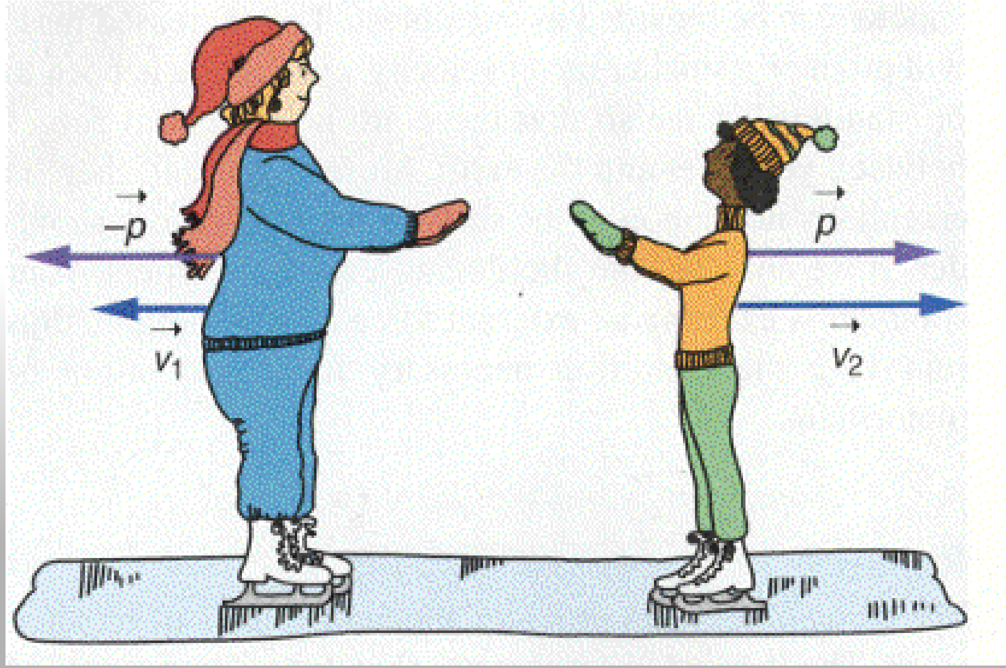
Chapter 9
Linear Momentum & Collisions



Units of Chapter 9

- Linear Momentum
- Momentum and Newton's Second Law
- Impulse
- Conservation of Linear Momentum
- Inelastic Collisions
- Elastic Collisions

Linear Momentum

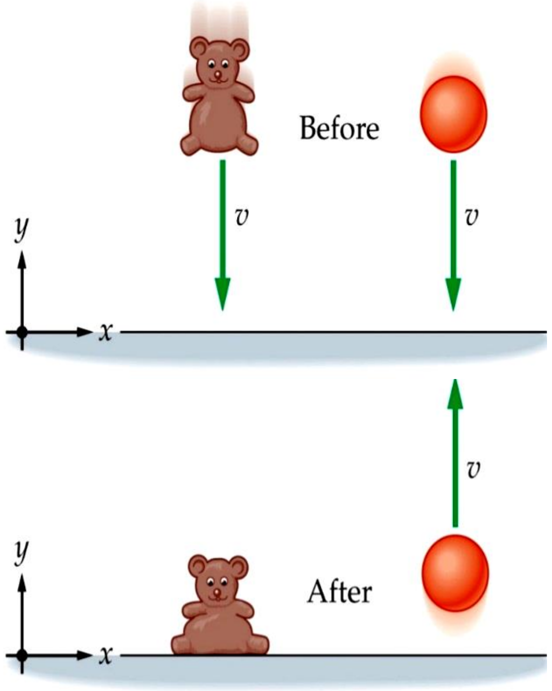
The linear momentum $\vec{P}$ of an object of mass m that is moving in a straight line with a velocity $\vec{v}$ is defined by:

$$\vec{P} = m\vec{v}$$

$\vec{P}$ *is a vector*
SI unit: $kg.m/s$

Change in Momentum: $\Delta\vec{p} = \vec{p}_f - \vec{p}_i$

- Teddy Bear: $\Delta p = 0 - (-mv) = mv$
- Bouncing Ball: $\Delta p = mv - (-mv) = 2mv$



Exercise 9-1

1. A 1180 kg car drives along a city street at 30.0 miles per hour (13.4 m/s). What is the magnitude of the car's momentum?

