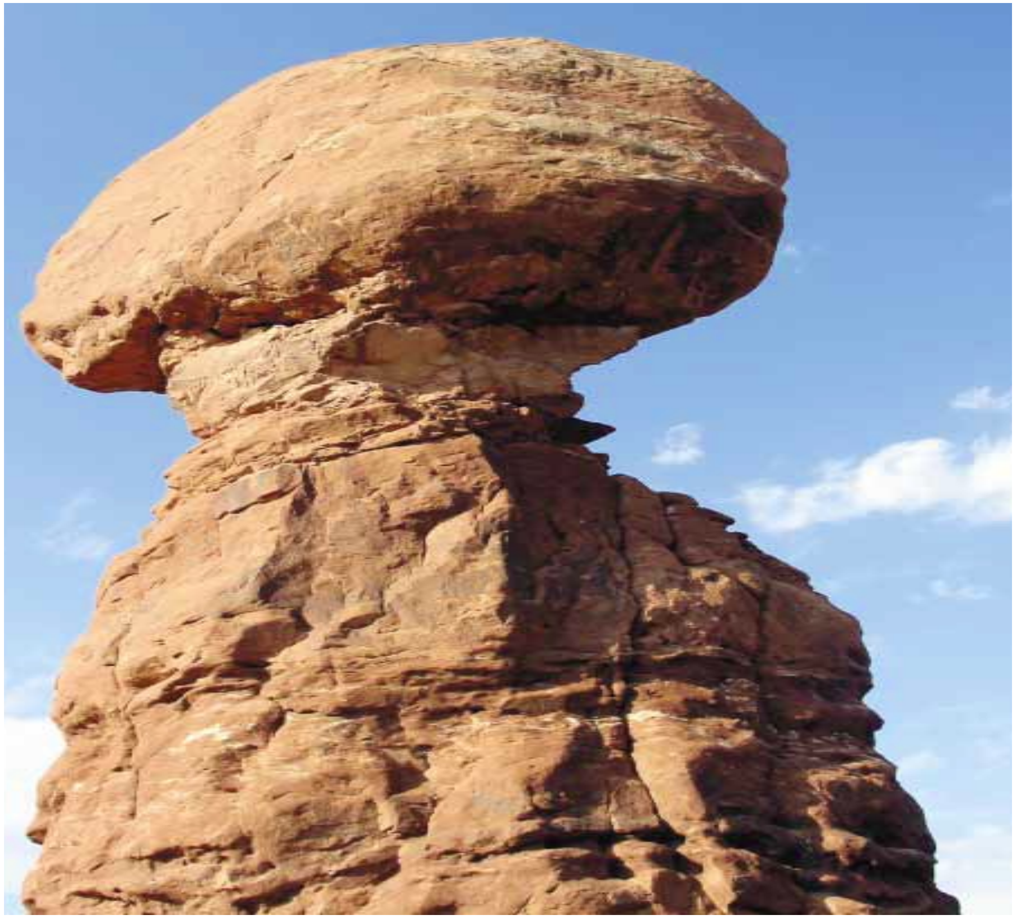
Static Equilibrium and Elasticity



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Elastic Properties of Solids

- Stress
- Strain
- Modulus of Elasticity
- Young's modulus
- Shear modulus
- Bulk modulus

****PLEASE NOTE, THE CONTENT IN THIS PRESENTATION IS JUST TO HELP YOU, AND IT IS NOT A SUBSTITUTE TO THE COURSE REFERENCES**

You can find the lecture topics in:

Course Reference: Physics for scientists and Engineers, Raymond Serway & et al 9th edition

Chapter(12):

- **rigied opjects in Equilibrium(12.1) [p363,p367]**
- **Elastic Properties of Solids(12.4)[p373-p377]**

