CHAPTER 3

Motion in Two or Three Dimensions

CONTENTS

1. POSITION AND VELOCITY VECTORS

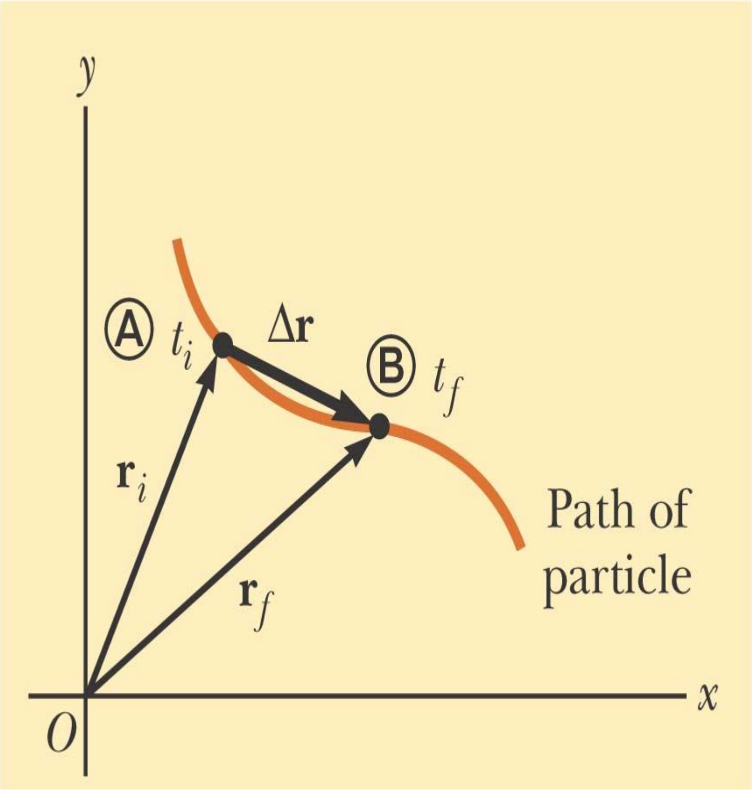
- **The position vector in two dimensions:**

$$\vec{r} = x\,\hat{\imath} + y\hat{\jmath}$$

- **The displacement vector of the object is defined as the change in its position:**

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath}$$

where $\vec{r}_2$ and $\vec{r}_1$ final and initial position



- **The average velocity vector during a time interval from t_1 to t_2:**

$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t} \qquad \text{(average velocity vector)}$$

$$v_{av-x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}, \qquad v_{av-y} = \frac{y_2 - y_1}{t_2 - t_1} = \frac{\Delta y}{\Delta t}$$

- **The instantaneous velocity vector:**

$$\vec{v} = \frac{d\vec{r}}{dt} \qquad \text{(instantaneous velocity vector)}$$

$$\vec{v} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} \quad \text{or} \quad \vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} \qquad \text{(the component of instantaneous velocity)}$$

  - the magnitude of the velocity: $v = \sqrt{v_x^2 + v_y^2}$
  - the direction of the velocity: $\theta = \tan^{-1}\left(\frac{|v_y|}{|v_x|}\right)$

Example 3.1:

A particle moves in the x-y plan. Its coordinates are given as function of time by: $x(t) = 4(m/s^2)t^2$ and $y(t) = 2(m/s)t$

(a) Find the position vector of the particle at times $t_1 = 0s$ and $t_2 = 2s$

$$\vec{r} = x\,\hat{\imath} + y\hat{\jmath} = 4t^2\hat{\imath} + 2\,t\,\hat{\jmath}$$

at $t_1 = 0s$: $\vec{r}_1 = \vec{0}$ (m)

at $t_2 = 2s$: $\vec{r}_2 = 4 \times 2^2\,\hat{\imath} + 2 \times 2\,\hat{\jmath} = 16\hat{\imath} + 4\hat{\jmath}$ (m)

(b) Find the average velocity vector of the particle in time interval $t_1 = 0s$ to $t_2 = 2s$

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = 16\hat{\imath} + 4\hat{\jmath} \quad (m)$$

