x = v_0 \cos\theta \qquad v_x^2 = v_0^2 \cos^2\theta$$

$$v_y = v_0 \sin\theta - gt \qquad v_y^2 = v_0^2 \sin^2\theta - 2g\Delta y$$

Exercise: 4-1

A projectile is launch from the origin with an initial speed of 20.0m/s at an angle of 35.00 above the horizontal. Find the x and y positions of the projectile at times

(a) t= 0.50 s Ans. x= 8.19 m, y=4.51 m

(b) t= 1.00 s Ans. x= 16.4 m, y= 6.57 m

(c) t= 1.50 s Ans. x= 24.6 m, y=6.17 m

Exercise 4-2

Referring to Example 4-1, find the velocity of the projectile at times

(a) t= 0.50 s

Ans. **v**= (16.4 m/s)i + (6.57 m/s)j

(b) t= 1.00 s

Ans. **v** = (16.4 m/s)i +(1.66 m/s)j

(c) t= 1.50 s

Ans. **v**=(16.4 m/s)i + (-3.24 m/s)j

Problem 31

A cork shoots out of a champagne bottle at an angle of 35.0° above the horizontal. If the cork travels a horizontal distance of 1.30 m in 1.25 s, what was its initial speed.

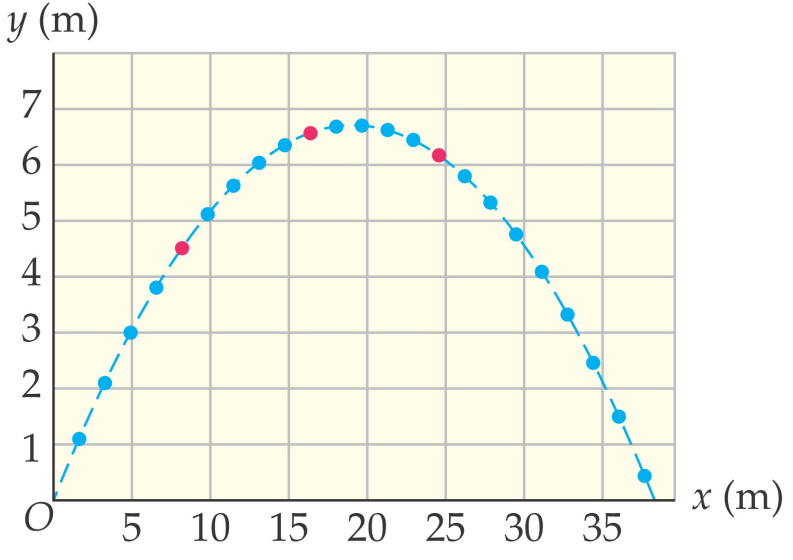
Answers:

