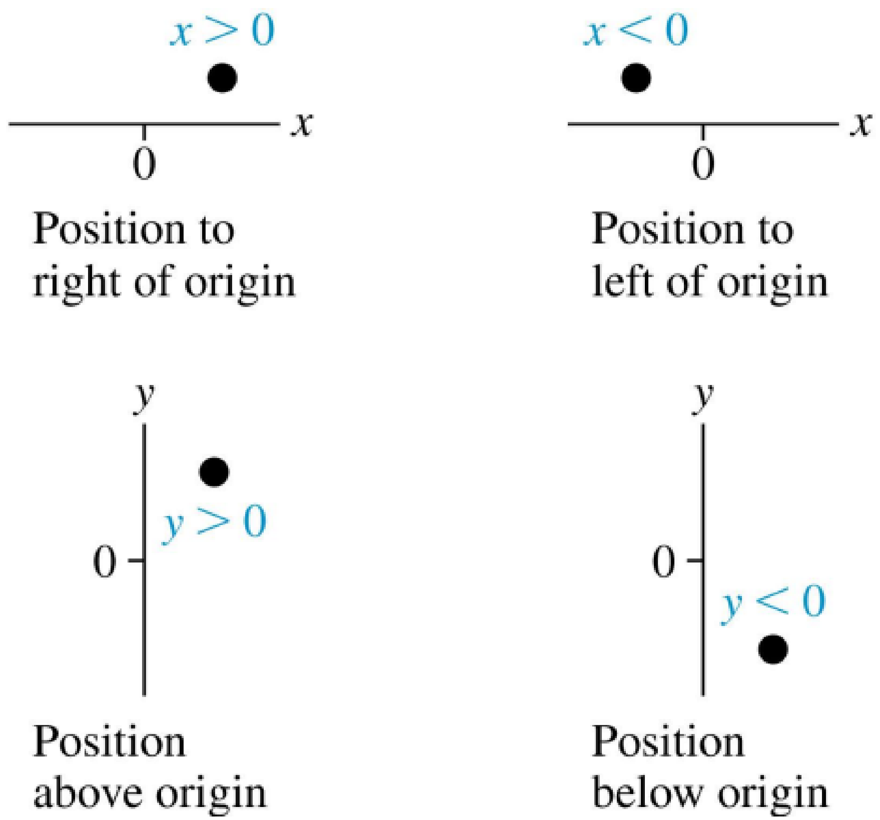
CHAPTER 2

Motion Along a Straight Line

CONTENTS

REPRESENTING POSITION

The position (x) of an object describes its location relative to some origin or other reference point.



1. DISPLACEMENT, TIME, AND AVERAGE VELOCITY

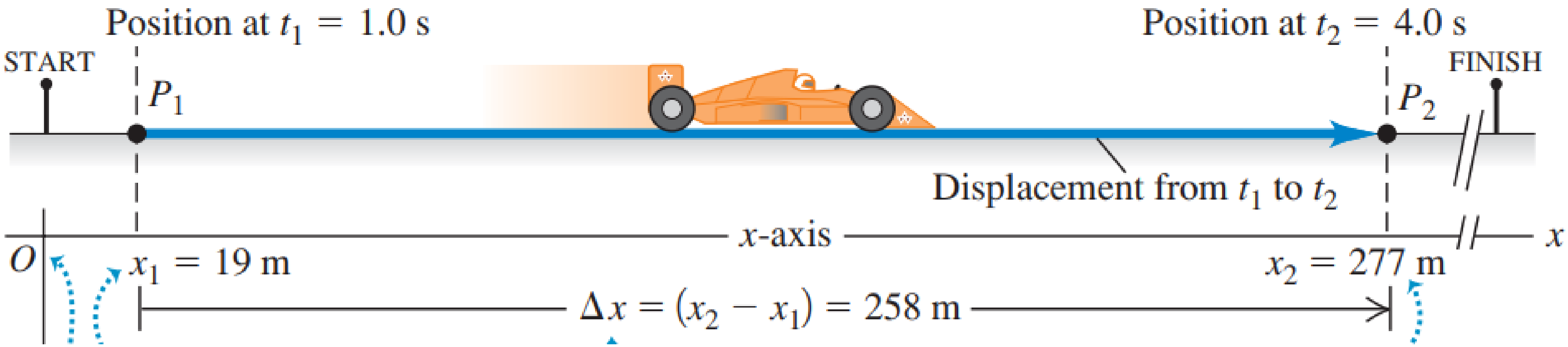
The displacement is the change in an object's position:

$$\Delta x = x_2 - x_1 \qquad \text{(m)}$$

The average velocity is the displacement, Δx, divided by the time interval, Δt, during which the displacement occurs:

$$v_{\text{av}-x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \qquad \text{(m/s)}$$

Example 2.1: A car is moving in a straight road, its initial position at time $t_1 = 1\,s$ is $x_1 = 19$ m and its final position at time $t_2 = 4\,s$ is $x_2 = 277$ m. (a) find the displacement travelled by the car? (b) find its average velocity?