$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t} = \frac{16\hat{\imath} + 4\hat{\jmath}}{2} = 8\hat{\imath} + 2\hat{\jmath} \quad (m/s)$$

(c) Find the instantaneous velocity vector of the particle at times $t_1 = 0s$ and $t_2 = 2s$

$$\vec{v} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} = \frac{d}{dt}(4t^2)\hat{\imath} + \frac{d}{dt}(2t)\hat{\jmath}$$

$$\vec{v} = 8t\hat{\imath} + 2\hat{\jmath} \qquad (m/s)$$

at $t_1 = 0s$: $\vec{v} = 2\hat{\jmath}$ (m/s)

at $t_2 = 2s$: $\vec{v} = 16\hat{\imath} + 2\hat{\jmath}$ (m/s)

<u>Remark</u> :

The speed of the particle at time $t_1 = 0s$:

$v = 2\,m/s$

The speed of the particle at time $t_2 = 2s$:

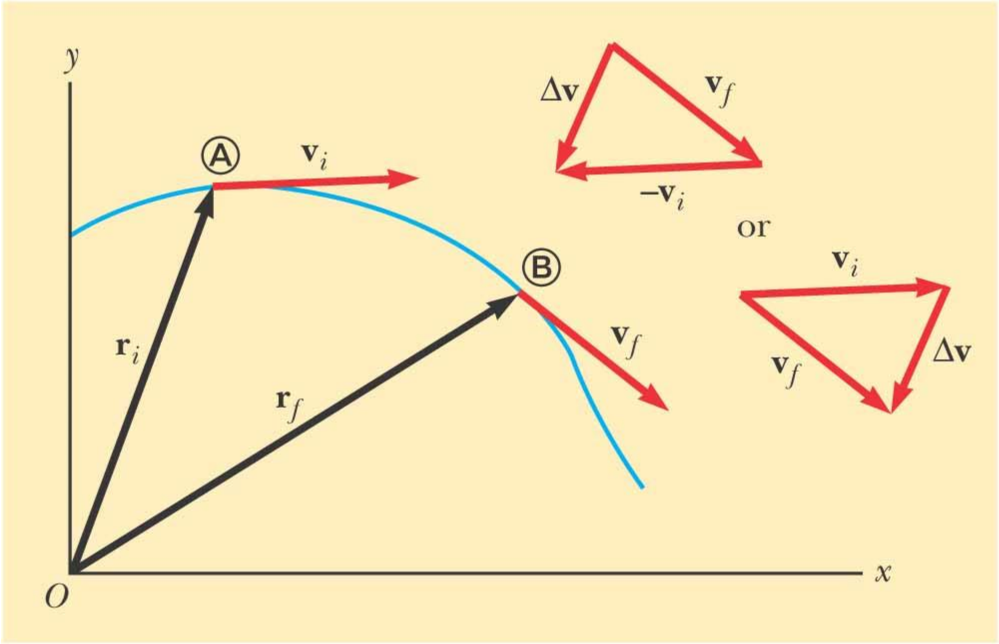
$v = \sqrt{16^2 + 2^2} = 16.12\,m/s$

2. THE ACCELERATION VECTOR

- The average acceleration during a time interval from t_1 to t_2 :

$$\vec{a}_{\text{av}} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}$$

(average acceleration vector)



- The Instantaneous acceleration vector :

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$a_x = \frac{dv_x}{dt} \qquad a_y = \frac{dv_y}{dt}$$

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath}$$ (components of the acceleration vector)

Example 3.2:

From example 3.1 the instantaneous velocity at any time is given by

$$\vec{v} = 8(m/s^2)t\hat{\imath} + 2(m/s)\hat{\jmath}$$

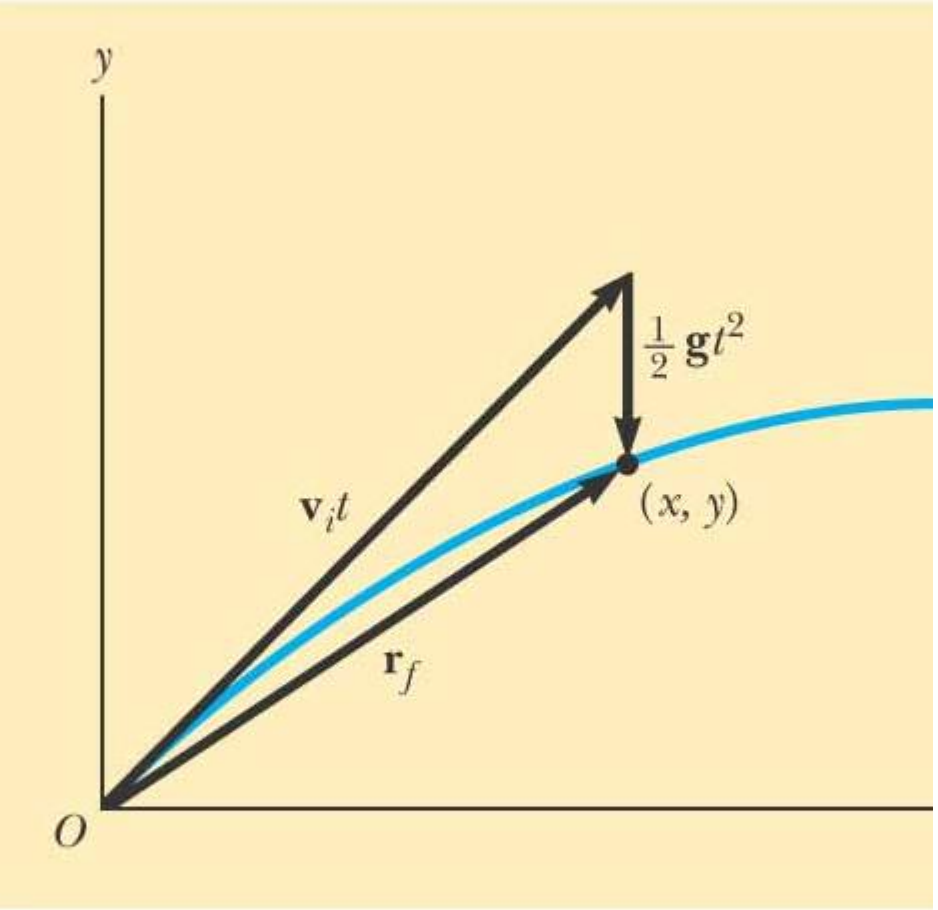
Find the instantaneous acceleration of the particle at time $t = 1s$

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath} = 8\hat{\imath} \quad (m/s^2)$$

at $t = 1s$: $\vec{a} = 8\hat{\imath} \ (m/s^2)$

3. PROJECTILE MOTION

- A projectile is any an object that is given an initial velocity and then follows a path determined entirely by the effects of gravitational acceleration and air resistance.
- The form of two-dimensional motion we will deal with is called projectile motion



- If air resistance is negligible, the trajectory of a projectile is a combination of horizontal motion with constant velocity and vertical motion with constant acceleration.
- The component initial velocity for the projectile from figure:

$$v_{x0} = v_0 cos\theta, \quad v_{y0} = v_0 sin\theta$$

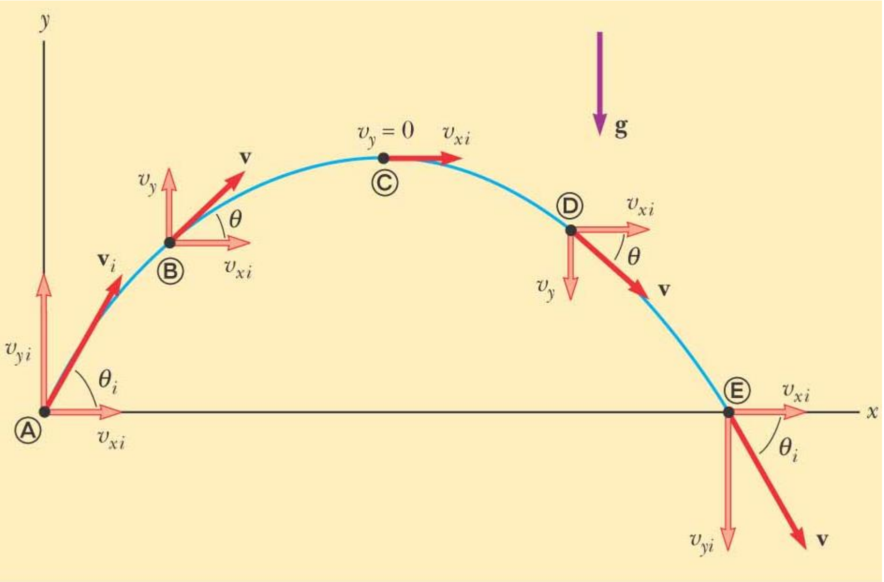
Example 3.3:

A ball is thrown with initial velocity of $18\, m/s$ and angle with x-axis $\theta = 30°$.

Find v_{x0}, v_{y0}

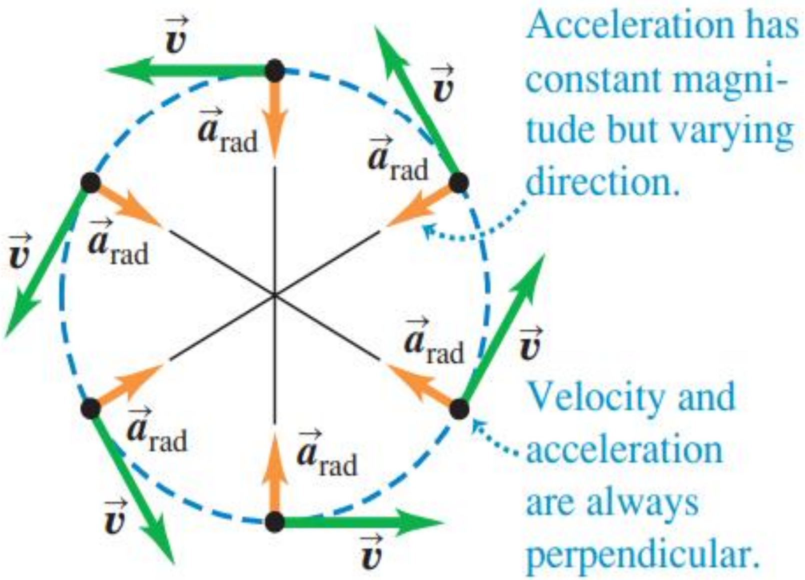
$$v_{x0} = v_0 cos\theta = 18\, cos30 = 15.6\, m/s$$

$$v_{y0} = v_0 sin\theta = 18\, sin\, 30 = 9\, m/s$$



4.MOTION IN A CIRCLE

- When a particle moves along a curved path, the direction of its velocity changes. As we saw in figure, this means that the particle must have a component of acceleration perpendicular to the path, even if its speed is constant.



Uniform Circular Motion:

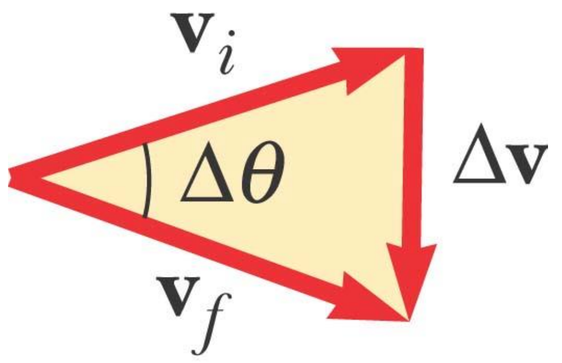
- When a particle moves in a circle with constant speed, the motion is called uniform circular motion

Centripetal acceleration:

- The magnitude of centripetal acceleration is given by:

$$a_c = \frac{v2}{R}$$ (uniform circular motion)

- Its direction toward the center of the circle of motion
- The figure shown the change in direction of the uniform speed on circle motion



Example 3.4:

A bicycle move on the circle path ($R = 6m$) with uniform speed is $5.8\,m/s$ find its centripetal acceleration.

$$a_c = \frac{v2}{R} = \frac{(5.8)^2}{6} = 5.61\ m/s^2$$