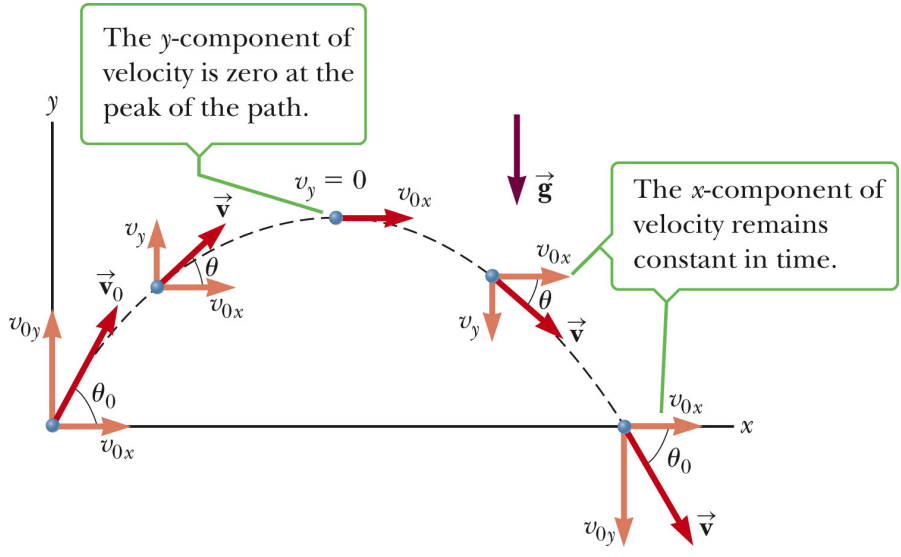
Vx = 1.04 m/s

V0 = 1.27 m/s

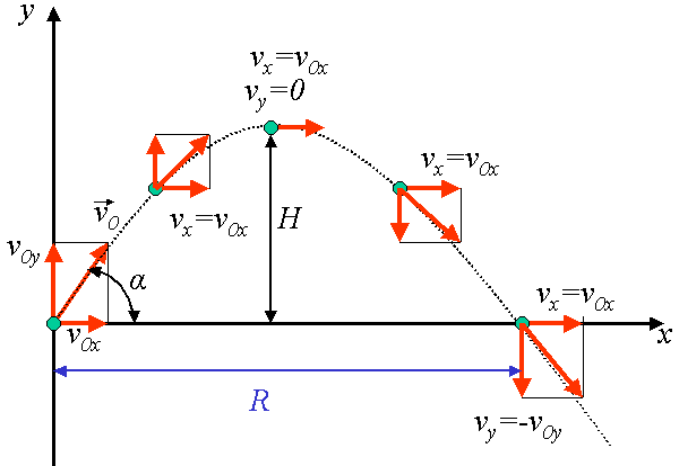
4-4- General Launch Angle

Snapshots of a trajectory; red dots are at $t = 1$ s, $t = 2$ s, and $t = 3$ s




The y-component of velocity is zero at the peak of the path.
The x-component of velocity remains constant in time.
y
x
$v_y = 0$
v_{0x}
$\vec{\mathbf{g}}$
$\vec{\mathbf{v}}$
$\vec{\mathbf{v}}_0$
v_y
θ
v_{0y}
θ_0

4-5- Projectile Motion: Key Characteristics



4-5- Projectile Motion: Key Characteristics

RANGE

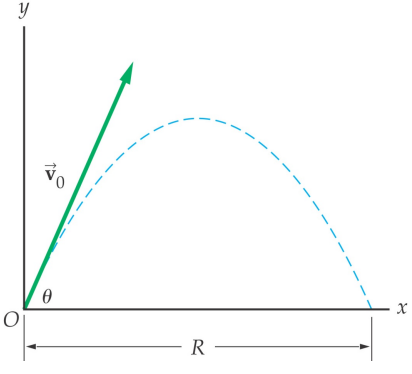
is the horizontal displacement of the projectile (dx)