<u>Answer</u>: 15,800 kg.m/s

2. A major-league pitcher can give a 0.142 kg baseball a speed of 101 mi/h (45.1 m/s). Find the magnitude of the baseball's momentum?

<u>Answer</u>: 6.40 kg.m/s

Important Note

A slow-moving object and an extremely-fast object can have the same momentum

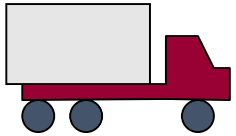
- Huge ship moving at a small velocity

$$P = m_v$$

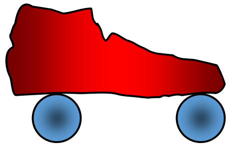
- High velocity bullet

$$P = {}_{m}V$$

- Slow moving truck



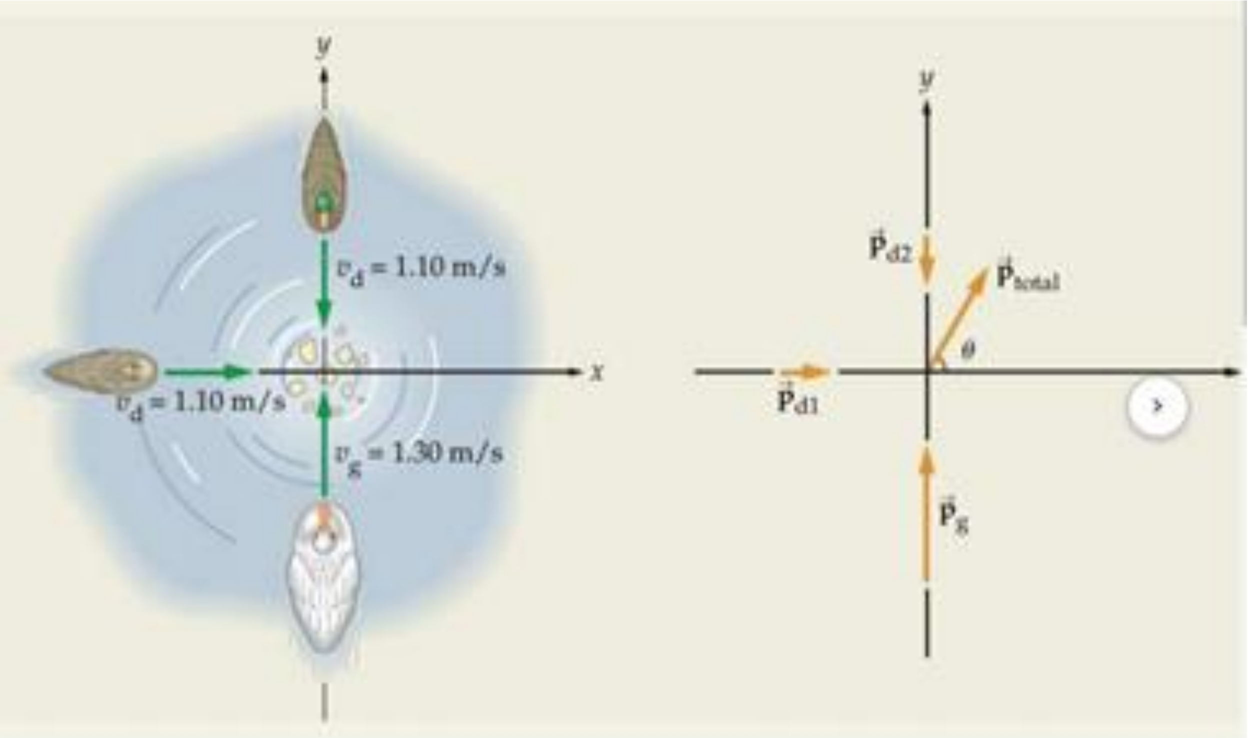
- Fast roller skate



Example 9-1

At a city park, a person throws some bread into a duck pond. Two 4.00 kg ducks and a 9.00 kg goose paddle rapidly toward the bread, as shown in the sketch. If the ducks swim at 1.1 m/s and the goose swims with a speed of 1.3 m/s, find the magnitude and direction of the total momentum of the three birds?

