CHAPTER 9

Rotation of Rigid Bodies

CONTENTS

RIGID BODY

A rigid body that the body has a perfectly definite and unchanging shape and size.

It is nondeformable; real-world bodies can be very complicated; the forces that act on them can deform them—stretching, twisting, and squeezing them.

1. ANGULAR VELOCITY AND ACCELERATION

The angular displacement:

$$\Delta\theta = \theta_2 - \theta_1 \qquad \text{(rad)}$$

The average angular velocity

$$\omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t} \qquad \text{(rad/s)}$$

The instantaneous angular velocity

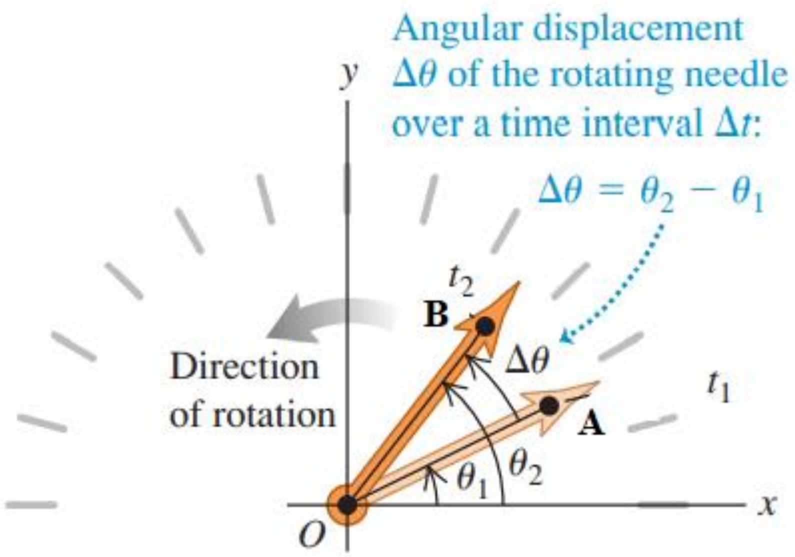
$$\omega = \frac{d\theta}{dt} \qquad \text{(rad/s)}$$

The average angular acceleration

$$\alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t} \qquad (\text{rad/s}^2)$$

The instantaneous angular acceleration

$$\alpha = \frac{d\omega}{dt} \qquad (\text{rad/s}^2)$$



Example 9.1:

The angular position of a wheel is given by

$$\theta = 2t^2$$

where θ in rad and t in seconds.

(a) find the angular displacement over the time interval from

$\mathrm{t} = 2.0\ \mathrm{s}$ to $\mathrm{t} = 4.0\ \mathrm{s}$?

$$t_1 = 2\ s: \qquad \theta_1 = 2(2)^2 = 2 \times 4 = 8\ \mathrm{rad}$$

$$t_2 = 4\ s: \qquad \theta_2 = 2(4)^2 = 2 \times 16 = 32\ \mathrm{rad}$$

$$\Delta\theta = \theta_2 - \theta_1 = 32 - 8 = 24\ \mathrm{rad}$$

(b) Find in the average angular velocity over the time intervals from $t = 2.0\ s$ to $t = 4.0\ s$?

$$\omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{32 - 8}{4 - 2} = \frac{24}{2} = 12\ \mathrm{rad/s}$$

(c) Find in the instantaneous angular velocity at $t = 4.0\ s$?

$$\omega = \frac{d\theta}{dt} = 2 \times 2\, t^{2-1} = 4t$$

$$\omega = 4t$$

$$t = 4.0\ s: \qquad \omega = 4(4) = 16\ \mathrm{rad/s}$$

2. ROTATION WITH CONSTANT ANGULAR ACCELERATION

Rotational Motion About a Fixed Axis	Translational Motion
$\omega_f = \omega_i + \alpha t$	$v_f = v_i + at$
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$	$x_f = x_i + v_i t + \frac{1}{2}at^2$
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$	$v_f^2 = v_i^2 + 2a(x_f - x_i)$
$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t$	$x_f = x_i + \frac{1}{2}(v_i + v_f)t$

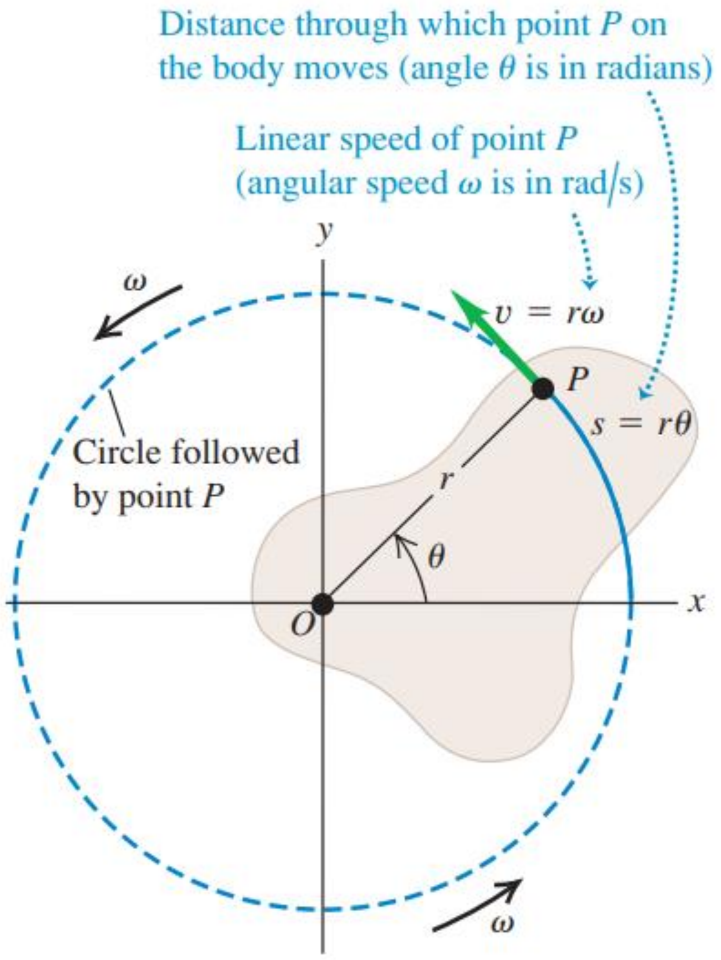
3. RELATING LINEAR AND ANGULAR KINEMATICS