MAXIMUM HEIGHT

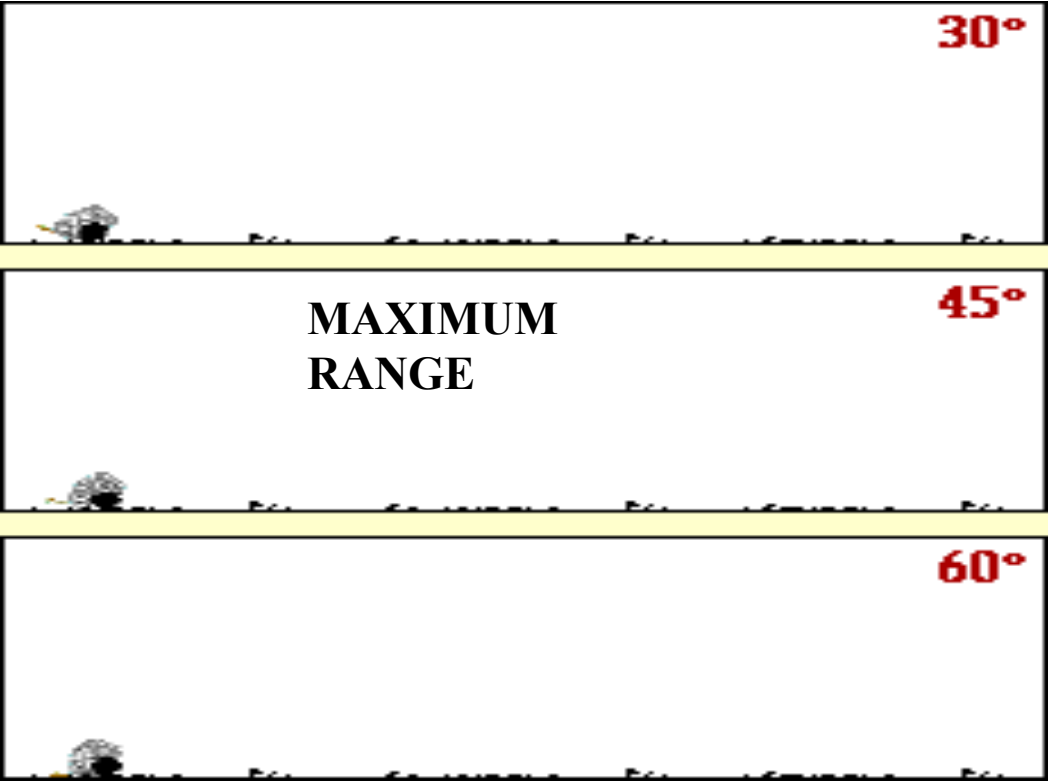
is the vertical displacement of the projectile (dy)

4-5- Projectile Motion: Key Characteristics

- **Range**: the horizontal distance a projectile travels
- If the initial and final elevation are the same:
- Range of projectile launched from origin with an initial speed v0 and a launch angle θ is:

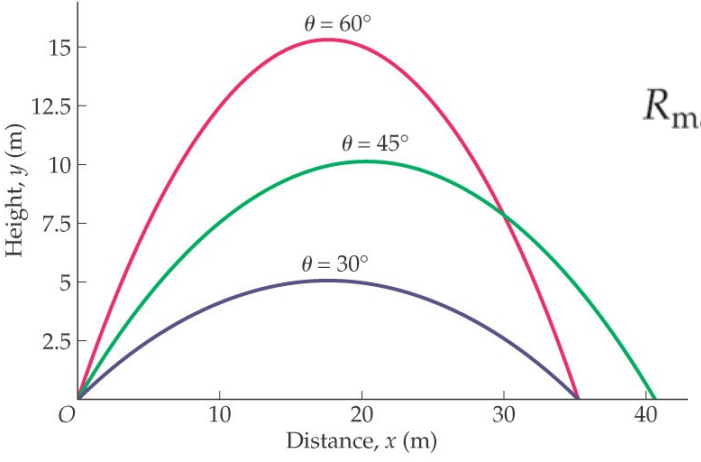


$$R = \left(\frac{{v_0}^2}{g}\right) \sin 2\theta$$


30°
MAXIMUM
RANGE
45°
60°

4-5- Projectile Motion: Key Characteristics

The range is a maximum when $\theta = 45°$:

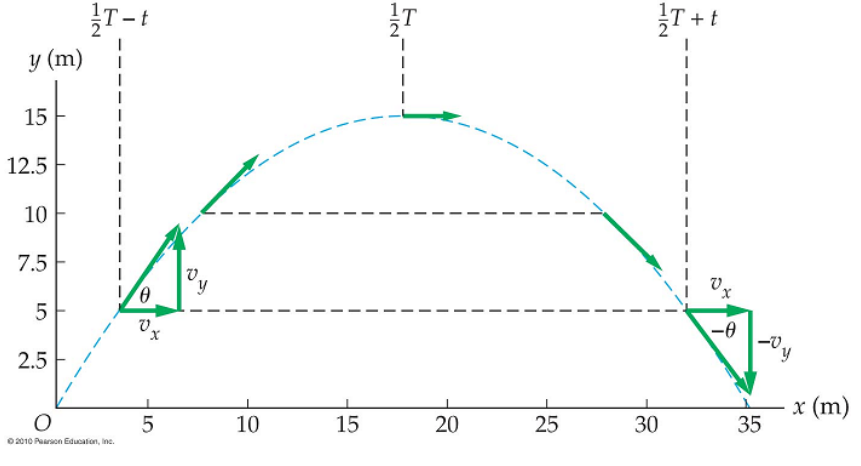


(b)