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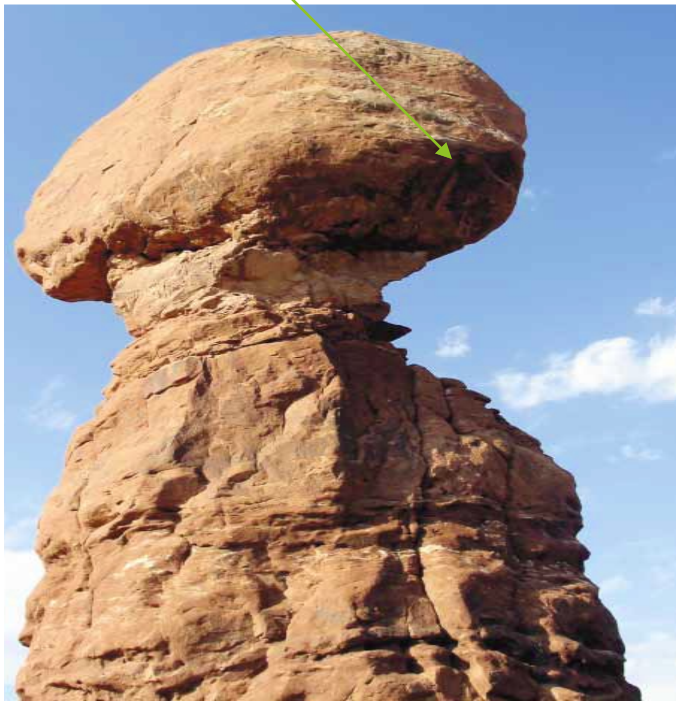


Static Equilibrium and Elasticity

Rigid Object in Equilibrium

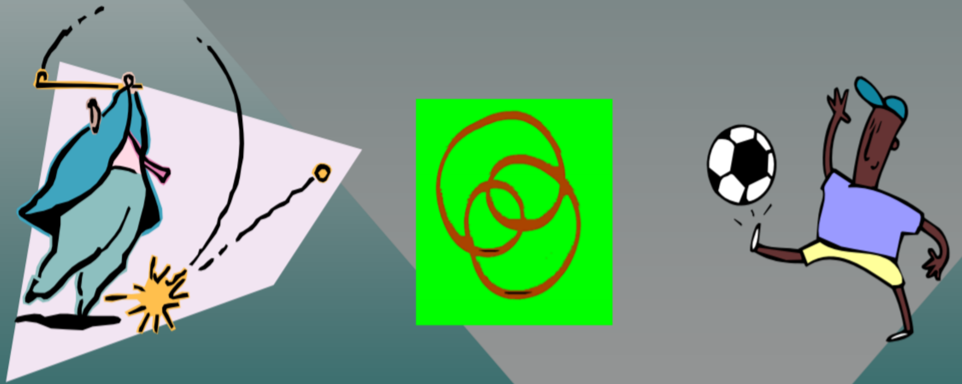
An object in mechanical equilibrium must satisfy the following two conditions:

1. **The net external force must be zero:** $\sum \vec{\mathbf{F}} = 0$ → Transitional equilibrium

2. **The net external torque must be zero:** $\sum \vec{\boldsymbol{\tau}} = 0$ → Rotational equilibrium

ELASTICITY

An **elastic body** is one that returns to its original shape after a deformation.



Golf Ball | Rubber Band | Soccer Ball

An **inelastic body** is one that does **not** return to its original shape after a deformation.

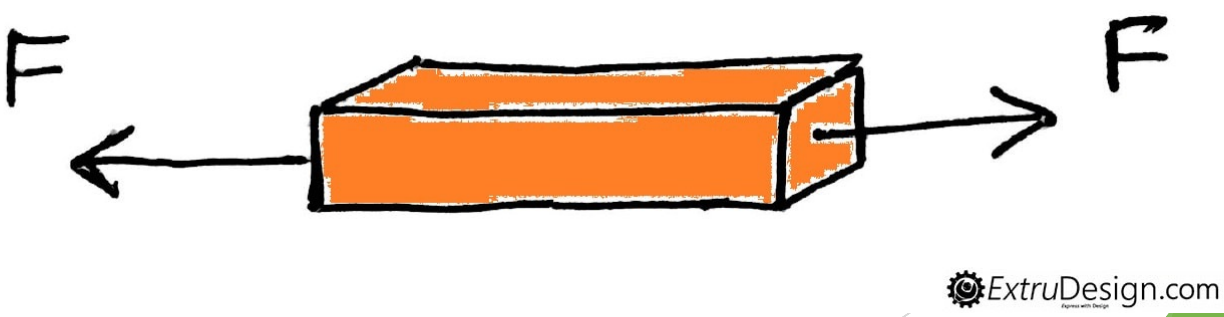
Dough or Bread | Clay | Inelastic Ball

Elastic Properties of Solids

All objects are deformable
when external forces act on it.