- P_{total} = (4.40 kg.m/s)i + (7.30 kg.m/s)j
- Magnitude of P = 8.52 kg.m/s
- $\theta = 58.9^{o}$



Problems

Problem 3: A 26.2 kg dog is running northward at 2.7 m/s while a 5.3 kg cat is running eastward at 3.04 m/s. Their 74.0 kg owners has the same momentum as the two pets taken together. Find the direction and magnitude of the owner's velocity?

Answers:

Direction =77.1^{o} north of east

Magnitude= 0.985 m/s

Problems

Problem 5: A 0.15 kg baseball is dropped from rest. If the magnitude of the baseball's momentum is 0.780 kg.m/s just before it lands on the ground, from what height was it dropped?

Answer:

$h = 1.38$ m

9.2 Momentum and Newton's Second Law

Newton's second law states that:

$$\sum \vec{F} = m\vec{a}$$

It is only valid for objects that have constant mass. The following is a more general form of Newton's second law, also useful when the mass is changing:

$$\sum \vec{F} = \frac{\Delta \vec{P}}{\Delta t}$$

9.3 Impulse

- An impulse is a **force** exerted over a finite period of **time**
- A **net impulse** will **change the momentum** of an object in accordance with Newton's second law

Definition of Impulse, $\vec{I}$

$$\vec{I} = \vec{F}_{av}\Delta t$$

SI unit: $N \cdot s = kg \cdot m/s$

$$\vec{F}_{avg} = m\vec{a} = \frac{m\Delta\vec{v}}{\Delta t} = \frac{\Delta\vec{P}}{\Delta t}$$

$$\vec{I} = \Delta\vec{P} = \vec{F}_{avg}\Delta t$$

- Impulse is a vector, in the same direction as the average force.
- The impulse is equal to the change in momentum.

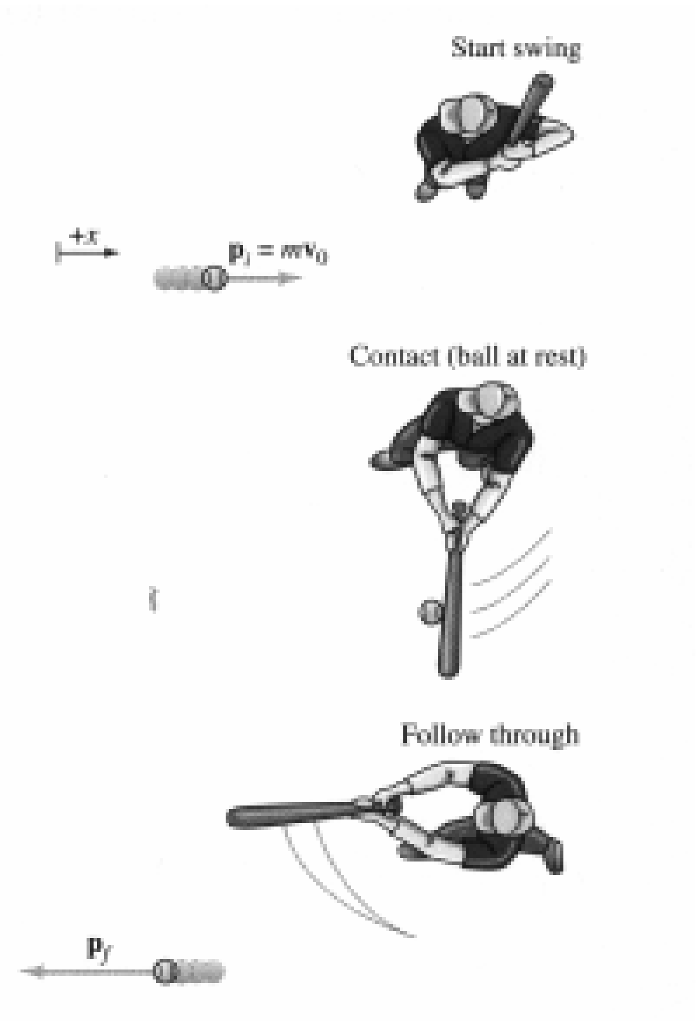
Exercise

A 0.2 kg baseball traveling at 40 m/s is hit and returned at 50 m/s in the opposite direction. If the ball and bat are in contact for 0.002 s, determine the **average force on the ball?**