$$R_{\max} = \frac{{v_0}^2}{g}$$

4-5- Projectile Motion: Key Characteristics

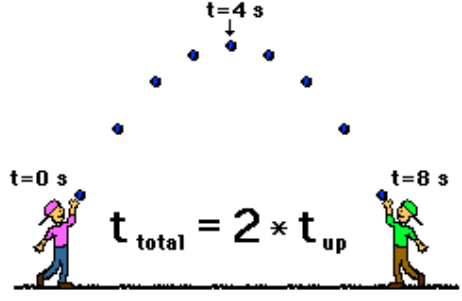
Symmetry in projectile motion: (concerning time and speed)



Time of flight and at the peak

- **1-Time of Flight :** The time ta
  launched and

$$t = (2$$



Maximum Height

- Maximum height of a p
site is

$$y_{max} = (v$$

Problems

Question 1: A body is projected and landed at the same level with a velocity of 20 m/s at 50o to the horizontal.

Find

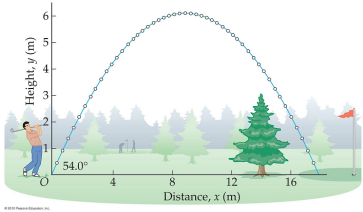
(i) Maximum height reached **Ans.= 11.97 m**
(ii) Time of flight **Ans.= 3.126 s**
(iii) Range of the projectile **Ans.= 40.196 m**

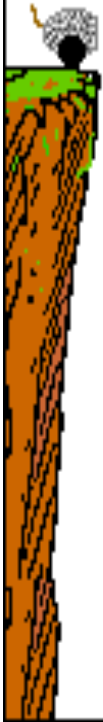
Example 5

- Chipping from the rough, a golfer sends a ball over a 3m high tree that is 14m away. The ball lands at the same level from which it was struck after traveling a horizontal distance of 17.8m-on green of course.

- (a) if the ball left the club 54.0 above the horizontal and landed on the green 2.24s later, what was its initial speed?
- (b)How high was the ball when it passed over the tree?

- Answers : a)13.5 m/s b) 4.03 m



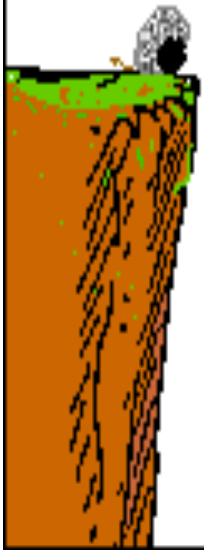


PROJECTILE I
(SIMPLE PROJECTILE)

$t = -- \ s$

$v_x = -- \ m/s \quad v_y = -- \ m/s$

PROJECTILE II (WITH ANGLE)



t= -- s

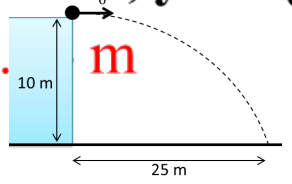
v_x= -- m/s v_y= -- m/s

Exercises

- **1)** John is on top of the building and ja
angle of 60° and with initial velocity 2(
after 2 s?

Vertical distance, $y = V_{oy}\, t - 1/2\, gt^2$

- Ans.= 15. m



2) A stone is thrown horizontally from
horizontal distance of 25 m before it hi

Summary of Chapter 4

- Components of motion in the x- and y-directions can be treated independently
- In projectile motion, the acceleration is $-g$
- If the launch angle is zero, the initial velocity has only an x-component
- The path followed by a projectile is a parabola
- The range is the horizontal distance the projectile travels