That is,

it is possible to change the shape or the size (or both) of an object by applying external forces.

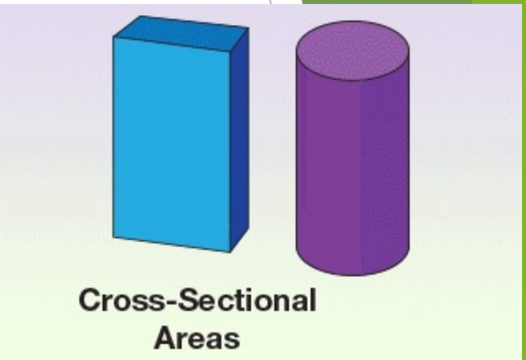


Elastic Properties of Solids

Stress

- is the external force acting on an object per unit cross-sectional area.

$$Stress = F \backslash A$$



- is a quantity that is proportional to the force causing a deformation.

- The unit of Stress in SI system is ……….

- The result of a stress is **strain**, which is a measure of the degree of deformation.

Elastic Properties of Solids

- The result of a stress is **strain**, which is a measure of the degree of deformation.
- Strain is proportional to stress. (Strain α Stress)
- The constant of proportionality (α) is called the elastic modulus.

Elastic Properties of Solids

The elastic modulus:

- Is defined as the ratio of the stress to the resulting: strain.

- **Elastic modulus = stress / strain**

- The elastic modulus relates what is done to a solid object (a force is applied) to how that object responds (it deforms to some extent).

Elastic Properties of Solids

The Types of an elastic modulus:

1. **Young's modulus, which measures the resistance of a solid to a change in its length.**

2. **Shear modulus, which measures the resistance to motion of the planes within a solid parallel to each other.**

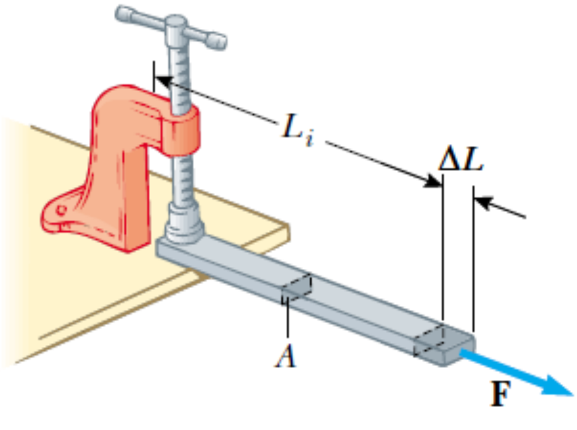
3. **Bulk modulus, which measures the resistance of solids or fluids to changes in their volume.**

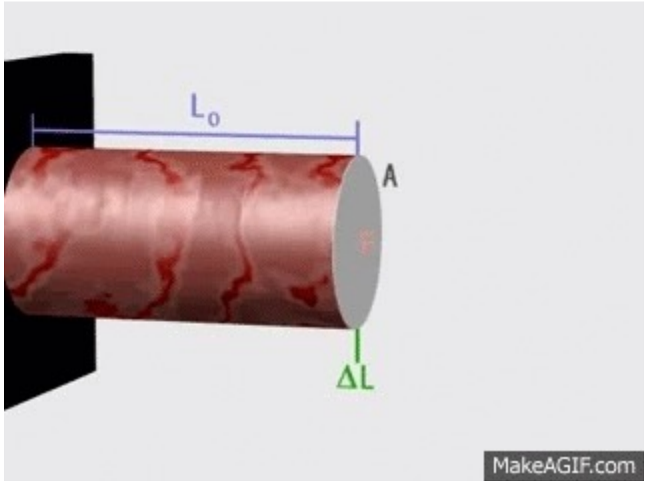
Elastic Modulus

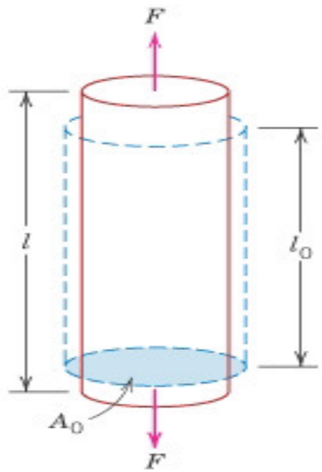
1- Young's Modulus: Elasticity in Length