(a) Displacement:

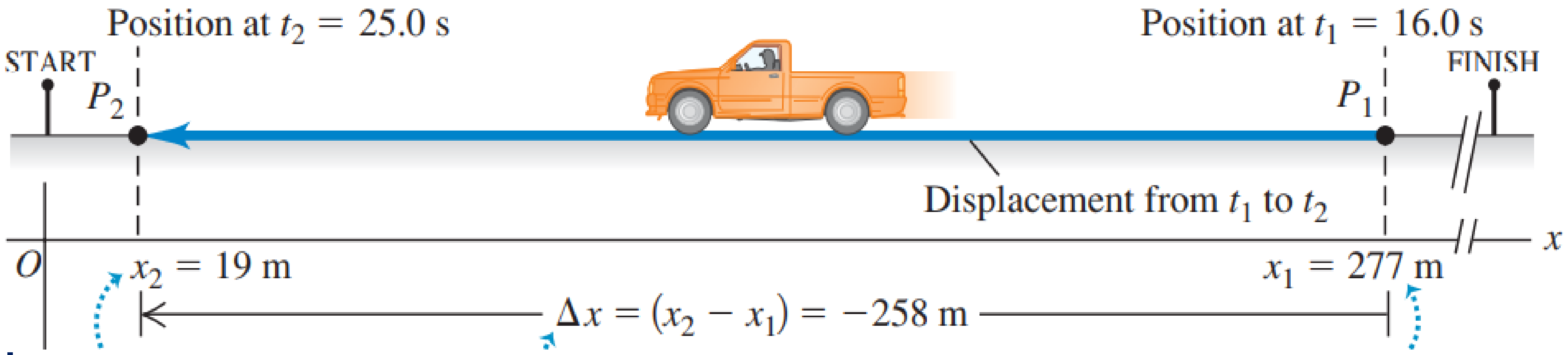
$$\Delta x = x_2 - x_1 = 277 - 19 = 258\text{ m}$$

(b) Average velocity:

$$v_{av-x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{277-19}{4-1} = \frac{258}{3} = +86\text{ m/s}$$

Remark 1: The velocity is positive when the car moves to the right (the position increases).

Example 2.2: A car is moving in a straight road, its initial position at time $t_1 = 16\ s$ is $x_1 = 277$ m and its final position at time $t_2 = 25\ s$ is $x_2 = 19$ m. (a) find the displacement travelled by the car? (b) find its average velocity?



(a) Displacement:

$$\Delta x = x_2 - x_1 = 19 - 277 = -258\ \mathrm{m}$$

(b) Average velocity:

$$v_{av-x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{19 - 277}{25 - 16} = \frac{-258}{9} = -29\ \mathrm{m/s}$$

Remark 2: The velocity is negative when the car moves to the left (the position decreases).

2. INSTANTANEOUS VELOCITY

The instantaneous velocity is the velocity at a specific instant of time or specific point along the path.

$$v_x = \frac{dx}{dt} \qquad (m/s)$$

<u>Example 2.3:</u> Suppose the position of the car at any time t is given by the equation

$$x = 20m + (5\ m/s^2)t^2$$

(a) find the car's displacement between $t_1 = 1.0\ s$ and $t_2 = 2.0\ s$?

(b) find its average velocity during that interval?

(c) find its instantaneous velocity at $t = 1.0\ s$ and at $t = 2.0\ s$?

$$\mathbf{x = 20 + 5t^2}$$

(a) displacement:

$$\mathbf{t_1 = 1.0\ s:} \qquad \mathbf{x_1 = 20 + 5\ (1)^2 = 25\ m}$$

$$\mathbf{t_2 = 2.0\ s:} \qquad \mathbf{x_2 = 20 + 5\ (2)^2 = 40\ m}$$

$$\mathbf{\Delta x = x_2 - x_1 = 40 - 25 = +15\ m}$$

(b) average velocity:

$$v_{av-x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{40-25}{2-1} = 15 \qquad (m/s)$$

(c) instantaneous velocity: $\frac{d}{dt}(a) = 0$ and $\frac{d}{dt}(b\,t^n) = n \times b\,t^{n-1}$, where **a** and **b** are constants

