CHAPTER 10

Dynamics of Rotational Motion

CONTENTS

1. TORQUE

Torque is the quantitative measure of the tendency of a force to cause or change a body's rotational motion.

$$\tau = F\,d = F\,r\,\sin\phi \quad (N.m)$$

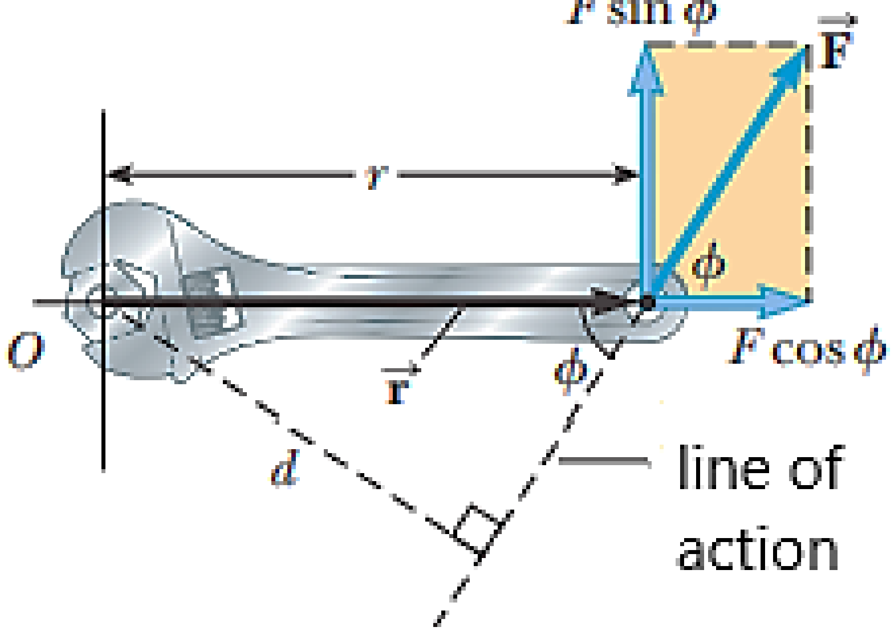
Where $d = r\sin\phi$ is the moment arm.

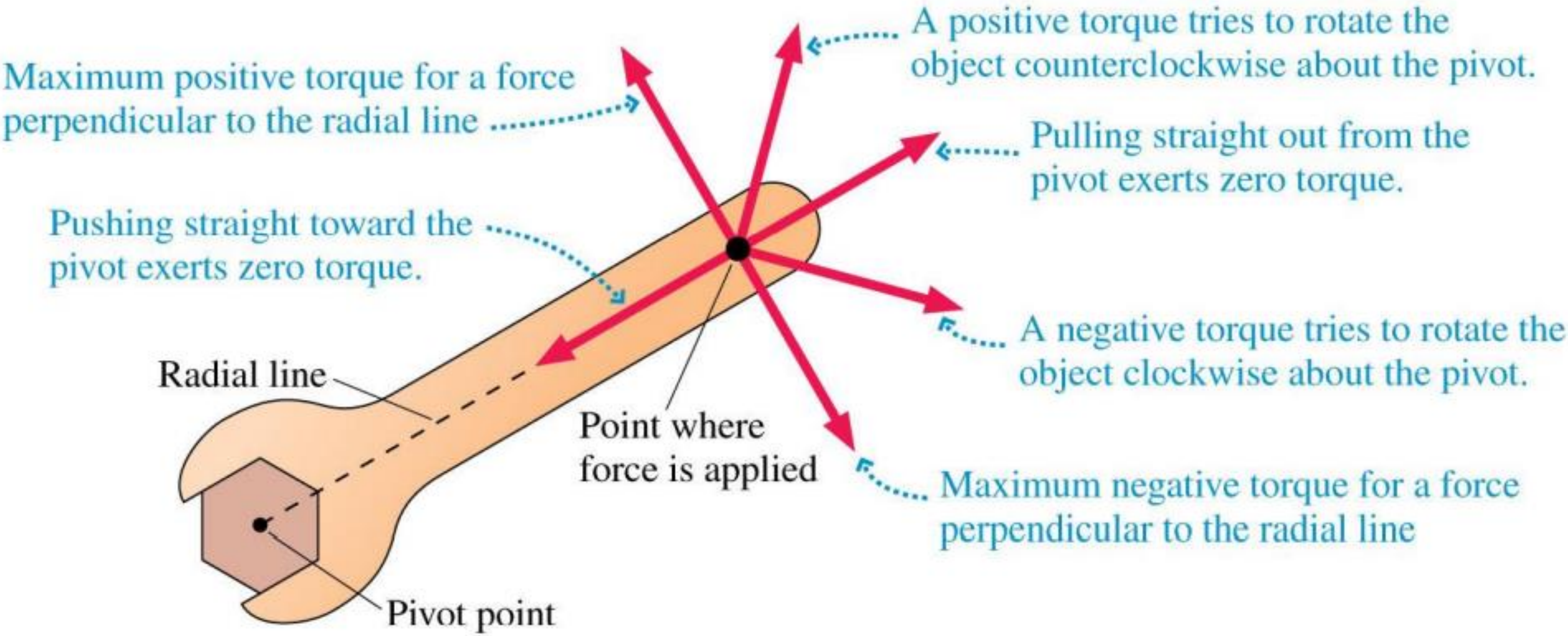
1. If the force rotates **clockwise**:

$$\tau = -F\,d \quad \text{(negative torque)}$$

2. If the force rotates **counter-clockwise**:

$$\tau = +F\,d \quad \text{(positive torque)}$$

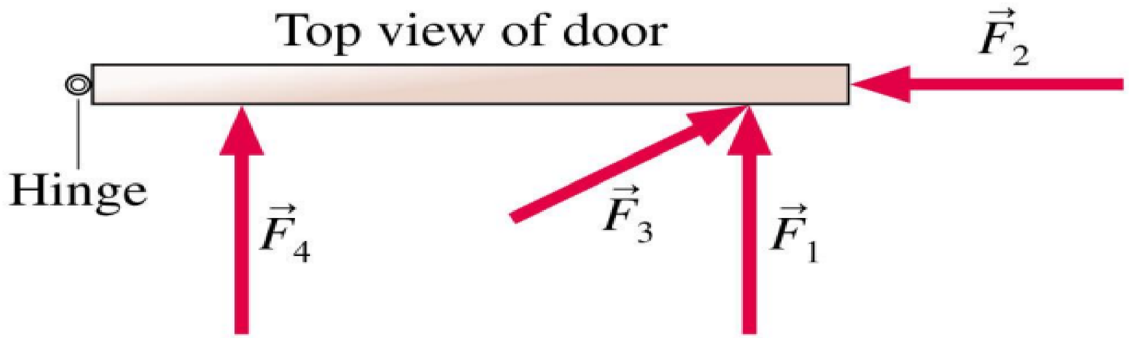



A positive torque tries to rotate the object counterclockwise about the pivot.
Maximum positive torque for a force perpendicular to the radial line
Pulling straight out from the pivot exerts zero torque.
Pushing straight toward the pivot exerts zero torque.
A negative torque tries to rotate the object clockwise about the pivot.
Radial line
Point where force is applied
Maximum negative torque for a force perpendicular to the radial line
Pivot point

Example 10.1:

- The four forces shown have the same strength. Which force would be most effective in opening the door?

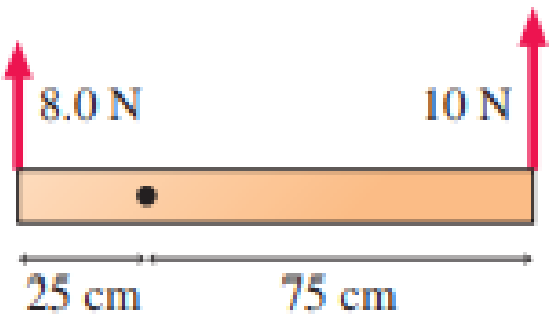
A. Force F_1
B. Force F_2
C. Force F_3
D. Force F_4
E. Either F_1 or F_3



The net torque:

$$\sum \vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 + \cdots$$

Example 10.2: What is the net torque on the bar shown in figure, about the axis indicated by the dot?



Solution:

$$d_1 = 25\ \text{cm} = 0.25\ \text{m}$$

$$d_2 = 75\ \text{cm} = 0.75\ \text{m}$$

$$\tau_1 = -F_1 d_1 = -8 \times 0.25 = -2\ \text{N.m}$$

$$\tau_2 = +F_2 d_2 = +10 \times 0.75 = +7.5\ \text{N.m}$$

$$\sum \tau = \tau_1 + \tau_2 = -2 + 7.5 = +5.5\ \text{N.m}$$

Remark: the net torque rotates in the direction of counter-clockwise.

2. TORQUE AND ANGULAR ACCELERATION FOR A RIGID BODY

The net torque acting on the particle is proportional to its angular acceleration, and the proportionality constant is the moment of inertia (I).

$$\sum \tau = I\,\alpha$$

This equation is the **rotational analog of Newton's second law for a rigid body.**

Example 10.3:

A certain wheel has a rotational moment of inertia of 12 $\mathrm{kg.m^2}$. As it turns at rate of 0.6 $\mathrm{rad/s^2}$, what is the value of the net torque?

$$\sum \tau = I\,\alpha = 12 \times 0.6 = 7.2\ \mathrm{N.m}$$

3. ANGULAR MOMENTUM

Angular momentum of a rigid body:

$$\vec{L} = \mathrm{I}\,\vec{\omega} \qquad (\mathrm{kg.m^2/s})$$

Example 10.4: A turbine fan in a jet engine has a moment of inertia of 2.5 $\mathrm{kg.m^2}$ about its axis of rotation. As the turbine starts up, its angular velocity is given by

$$\omega = 40\,\mathrm{t}^2$$

Find the fan's angular momentum as a function of time, and find its value at $t = 3.0\ s$?

$$L = \mathrm{I}\,\omega = 2.5 \times 40\,\mathrm{t}^2 = 100\,\mathrm{t}^2$$

At $\quad \mathrm{t} = 3.0\ \mathrm{s}: \qquad L = 100\,(3)^2 = 900\ \mathrm{kg.m^2/s}$

4. CONSERVATION OF ANGULAR MOMENTUM

The net torque is related to the rate of change of the angular momentum:

$$\sum \vec{\tau} = \frac{d\vec{L}}{dt}$$

When the net torque acting on a system is zero,

$$\sum \vec{\tau} = 0 \; ,$$

the total angular momentum of the system is constant (conserved).

$$\frac{d\vec{L}}{dt} = 0 \;\Rightarrow \vec{L} = \text{constant}$$

$$L_i = L_f$$