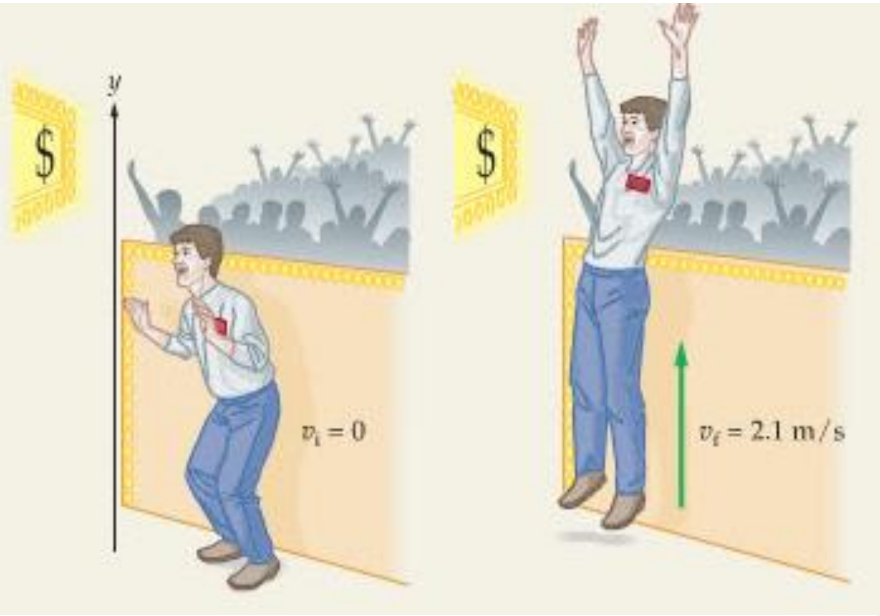
Given: $m = 0.2\ kg;\ v_i = \frac{40m}{s};\ v_f = 50\frac{m}{s};\ t = 0.002s$

Answer:

$F = -9000\ N$



Example



Example 9.2: After wining a prize on a game shown, a 72 kg contestant jumps for joy.

a) If the jump results in an upward speed of 2.1 m/s, what is the impulse experienced by the contestant?

b) Before the jump, the floor exerts an upward force of (mass×g) on the contestant. What additional average upward force does the floor exert if the contestant pushes down on it for 0.36 s during the jump?

Given: $m = 72kg, \quad v_f = 2.1m/s \,, \quad t = 0.36s$

Answer:
$F = (420\hat{y})\ N$

9-4 Conservation of Linear Momentum

The net force acting on an object is the rate of change of its momentum:

$$\sum \vec{F} = \frac{\Delta \vec{P}}{\Delta t}$$

If the net force is zero, the momentum does not change.

Conservation of Momentum:

If the net force acting on an object is zero, its momentum is conserved; that is ,

