CHAPTER 6 | Work and Kinetic Energy

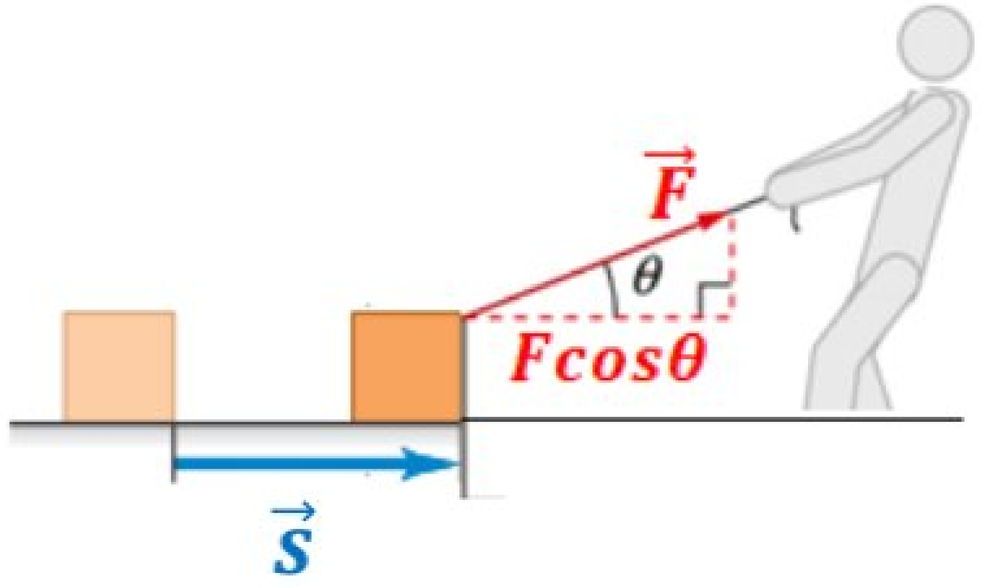
CONTENTS

1. WORK

- The work is defined as the ability to perform a force along a certain displacement

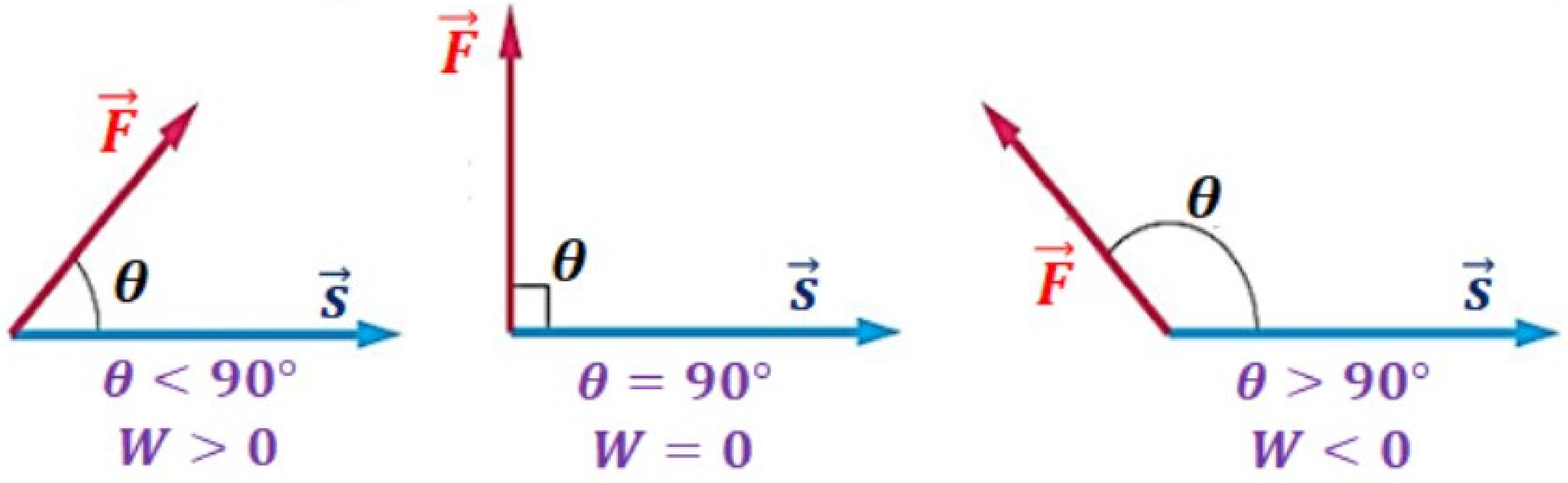
- The work done by the constant force $\vec{F}$ is given by the scalar product of the force $\vec{F}$ and the displacement $\vec{s}$:

- $W = \vec{F}.\vec{s} = F\,s\cos\theta$ (The $S.I$ unit of work is ($N.m$) or (J))

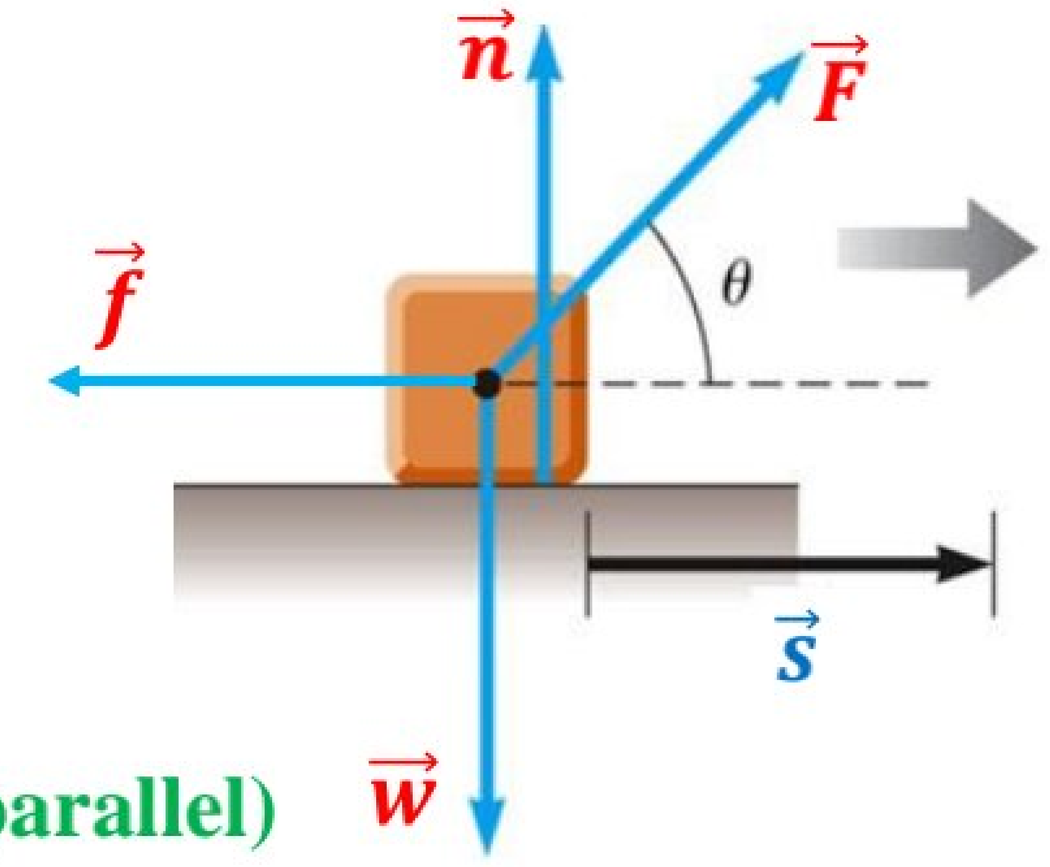


where θ is the angle between the force and displacement

- The work done is a scalar quantity and can be zero, positive or negative value, depending on the angle θ between the force and the displacement



- If more than one force acts on an object, then the total work is equal to the algebraic sum of the work done by the individual forces



$$W_{net} = W_w + W_n + W_F + W_f = F\,s\cos\theta - f s$$

$W_w = W_n = 0$ (because the angle between the force and displacement is 90º)

$W_f = -fs$ (because the angle between the force and displacement is 180º (anti-parallel)

(Be careful not to confuse the work W with the weight w)

Example 6.1:

A man applies a force (F) of $700\ N$ to a crate and pushes it through a distance of block $2m$. Calculate the amount of work done by the man.

