CHAPTER 1

Units, Physical Quantities, and Vectors

CONTENTS

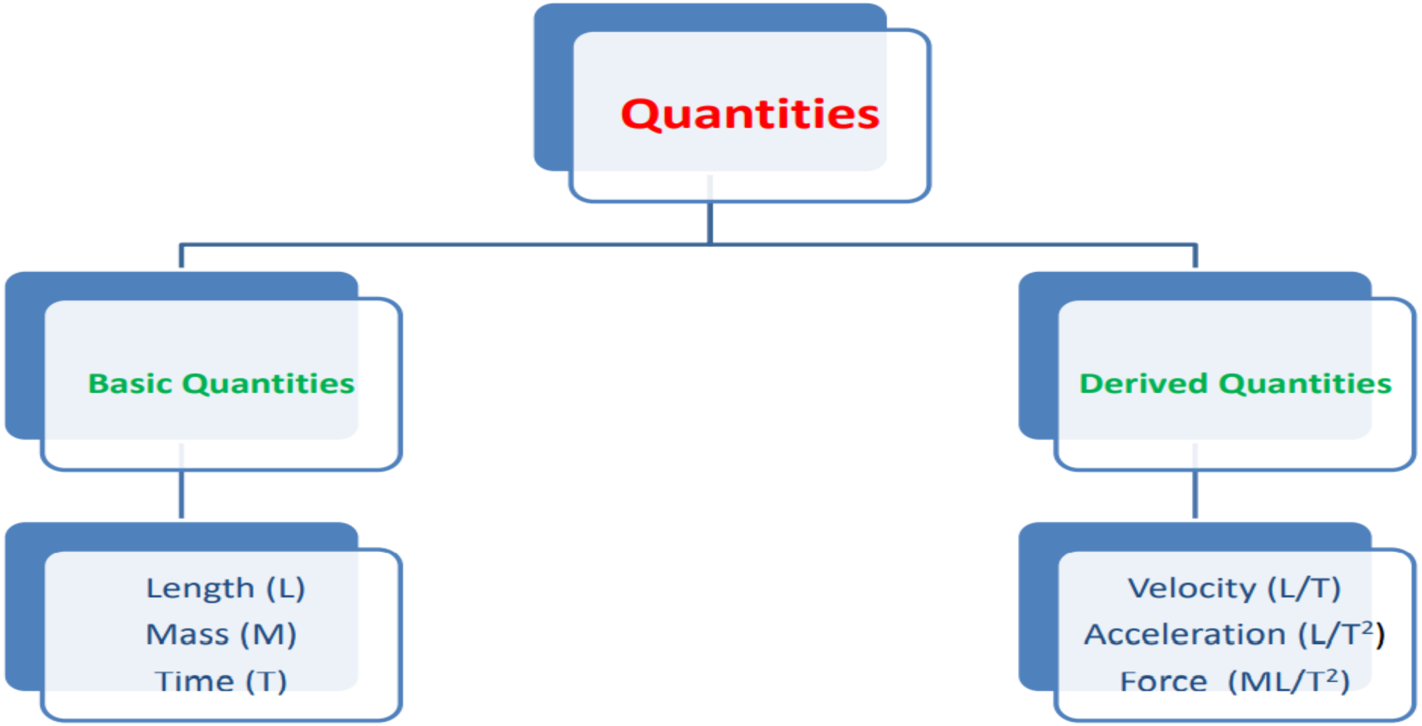
PHYSICAL QUANTITY

Physical quantity = numerical value × unit

Examples:

mass: m=70 kg

force: F=50 N



1. STANDARDS AND UNITS

SI units (International System of units)

Quantity	Unit	Symbol
Length	Meter	m
Mass	Kilogram	Kg
Time	Second	s
Electric current	Ampere	A
Temperature	Kelvin	K
Amount of substance	Mole	mol
Luminous intensity	Candela	cd

Examples:

$10^{-9}\text{m} = 1\text{ nm}$

$10^{-6}\text{m} = 1\ \mu\text{m}$

$10^{-3}\text{m} = 1\text{ mm}$

$10^{-2}\text{m} = 1\text{ cm}$

$10^{3}\text{m} = 1\text{ km}$

Prefixes for Powers of Ten

Power	Prefix	Abbreviation
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T

2. USING AND CONVERTING UNITS

Example 1.1:

1. Express the area of $6.0\ \mathrm{cm}^2$ in m^2

$$6.0\ \mathrm{cm}^2 = (6.0)\left(10^{-2}\mathrm{m}\right)^2 = 6.0 \times 10^{-4}\mathrm{m}^2$$

2. Express the speed of $130\ \mathrm{km/h}$ in $\mathrm{m/s}$

$$1\ \mathrm{km} = 10^3\mathrm{m} \quad \text{and} \quad 1\ \mathrm{h} = 3600\ \mathrm{s}$$

$$130\frac{\mathrm{km}}{\mathrm{h}} = \left(130\frac{\mathrm{km}}{\mathrm{h}}\right)\left(\frac{1000\ \mathrm{m}}{\mathrm{km}}\right)\left(\frac{\mathrm{h}}{3600\ \mathrm{s}}\right) = 36.1\ \mathrm{m/s}$$

3. VECTORS AND VECTOR ADDITION

Physical Quantities

Scalar

- Magnitude
- No direction

Examples:

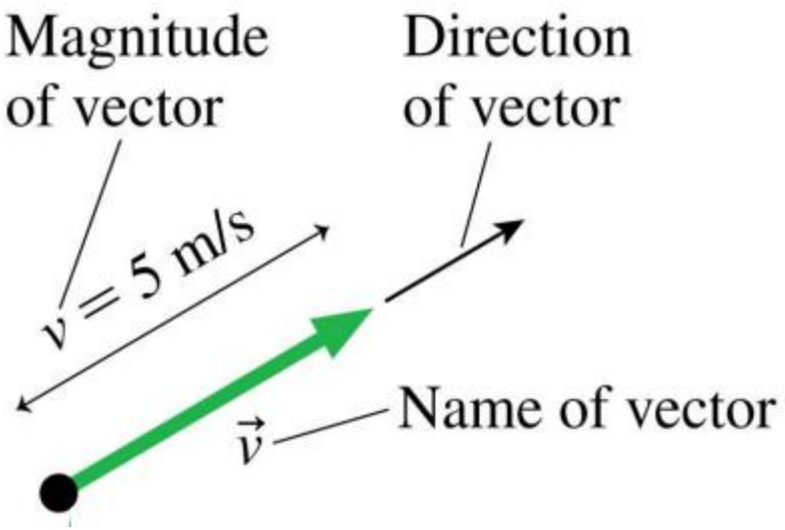
Length, mass, time, electric current, temperature, area, distance, speed, energy, density, pressure, power