Young's modulus *is defined* as :

$$Y \equiv \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{\Delta L/L_i}$$







1- Young's Modulus: Elasticity in Length

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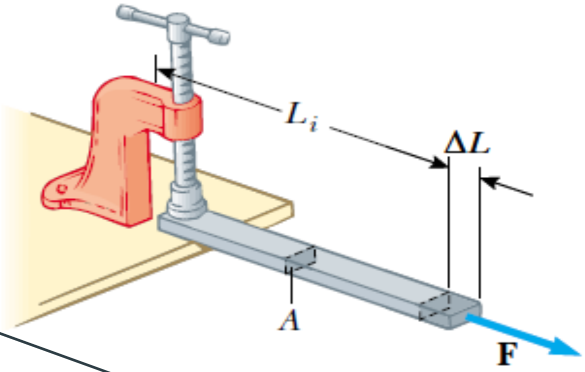


- **Tensile stress**: the ratio of the magnitude of the external force F to the cross-sectional area A.
- **Tensile strain**: in this case the ratio of the change in length ΔL to the original length Li.

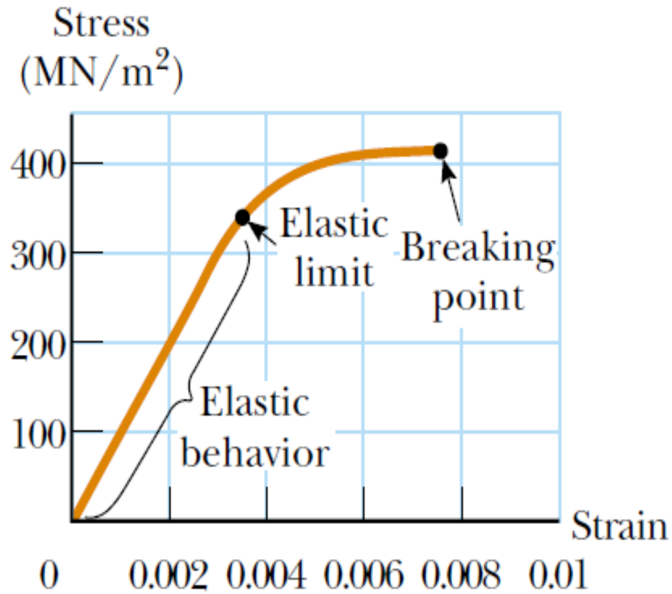
*note that, strain is Dimensionless quantity

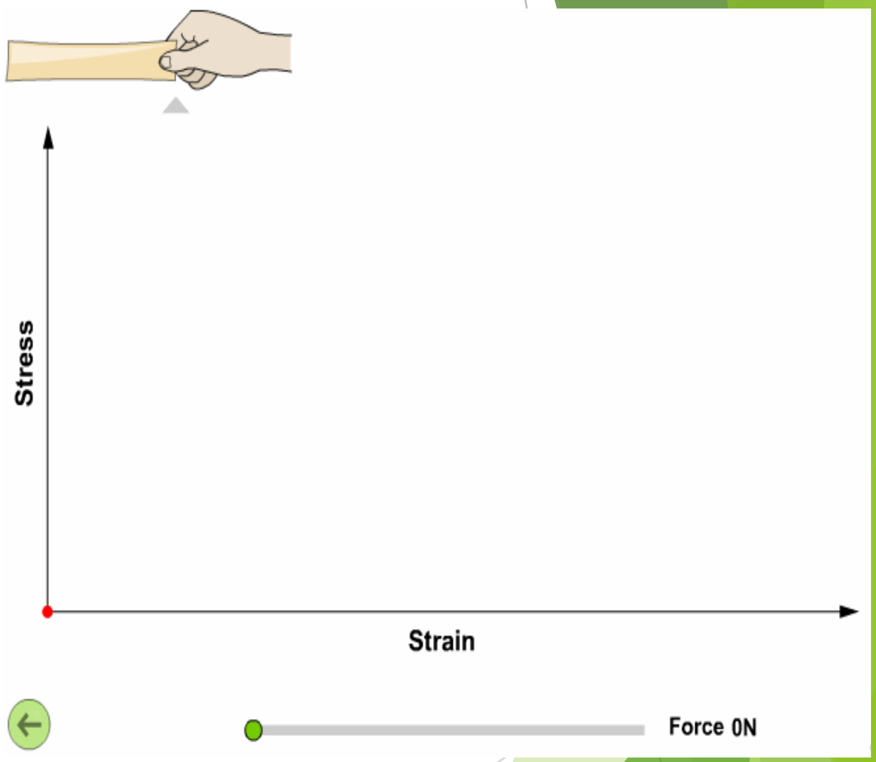
The unit of young 's modulus is the ratio of that for force to that for area. (N \ m^2)