While the force $\vec{F}$ and displacement $\vec{s}$ are parallel $(\theta = 0)$, the work done is:

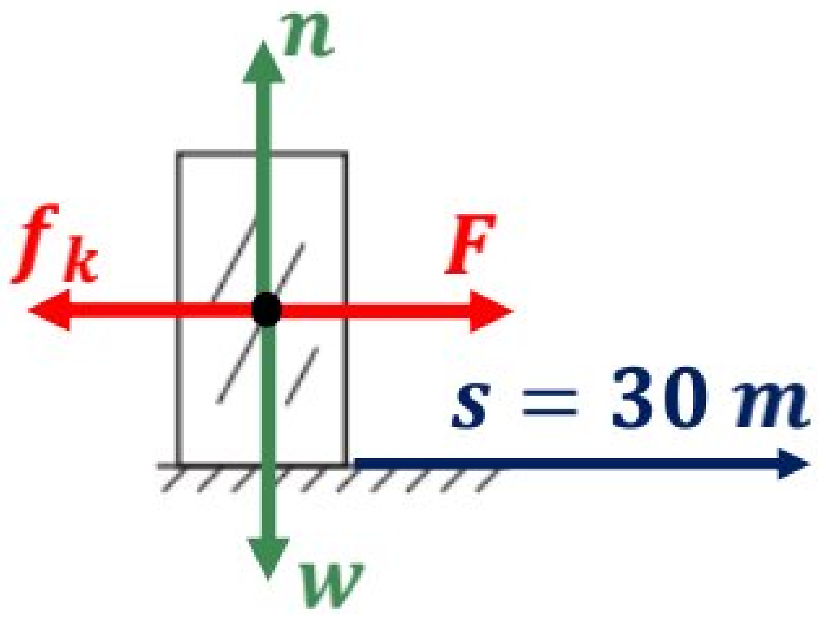
$$W_F = F\,s = 700 \times 2 = 1400\,J$$

Example 6.2:

A $40\,kg$ box is pulled $30\,m$ on a horizontal floor by applying a horizontal force (F) of magnitude $100\,N$. If the floor exerts a friction force (f_k) of magnitude $20\,N$. Calculate:

(a) The work done by the Force F.

While the force F is in the same direction of displacement $(\theta = 0)$, the work done is:



$$W_F = Fs = 100 \times 30 = 3000\,J$$

(b) The work done by the weight.

The weight w does no work because it is perpendicular to the displacement ($\theta = 90°$), therefore:

$$W_w = w\,s\cos 90 = 0$$

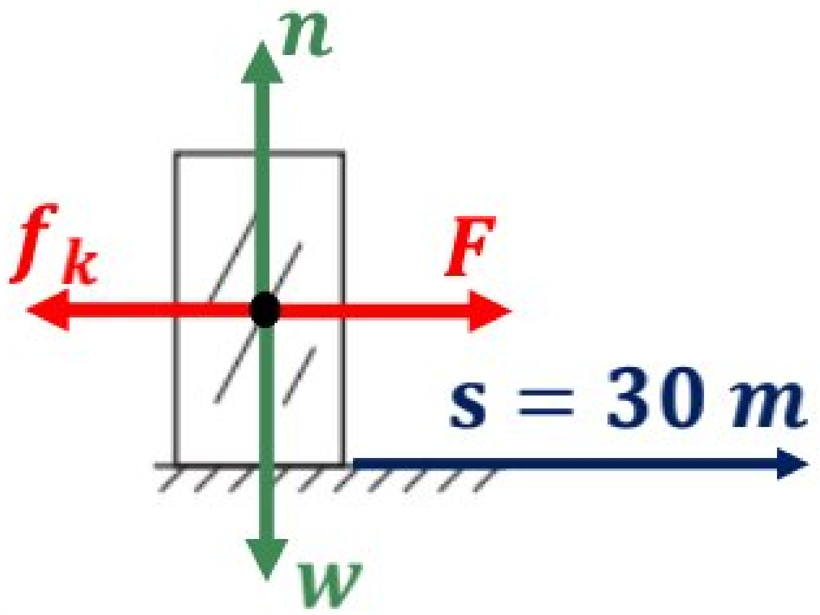
(c) The work done by the normal force.

The normal force n is perpendicular to the displacement ($\theta = 90°$):

$$W_n = n\,s\cos 90 = 0$$

(d) The work done by the friction force.