$$\vec{P}_f = \vec{P}_i$$

Example

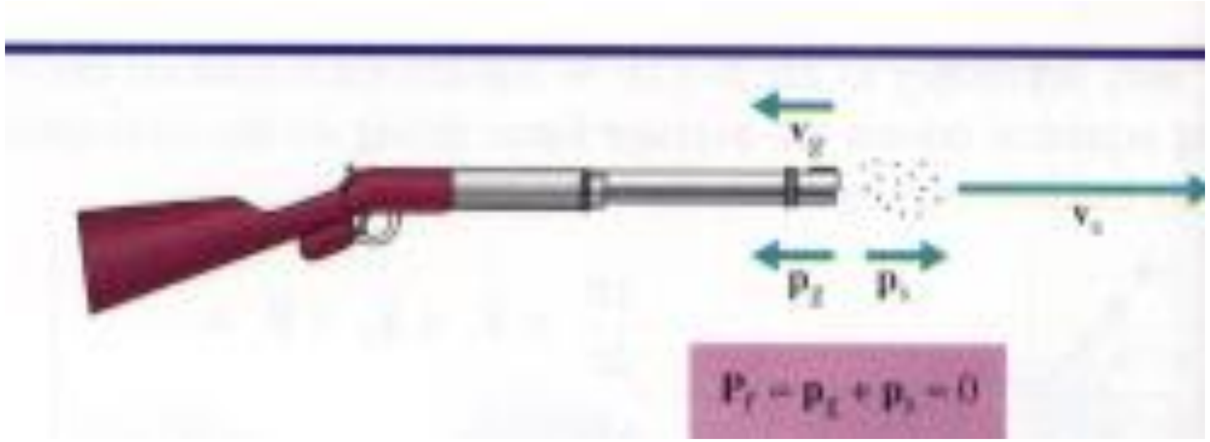
Example : If a shotgun of mass 3.0 kg fires shot having a total mass of 0.05 kg with a muzzle velocity of 525 m/s, what is its recoil velocity?

Note: (When you fire a bullet from the gun, the gun experience a force in the backward direction. Because of this force, **the gun attains a velocity in the backward direction**. This velocity is known as recoil velocity).

Given: $m_g = 3.0kg,$ $m_s = 0.05kg,$ $v_s = 525m/s$, $v_g = ???$

$$\vec{P}_f = \vec{P}_i$$

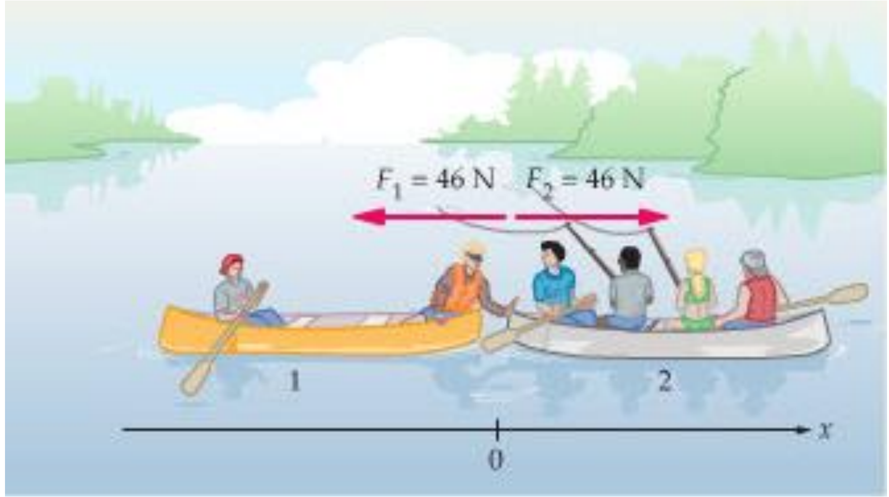
$$(m_g + m_s)v_i = m_g v_g + m_s v_s$$



Answer:

$v_g =$ - 9 m/s

Example



Example 9.3: Two groups of canoeists meet in the middle of the lake. After a brief visit, a person in canoe 1 pushes on canoe 2 with a force of 46 N to separate the canoes. If the mass of the canoe 1 and its occupants is m_1 = 130 kg, and the mass of the canoe 2 and its occupants is m_2 = 250 kg. find the momentum of each canoes after 1.2 s of pushing?

Given: $m_1 = 130\ kg,\ \ m_2 = 250\ kg\ \ ,\ \ t = 1.2\ s$

Answers:

P_1= - 55 kg. m/s , P_2= 55 kg. m/s

Problems

Problem 22

Two ice skaters stand at rest in the center of an ice rink. When they push off against one another the 45 kg skater acquires a speed of 0.62 m/s. If the speed of the other skater is 0.89 m/s, what is this skater's mass.