1- Young's Modulus: Elasticity in Length

The elastic limit

The **elastic limit** of a substance is defined as the maximum stress that can be applied to the substance before it becomes permanently deformed and does not return to its initial length.





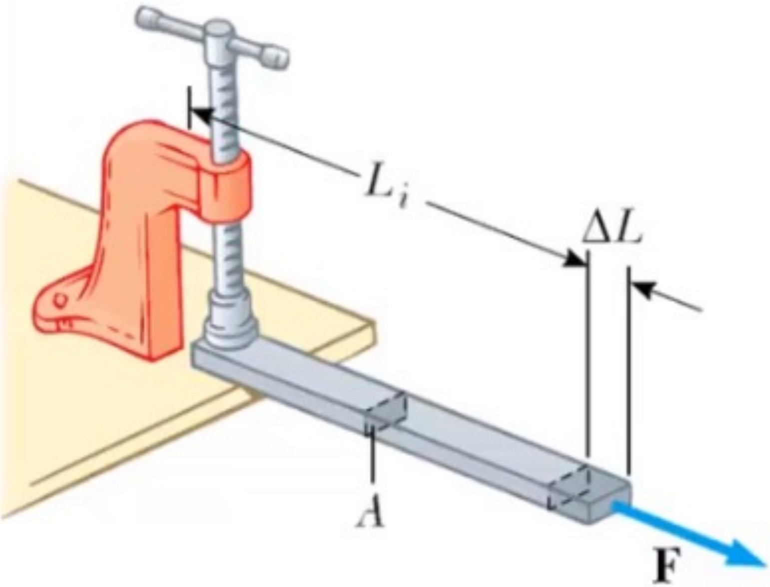
Example

- A bar has dimensions 1cm by 1cm by 20cm. It is subjected to a 10000N tension force and stretches 0.01cm. Find

(a) the stress;

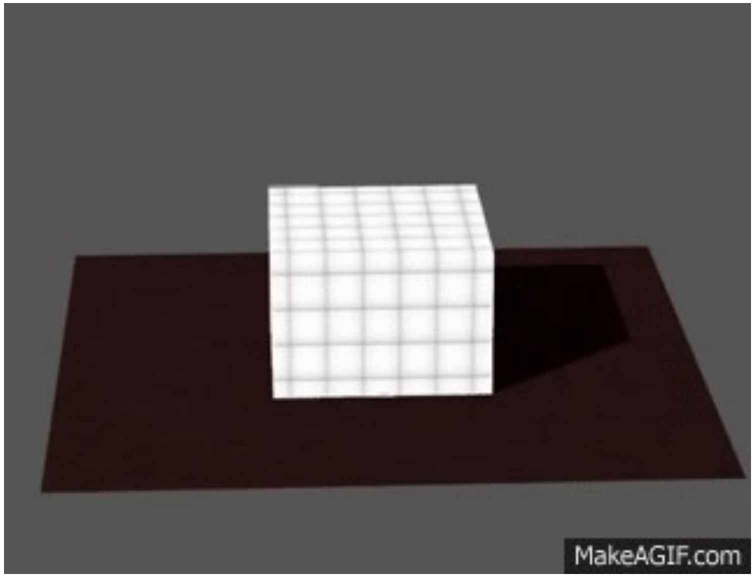
(b) the strain;