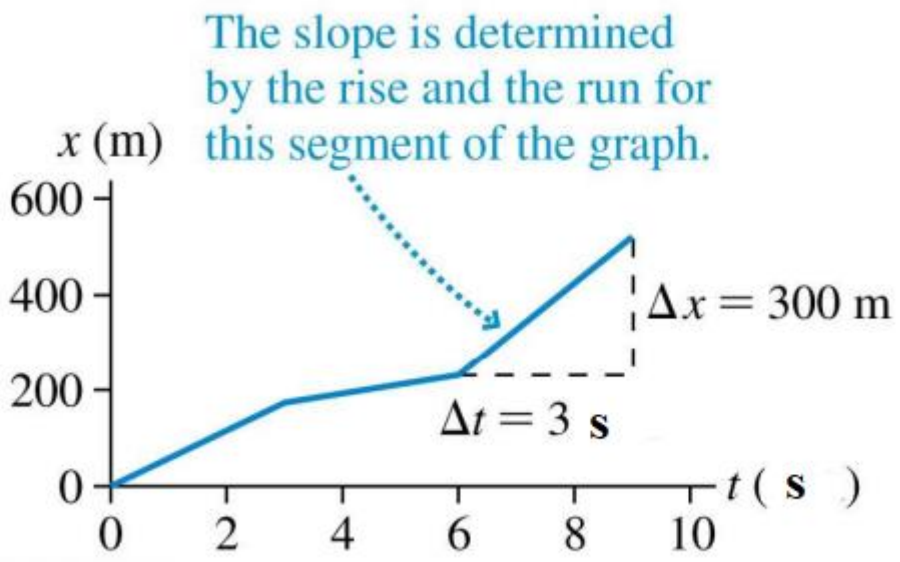
$$\frac{dx}{dt} = 0 + 2 \times 5\,t^{2-1} = 10\,t$$

$$v_x = 10\,t$$

$$\mathbf{t_1 = 1.0\ s:} \qquad v_x = 10 \times 1 = 10\ m/s$$

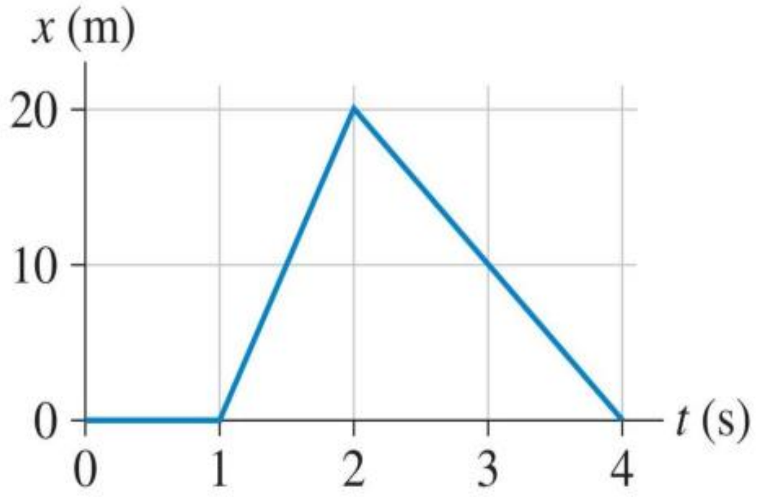
$$\mathbf{t_2 = 2.0\ s:} \qquad v_x = 10 \times 2 = 20\ m/s$$

The slope of an object's position-versus-time graph is the object's instantaneous velocity at that point in the motion.



$$v_x = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{300}{3} = 100\ \text{m/s}$$

Example 2.4: Here is a position graph of an object:



(a) what is the object's velocity at $t = 1.5$ s?

$$v_x = \frac{\text{rise}}{\text{run}} = \frac{20-0}{2-1} = +10\ \text{m/s}$$

(b) what is the object's velocity at $t = 3.0$ s?

$$v_x = \frac{\text{rise}}{\text{run}} = \frac{0-20}{4-2} = -10\ \text{m/s}$$

3. AVERAGE ACCELERATION

Average acceleration is the change in the velocity, Δv_x, divided by the time interval, Δt:

$$a_{av-x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{\Delta v_x}{\Delta t} \qquad (\mathrm{m/s^2})$$

Example 2.5:

A car accelerates along a straight road from rest to 60 km/h in 5 seconds. What is the magnitude of the average acceleration?