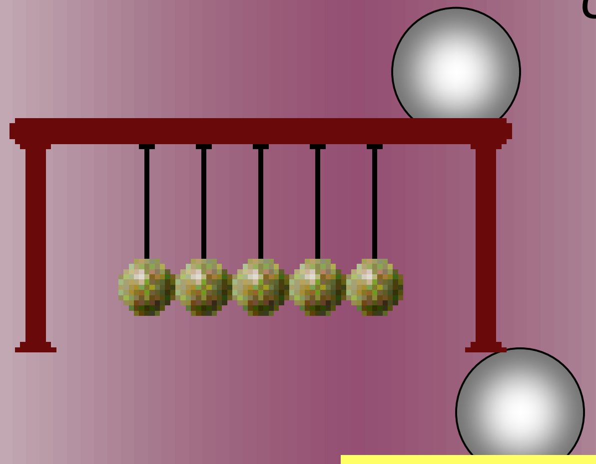
Answer

M = 31 kg

COLLISIONS

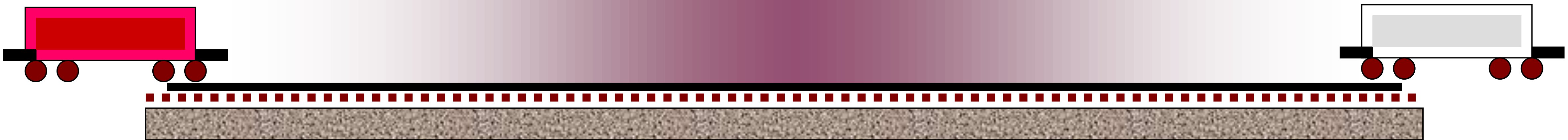
- **ELASTIC COLLISIONS**

Momentum transfer from one Object to another .



- **INELASTIC COLLISIONS**

Is a Newton's cradle like the one Pictured here, an example of an elastic or inelastic collision?



Conservation of Momentum in Collisions

- Note: the time of collision is short enough that external forces may be ignored

  Hence, momentum is conserved.

- Two types of collisions:

1. **Inelastic Collision:**

$$\vec{p}_f = \vec{p}_i \text{ but } K_f \neq K_i$$

- Completely inelastic collision: objects stick together afterwards: $\vec{p}_f = \vec{p}_{i,1} + \vec{p}_{i,2}$