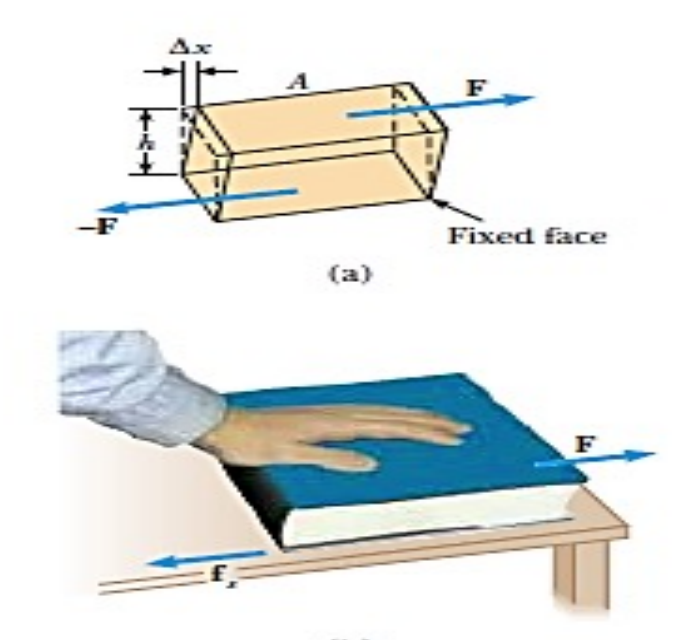
(c) If the stress-strain graph is straight line, how much does the bar stretch when the applied force is increased to 50000N?



2- Shear Modulus: Elasticity of Shape

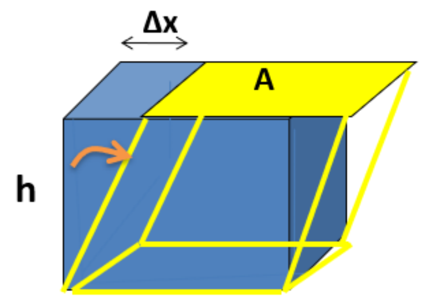
Another type of deformation occurs when an object is subjected to a force parallel to one of its faces while the opposite face is held fixed by another force.

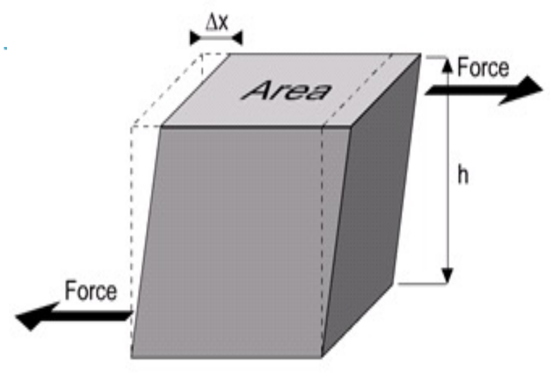




2- Shear Modulus: Elasticity of Shape

$$S \equiv \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x/h}$$





Shear stress: ratio of the tangential force to the area A of the face being sheared.