Vector

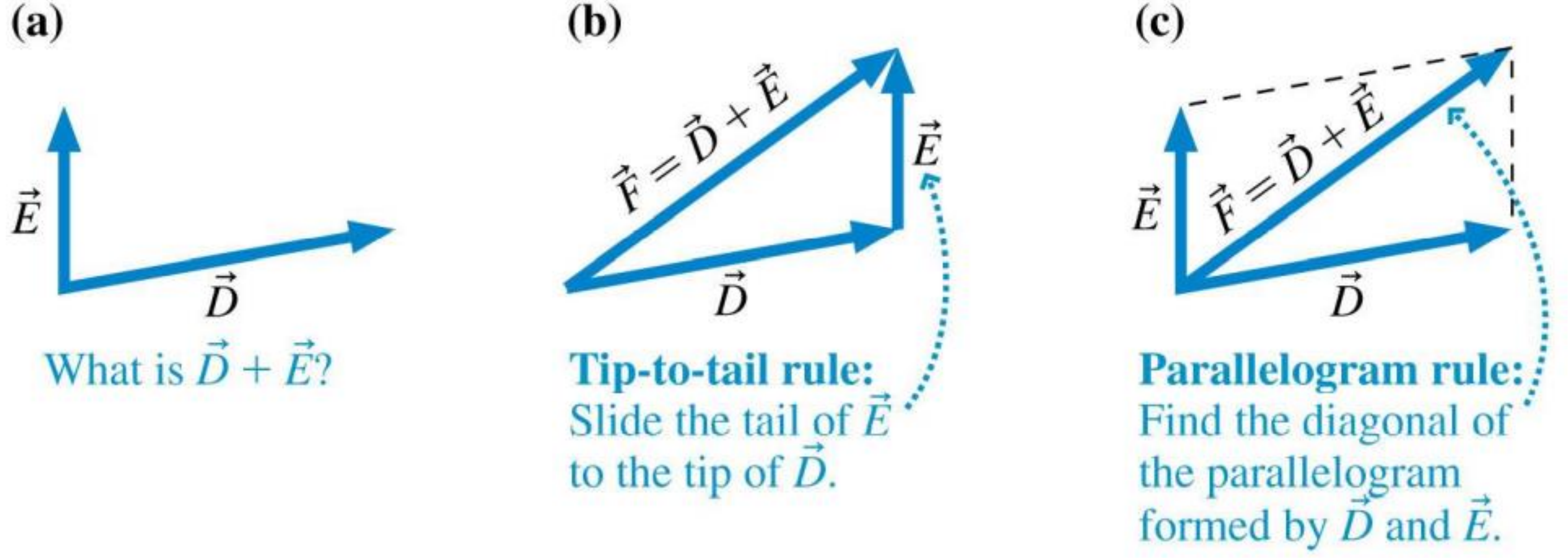
- Magnitude
- Direction

Examples:

Displacement, velocity, force, acceleration, momentum, weight impulse



VECTOR ADDITION



4. COMPONENTS OF VECTORS

A vector,

$\vec{A} = (A_x, A_y)$

The magnitude of a vector,

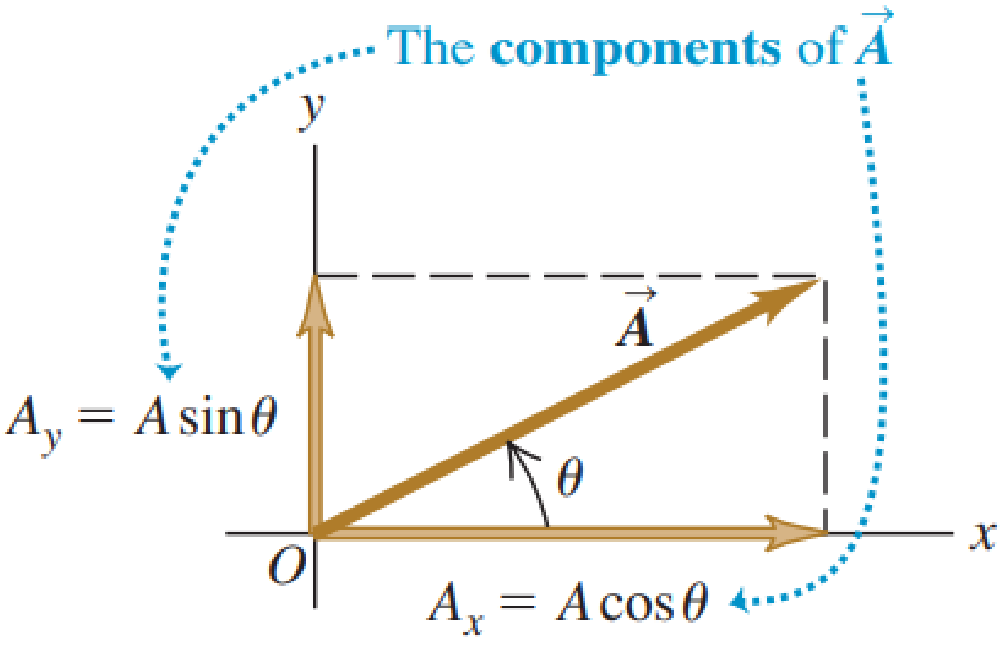
$A = \sqrt{A_x^2 + A_y^2}$

The x-component of a vector,

$A_x = A\cos\theta$

The y-component of a vector,

$A_y = A\sin\theta$



The direction of a vector

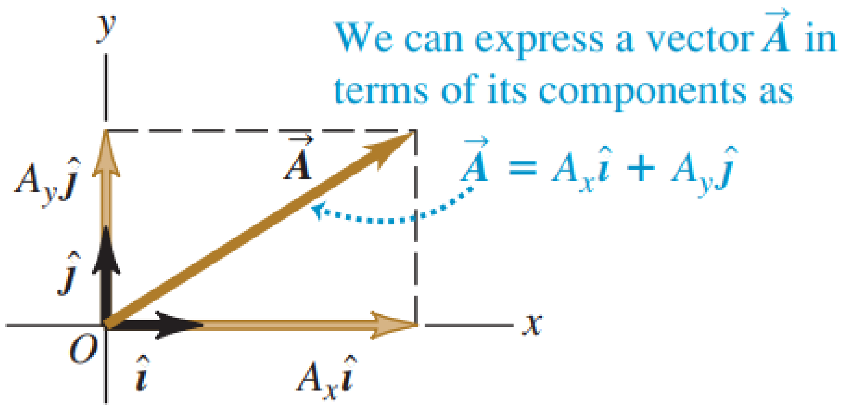
$$\theta = \tan^{-1}\left(\frac{|A_y|}{|A_x|}\right)$$

Quadrant / Component	1st	2nd	3rd	4th
A_x	(+)	(-)	(-)	(+)
A_y	(+)	(+)	(-)	(-)
Direction	θ	$180 - \lvert\theta\rvert$	$180 + \lvert\theta\rvert$	$360 - \lvert\theta\rvert$

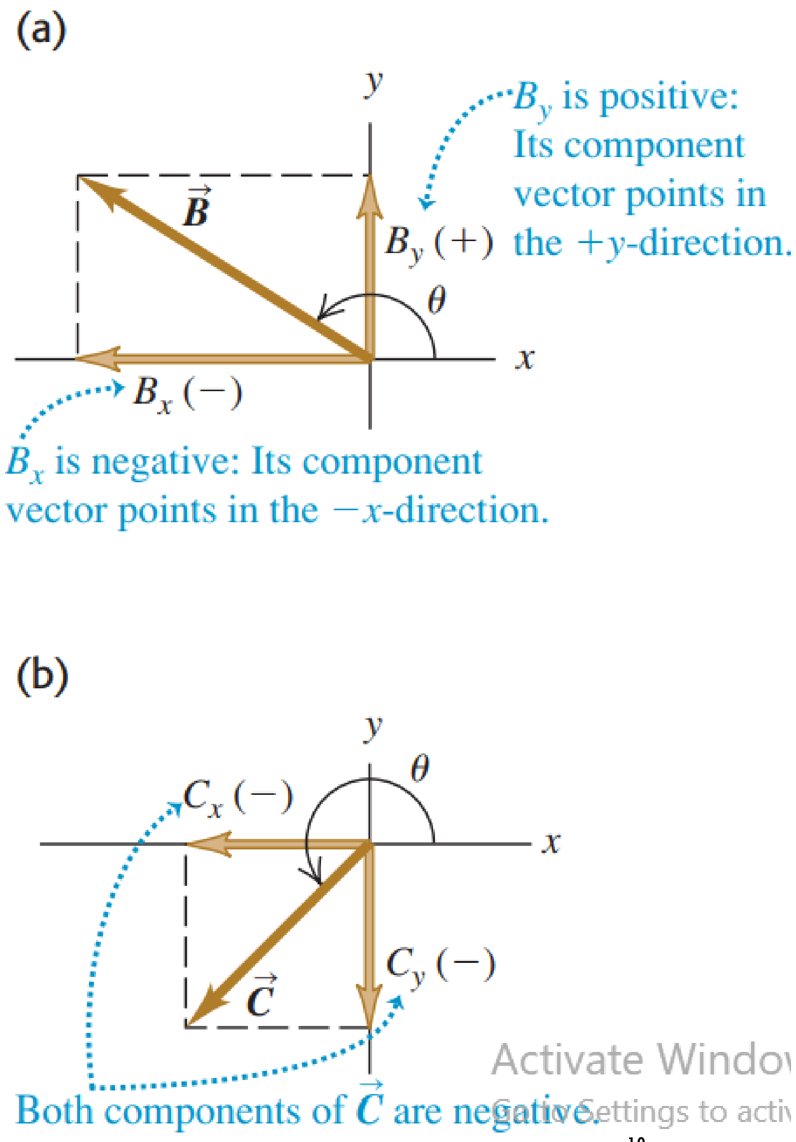
5. UNIT VECTORS:

$|\hat{\imath}| = |\hat{\jmath}| = 1$

$\vec{A} = A_x\,\hat{\imath} + A_y\,\hat{\jmath}$



The components of a vector may be positive or negative numbers.