1. **Elastic Collision:**

$$\vec{p}_f = \vec{p}_i \text{ and } K_f = K_i$$

Inelastic Collisions in One Dimension

For a **completely inelastic** collision

$$p_i = m_1 v_{1,i} + m_2 v_{2,i}$$
$$p_f = (m_1 + m_2) v_f$$

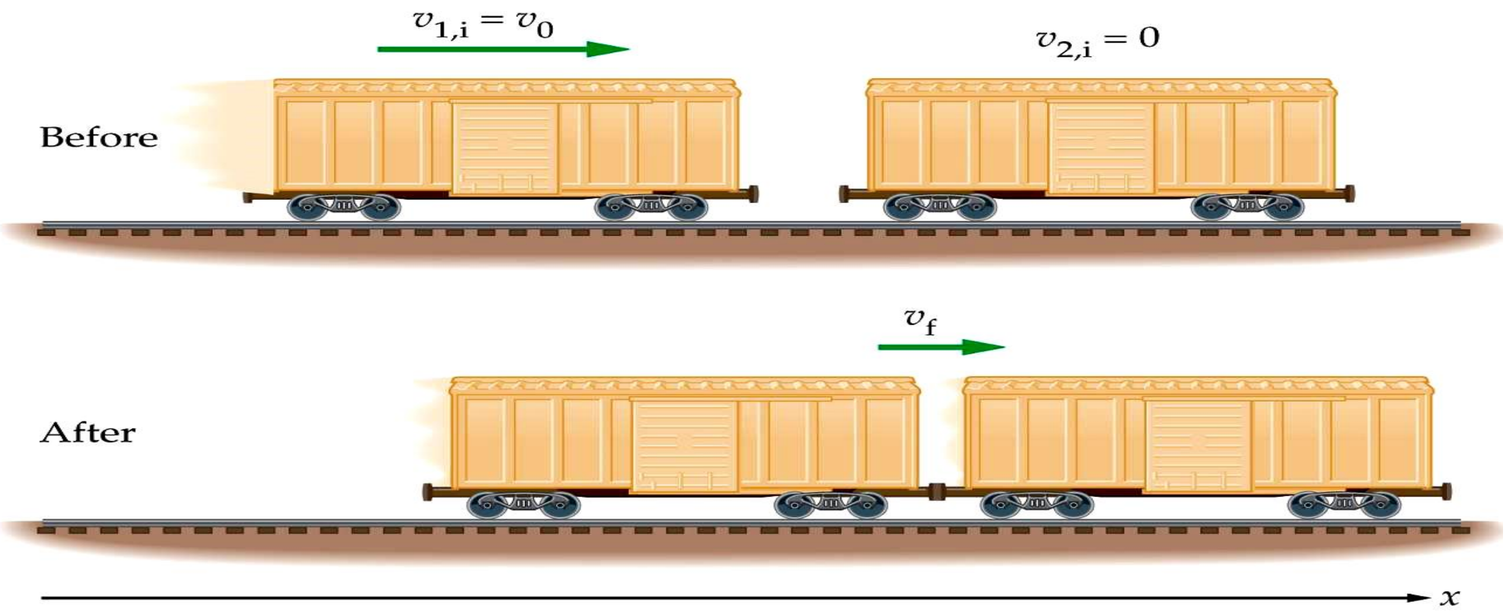
$$v_f = \frac{m_1 v_{1,i} + m_2 v_{2,i}}{m_1 + m_2}$$

Example:

A special case of a completely inelastic collision is when $v_{2,i} = 0$

we obtain:

$$v_f = \frac{m_1}{m_1 + m_2} v_{1,i}$$



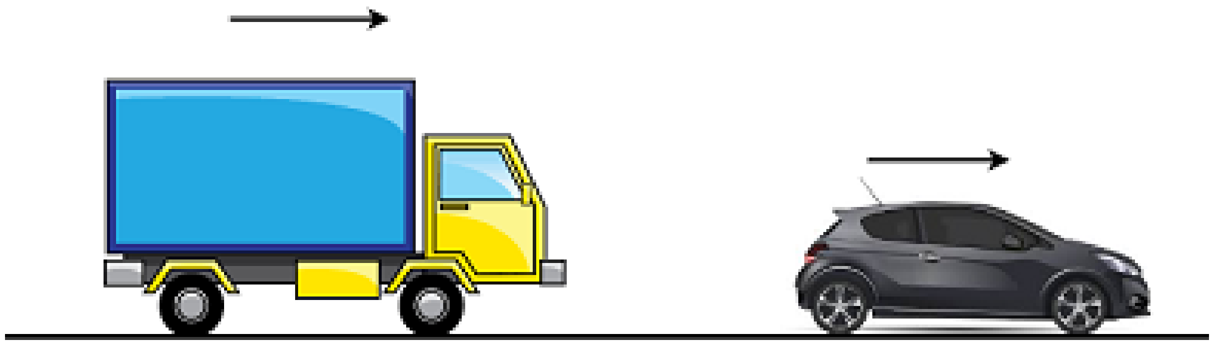
Exercise 9-2

A 1200 kg car moving at 2.5 m/s is struck in the rear by a 2600 kg truck moving at 6.2 m/s.