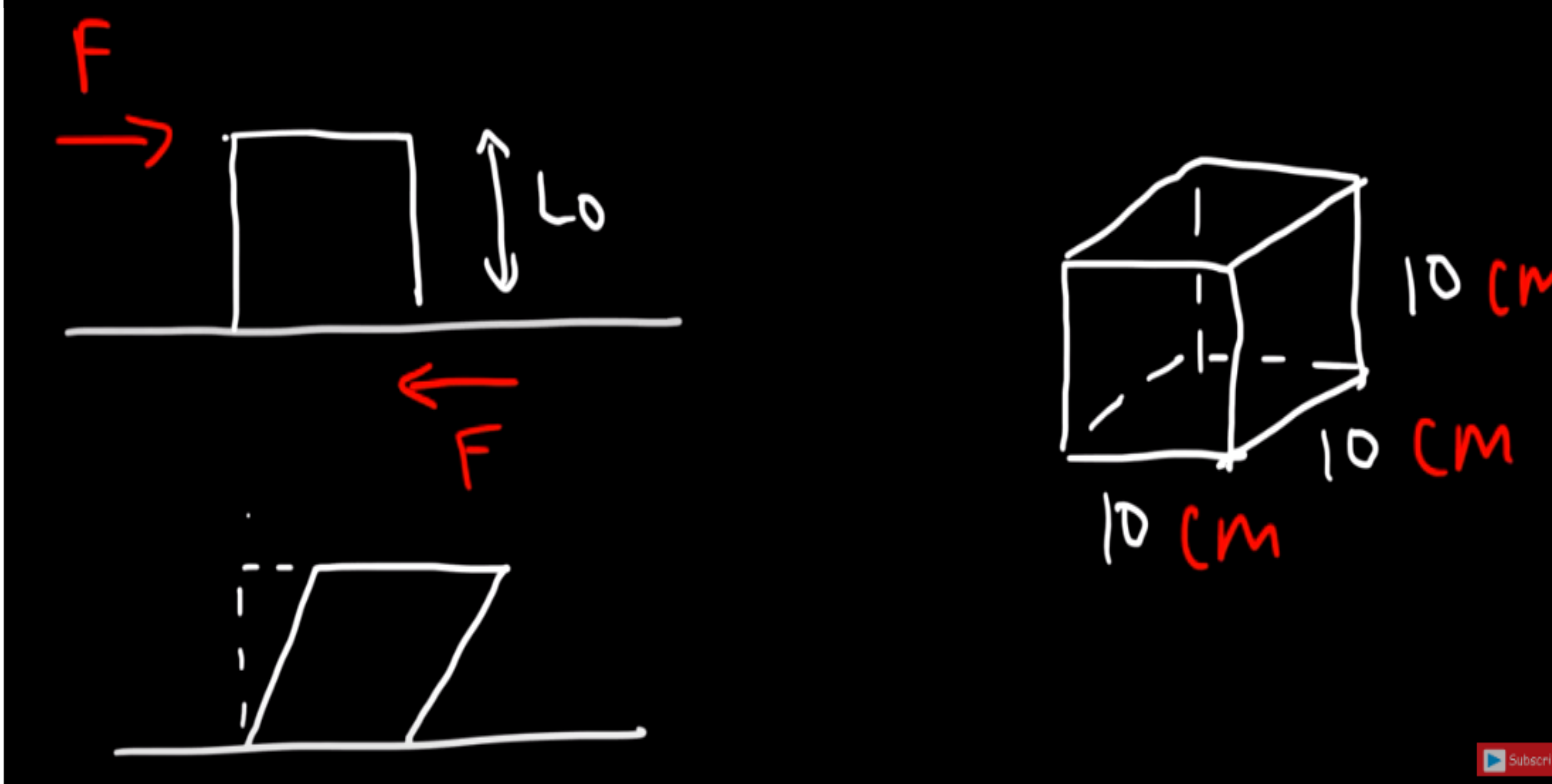
Shear strain: ratio $\Delta x/h$, where Δx is the horizontal distance that the sheared face moves and h is the height of the object.

The unit of shear modulus is

the ratio of that for force to that for area.

Example2:

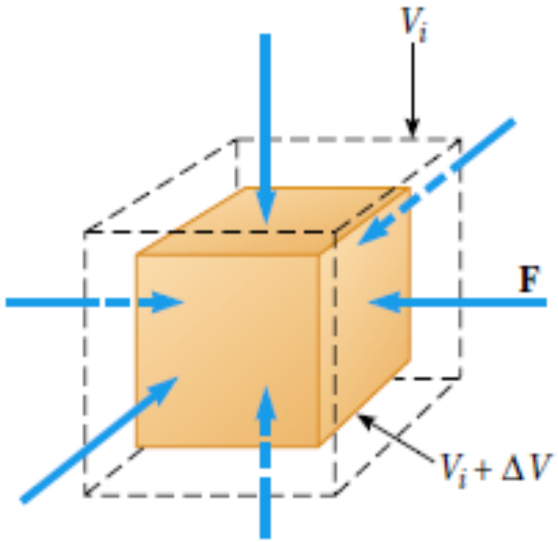
A cube with a side length of 10cm is made up of aluminum metal (G= 25 x 10^9 N/m2) A horizontal shear force of 50000 N is applied to this cube. (a) How far will the top of the cube move in the horizontal direction relative to the bottom of the cube? (b) What is the shear stress applied to the cube? (c) Calculate the shear strain on the cube.