The length arc and the angle:

$$s = r\,\theta \qquad (1)$$

Linear and angular speed:

$$v = r\,\omega \qquad (2)$$



Example 9.2: A racing car travels on a circular track of radius 250 m. Assuming the car moves with a constant speed of 45.0 m/s, find its angular speed?

Solution:

$$v = r\,\omega$$

$$45 = 250\,\omega$$

$$\omega = \frac{45}{250} = 0.18 \text{ rad/s}$$

Tangential acceleration:

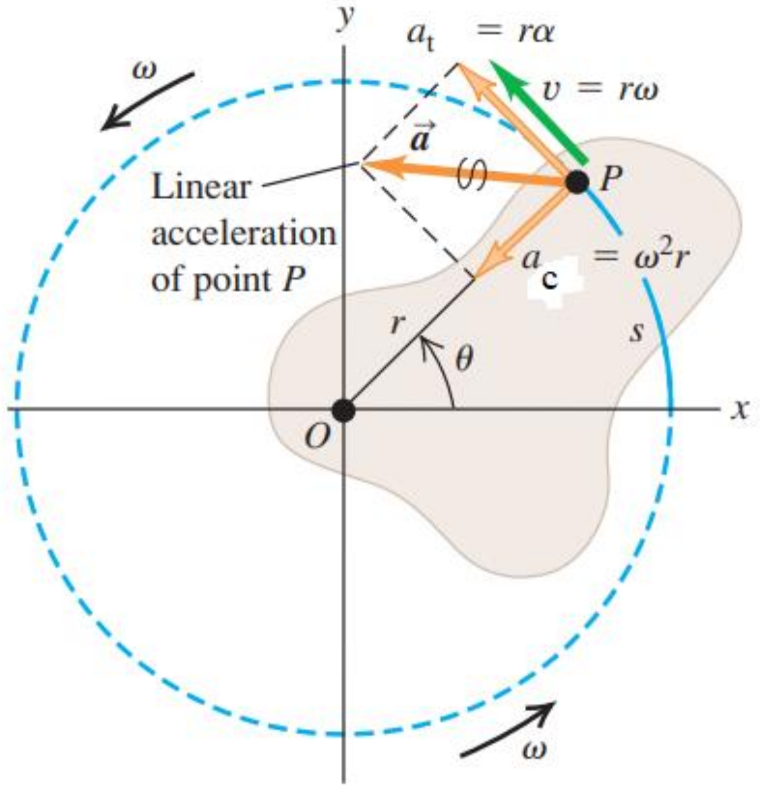
$$a_t = r\,\alpha \qquad (3)$$

Centripetal acceleration:

$$a_c = r\,\omega^2 \qquad (4)$$

Linear acceleration of point P:

$$a = \sqrt{a_t^2 + a_c^2} = r\sqrt{\alpha^2 + \omega^4} \qquad (5)$$



Example 9.3: A racing car travels on a circular track of radius 250 m. Assuming the car moves with an angular speed of $0.18\ \text{rad/s}$, find its centripetal acceleration?

Solution:

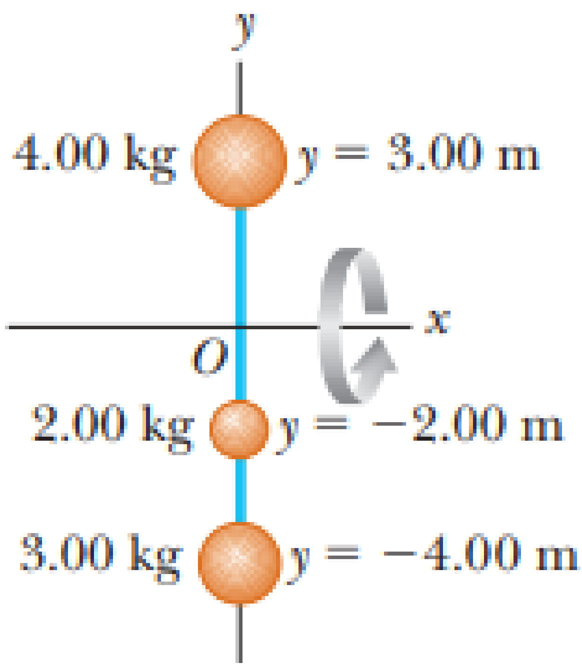
$$a_c = r\,\omega^2 = 250 \times (0.18)^2 = 8.1\ \text{m/s}^2$$

4. MOMENT -OF-INERTIA CALCULATIONS

Moment of inertia of rigid body about an axis:

$$I = m_1 d_1^2 + m_2 d_2^2 + m_3 d_3^2 + \cdots \quad (kg.m^2)$$

Here d is the distance between the mass and the axis of rotation.



Example 9.4:

Rigid rod of negligible mass lying along the y axis connect three particles as in figure. The system rotate about the x axis. Calculate the moment of inertia about the x axis?

$$I_x = 4 \times (3)^2 + 2 \times (2)^2 + 3 \times (4)^2 = 92\ kg.m^2$$