Example 1.2:

Find the x- and y- components of the vector $\vec{D}$?

The magnitude of the vector is $D = 3.0$ m, and the angle $\alpha = 45^o$.

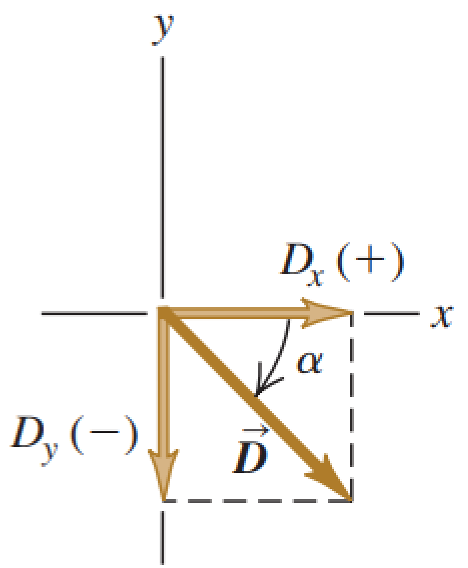
The direction of the vector $\vec{D}$ in 4th quadrant is:

$$\theta = 360 - \alpha = 360 - 45 = 315^o$$

Then,

$$D_x = D\cos\theta = 3\cos 315 = +2.1$$

$$D_y = D\sin\theta = 3\sin 315 = -2.1$$



Sum and subtraction of two vectors:

$$\vec{C} = \vec{A} \pm \vec{B} \Longrightarrow C_x = A_x \pm B_x \text{ and } C_y = A_y \pm B_y$$

Example 1.3: $\vec{A} = 2\,\hat{\imath} - 2\,\hat{\jmath}$ and $\vec{B} = 4\,\hat{\imath} - 6\,\hat{\jmath}$. Find:

a) $\vec{C} = \vec{A} + \vec{B}$. b) $\vec{D} = \vec{A} - \vec{B}$. c) the magnitudes of $\vec{C}$ and $\vec{D}$.

e) the directions of $\vec{C}$ and $\vec{D}$.

a) $C_x = 2 + 4 = 6$ and $C_y = -2 - 6 = -8 \Longrightarrow \vec{C} = 6\,\hat{\imath} - 8\,\hat{\jmath}$

b) $D_x = 2 - 4 = -2$ and $D_y = -2 + 6 = 4 \Longrightarrow \vec{D} = -2\,\hat{\imath} + 4\,\hat{\jmath}$

c) $\mathrm{C} = \sqrt{(6)^2 + (-8)^2} = 10$ and $\mathrm{D} = \sqrt{(-2)^2 + (4)^2} = 4.47$

e) $\theta_C = \tan^{-1}\left(\frac{8}{6}\right) = 53^o \Longrightarrow$ direction of $\vec{C} = 360 - 53^o = 307^o$

$\theta_D = \tan^{-1}\left(\frac{4}{2}\right) = 63^o \Longrightarrow$ direction of $\vec{D} = 180 - 63^o = 117^o$

6. PRODUCTS OF VECTORS

The scalar product of two vectors:

$$\vec{A} = A_x\,\hat{\imath} + A_y\,\hat{\jmath} \qquad \vec{B} = B_x\,\hat{\imath} + B_y\,\hat{\jmath}$$

$$\hat{\imath}\,.\,\hat{\imath} = \hat{\jmath}.\hat{\jmath} = 1 \text{ and } \hat{\imath}\,.\,\hat{\jmath} = \hat{\jmath}.\hat{\imath} = 0$$

$$\vec{A}\,.\,\vec{B} = A_x B_x + A_y B_y$$

Example 1.4:

Calculate the scalar product of the two vectors:

$$\vec{A} = 2\,\hat{\imath} + 3\,\hat{\jmath} \qquad \vec{B} = 4\,\hat{\imath} - 5\,\hat{\jmath}$$

$$\vec{A}\,.\,\vec{B} = 2 \times 4 + 3 \times (-5) = 8 - 15 = -7$$