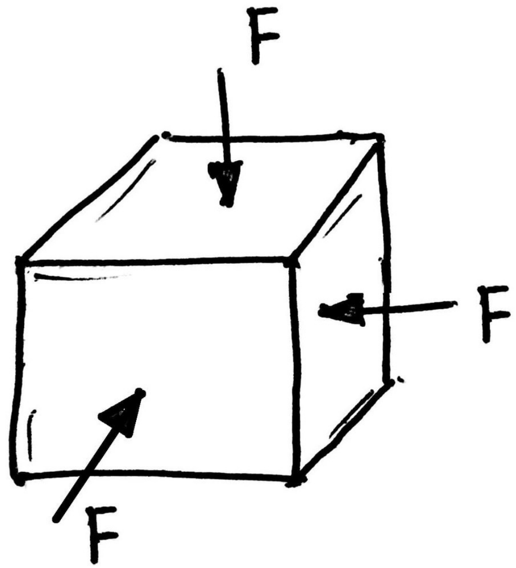


3- Bulk Modulus: Volume Elasticity

- is defined as:

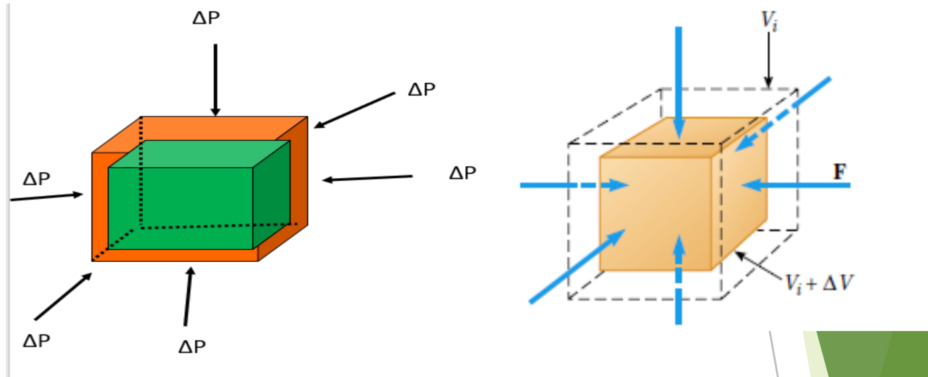
$$B = \frac{\text{volume stress}}{\text{volume strain}} = -\frac{\Delta F/A}{\Delta V/V_i} = -\frac{\Delta P}{\Delta V/V_i}$$





3- Bulk Modulus: Volume Elasticity

$$B = \frac{\text{volume stress}}{\text{volume strain}} = -\frac{\Delta F / A}{\Delta V / V_i} = -\frac{\Delta P}{\Delta V / V_i}$$



Volume stress: ratio of magnitude of total force F exerted on a surface to the area A of the surface.
P = F/A is called pressure. **If it changes by an amount ΔP = ΔF/A,** then the object will experience a volume change ΔV.

Volume strain: is equal to the change in volume ΔV divided by the initial volume Vi.

- The unit of bulk modulus is the ratio of that for force to that for area.

3- Bulk Modulus: Volume Elasticity

- Note that both solids and liquids have a bulk modulus. However, no shear modulus and no Young's modulus are given for fluids. (Why)

Answer:

Because a liquid does not sustain a shearing stress or a tensile stress.

If a shearing force or a tensile force is applied to a liquid, the liquid simply flows in response.

Table 12.1

Typical Values for Elastic Moduli

Substance	Young's Modulus (N/m^2)	Shear Modulus (N/m^2)	Bulk Modulus (N/m^2)
Tungsten	35×10^{10}	14×10^{10}	20×10^{10}
Steel	20×10^{10}	8.4×10^{10}	6×10^{10}
Copper	11×10^{10}	4.2×10^{10}	14×10^{10}
Brass	9.1×10^{10}	3.5×10^{10}	6.1×10^{10}
Aluminum	7.0×10^{10}	2.5×10^{10}	7.0×10^{10}
Glass	$6.5–7.8 \times 10^{10}$	$2.6–3.2 \times 10^{10}$	$5.0–5.5 \times 10^{10}$
Quartz	5.6×10^{10}	2.6×10^{10}	2.7×10^{10}
Water	—	—	0.21×10^{10}
Mercury	—	—	2.8×10^{10}

Example3:

A pressure of 3×10^8 Pa is applied to a block of volume 0.500 m^3. If the volume decreases by 0.004 m^3, what is the bulk modulus? What is the compressibility?

* **Note that: Pa means Pascal=N.m^{-2}**

Physics is Awesome ☺

YOURS,
TEACHING TEAM