The angle between f_k and displacement is $\theta = 180°$ because they point in opposite directions, therefore:



$$W_{f_k} = -f_k s = -20 \times 30 = -600\,J$$

(e) The net work done on the box

$$W_{net} = \underbrace{W_w}_{0} + \underbrace{W_n}_{0} + W_F + W_f = 0 + 0 + 3000 - 600 = 2400\,J$$

2. KINETIC ENERGY AND THE WORK–ENERGY THEOREM

- **The Kinetic Energy is energy associated with the state of motion of an object.**
- **For an object of mass m and moving with a speed of v its kinetic energy (K) is:**

$$K = \frac{1}{2}mv^2$$

- **The KE is a positive scalar quantity with the same unit as work ($N.m$) or Joule (J)**
- **Work-Energy Theorem state that: The net (total) work done on an object is equal to its change in kinetic energy:**

$$W_{net} = F_{net}\,s = \Delta K = \frac{1}{2}mv_f^2 - mv_i^2$$

Example 6.4:

1. A $4\ kg$ mass has a speed of $25\ m/s$, what is its kinetic energy?

$$K = \frac{1}{2}mv^2 = \frac{1}{2} \times 4 \times 25^2 = 1250\ J$$

2. The kinetic energy of an object of mass m moving with a velocity of $5 m/s$ is $25\ J$. What is the mass m of the object?

$$K = \frac{1}{2}mv^2 \quad \Rightarrow m = \frac{2K}{v^2} = \frac{2 \times 25}{5^2} = 2\ kg$$

2. If the mass of a moving object was doubled, but its speed remained the same. By what factor is its kinetic energy changed?

$$(m_1, v_1) \Rightarrow \ K_1 = \frac{1}{2}m_1 v_1^2,$$

$$(m_2 = 2m_1, v_2 = v_1) \Rightarrow \ K_2 = \frac{1}{2}m_2 v_2^2 = \frac{1}{2} \times 2m_1\ v_1^2 = 2 \times \frac{1}{2}m_1 v_1^2 = 2K_1$$

Doubling the mass of the object doubles the kinetic energy. The factor that Kinetic energy change is 2.

2. If the speed of a moving object was doubled, but its mass remained the same. By what factor is its kinetic energy changed?

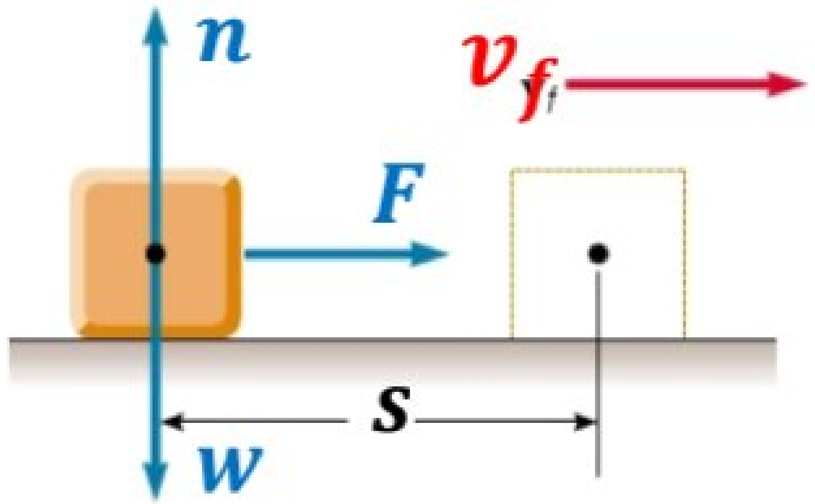
$$(m_1, v_1) \Rightarrow K_1 = \frac{1}{2} m_1 v_1^2,$$

$$(m_2 = m_1, v_2 = 2v_1) \Rightarrow K_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 (2v_1)^2 = 4 \times \frac{1}{2} m_1 v_1^2 = 4K_1$$

Doubling the speed of the object quadruples the kinetic energy. The factor that Kinetic energy change is **4**.

Example 6.5:

1. A $6.0\ \mathrm{kg}$ block initially at rest is pulled to the right along a horizontal, frictionless surface by a constant horizontal force of $12\ N$. Find the speed of the block after it has moved $3.0\ m$.



$$W_{net} = \underbrace{W_w}_{0} + \underbrace{W_n}_{0} + W_F = W_F = F\,s = 12 \times 3 = 36\ J$$

$$W_{net} = \Delta K = \frac{1}{2} m v_f^2 - m \underbrace{v_i^2}_{0} = \frac{1}{2} m v_f^2 \Longrightarrow v_f^2 = \frac{2W_{net}}{m} \Longrightarrow v_f = \sqrt{\frac{2W_{net}}{m}} = \sqrt{\frac{2 \times 36}{6}} = 3.5\ m/s$$