At rest: $v_{1x} = 0\ \mathrm{m/s}$

$$v_{2x} = \left(60\,\frac{\mathrm{km}}{\mathrm{h}}\right)\left(\frac{1000\ \mathrm{m}}{\mathrm{km}}\right)\left(\frac{\mathrm{h}}{3600\ \mathrm{s}}\right) = 16.7\ \mathrm{m/s}$$

$$a_{av-x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{16.7 - 0}{5 - 0} = 3.34\ \mathrm{m/s^2}$$

INSTANTANEOUS ACCELERATION

$$a_x = \frac{\mathrm{d}v_x}{\mathrm{d}x}$$

Example 2.6:

Suppose the velocity of a car at any time t is given by the equation

$$v_x = 60\ \mathrm{m/s} + \left(0.5\ \mathrm{m/s^3}\right)t^2$$

Find the instantaneous acceleration at times t=1.0 s and t=3.0 s?

$$a_x = \frac{\mathrm{d}v_x}{\mathrm{d}x} = 0 + 2 \times 0.5\ t^{2-1}$$

$$a_x = t$$

When $t = 1.0\ \mathrm{s}$:

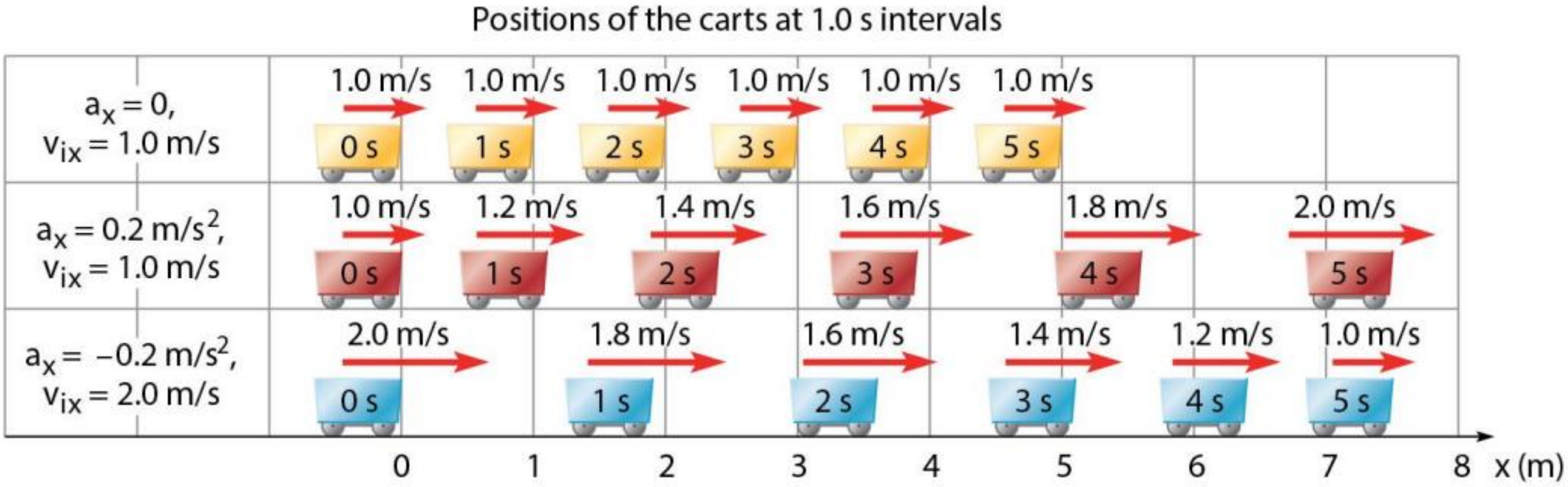
$$a_x = 1.0\ \mathrm{m/s^2}$$

When $t = 3.0\ \mathrm{s}$:

$$a_x = 3.0\ \mathrm{m/s^2}$$

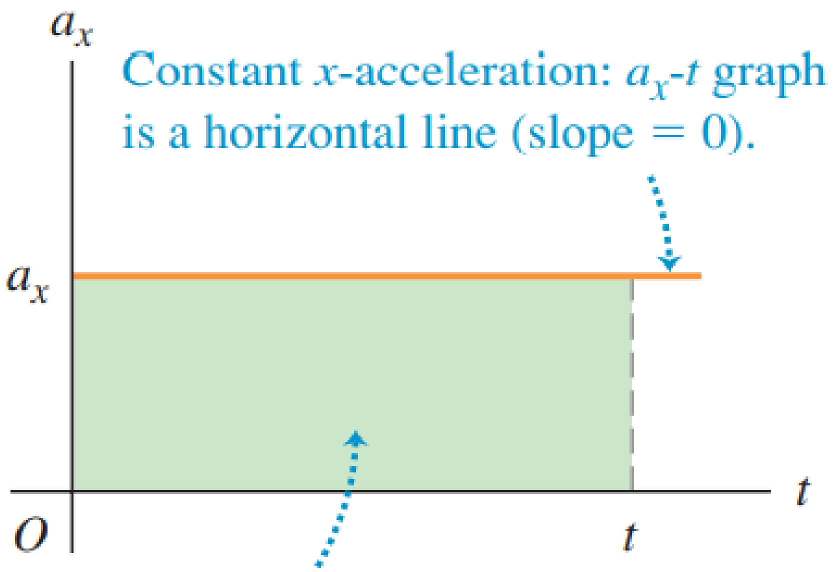
4. MOTION WITH CONSTANT ACCELERATION

Motion diagrams for three carts. Each cart is shown at 1.0 s time intervals, and each has a (different) constant acceleration.



KINEMATIC EQUATIONS

Equation		Includes Quantities			
$v_x = v_{0x} + a_x t$	(1)	t		v_x	a_x
$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$	(2)	t	x		a_x
$v_x^{\,2} = v_{0x}^{\,2} + 2a_x(x - x_0)$	(3)		x	v_x	a_x
$x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right) t$	(4)	t	x	v_x	



Where:

$$v_{0x} = \text{initial velocity}$$
$$v_x = \text{final velocity}$$

Example 2.7:

An object starts from **rest** and uniformly accelerates at a rate of 2 $\mathrm{m/s^2}$ for 5 seconds. What is the object's final velocity?

$$v_x = v_{0x} + at = 0 + 2 \times 5 = 10\ \mathrm{m/s}$$

Example 2.8: :

A car is traveling at 24.0 m/s when the driver suddenly applies the brakes, causing the car to slow down with constant acceleration. The car **comes to a stop** in 4 s. What is the acceleration of the car?

$$v_x = v_{0x} + at$$

$$0 = 24 + 4\,a$$

$$a = -6.0\ \mathrm{m/s^2}$$

Example 2.9:

A car accelerates from 5.0 m/s to 21 m/s at a constant rate of 3.0 $\mathrm{m/s^2}$.

How far does it travel while accelerating?

$$v_x^2 = v_{0x}^2 + 2a(x - x_0)$$

$$21^2 = 5^2 + 2 \times 3 \times \Delta x$$

$$\Delta x = \frac{21^2 - 5^2}{6} = 69.3 \text{ m}$$

Example 2.10:

If a car moves with initial velocity 40 m/s and constant acceleration 12 $\mathrm{m/s^2}$

for a total time of 10s , what total distance does it travel?

$$x = x_0 + v_{0x}t + \frac{1}{2}a\,t^2$$

$$\Delta x = 40 \times 10 + \frac{1}{2} \times 12 \times (10)^2 = 1000 \text{ m}$$

5. FREELY FALLING OBJECTS

If no forces act on an object other than the gravitational force, we say that the object is in free fall.

For example, a stone dropped from the edge of a cliff—if air resistance can be ignored, the stone is in free fall. Or a ball thrown upward—if air resistance is ignored, the ball is in free fall.

An object in free fall has constant downward acceleration, denoted by the symbol (g) .

Free fall is an example of motion with constant acceleration.

The g is the magnitude of a vector, it is always positive number.

$$a_{\text{free fall}} = -\mathbf{g} = -9.8\ \mathbf{m/s^2}$$

Example 2.11:

A ball is thrown upward at a velocity of 19.6 m/s. What is its velocity after 3.0 s?

$$v_y = v_{0y} - \mathbf{g}t = 19.6 - 9.8 \times 3 = -9.8\ \mathbf{m/s}$$

Example 2.12:

A stone is dropped from rest from the top of a tall building, After 3.00 s of free-fall. What is the displacement y of the stone?

$$\Delta y = v_{0y}t - \frac{1}{2}\mathbf{g}t^2 = 0 - \frac{1}{2} \times 9.8 \times (3)^2 = -44.1\mathbf{m}$$

Remark 3: The sign of velocity and displacement is –ve when the object moves downward and is +ve when the object moves upward.

Example 2.13:

A stone is thrown upward vertically with initial velocity 10 m/s. Calculate the **maximum height** that the stone can be reached?

At maximum height the final velocity is zero,

$$v_y^2 = v_{0y}^2 - 2g\,\Delta y$$

$$0^2 = 10^2 - 2 \times 9.8 \times \Delta y$$

$$\Delta y = \frac{0-100}{-19.6} = 5.1\ \text{m}$$

Remark 4: **If the object is thrown upward , then , it reaches to maximum height at final velocity equals zero.**