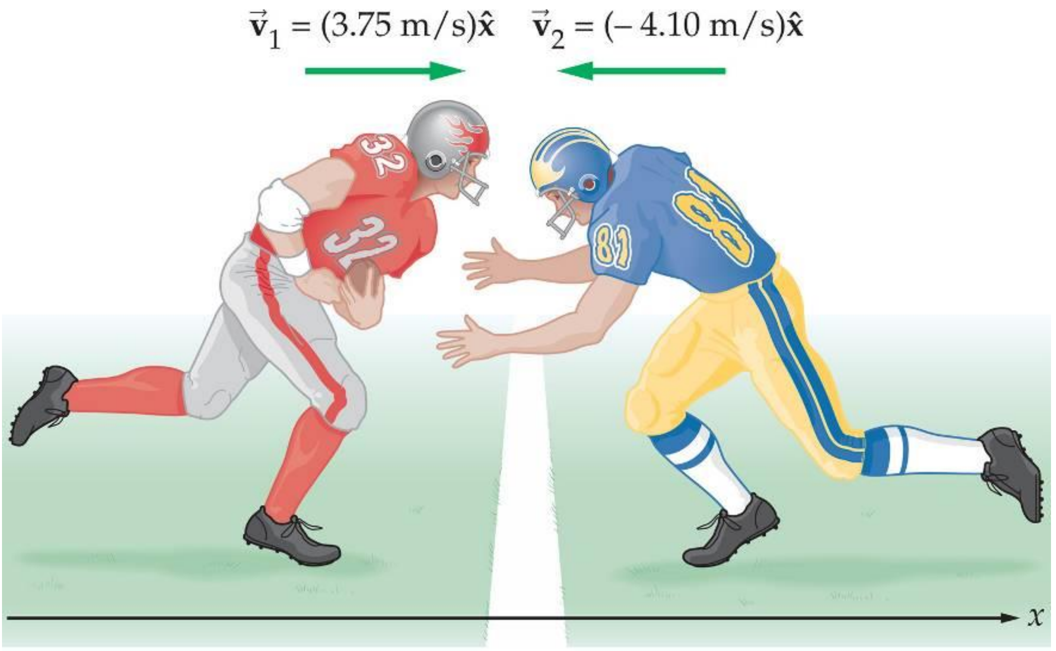
If the vehicles stick together after the collision, what is their speed immediately after colliding ?

(Assume that external forces may be ignored)

Answer: 5.0 m/s



Inelastic Collisions in One Dimension



Example: On a touchdown attempt, a 95.0 kg running back runs toward the end zone at 3.75 m/s. A 111 kg line backer moving at 4.10 m/s meets the runner in a head-on collision and locks his arms around the runner.

(a) Find their velocity immediately after the collisions.

(b) Find the initial and final kinetic energies and the energy lost in the collision.

Answers:

$v_f = -0.48 \text{ m/s}$ $K_i = 1600 \text{ J}$ $K_f = 23.6 \text{ J}$ $\Delta K = -1576 \text{ J}$

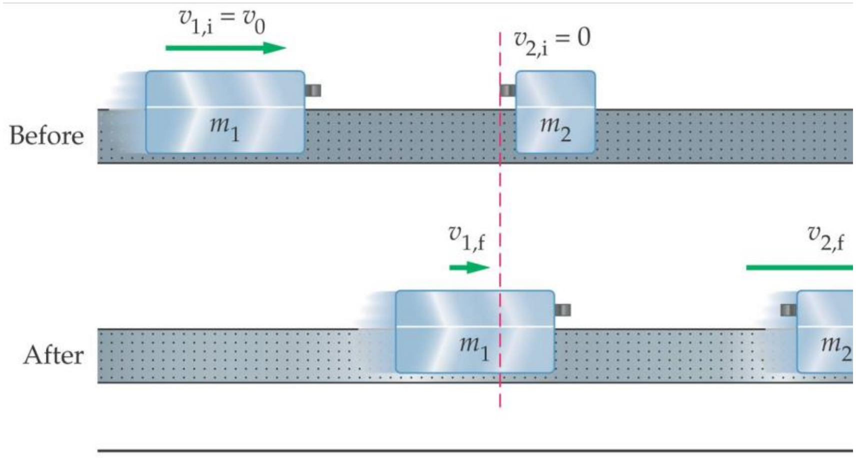
Elastic Collisions in One Dimension

Special case: If $\boldsymbol{v}_{1,i} = \boldsymbol{v}_0$ and $\boldsymbol{v}_{2,i} = 0$:
(that is, initially one mass is moving and the second mass is at rest)

This leads to two equations (conservation of momentum and kinetic energy equations) and two unknowns (the final speeds). Solving for the final speeds, we get:

$$v_{1,f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_0$$

$$v_{2,f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_0$$



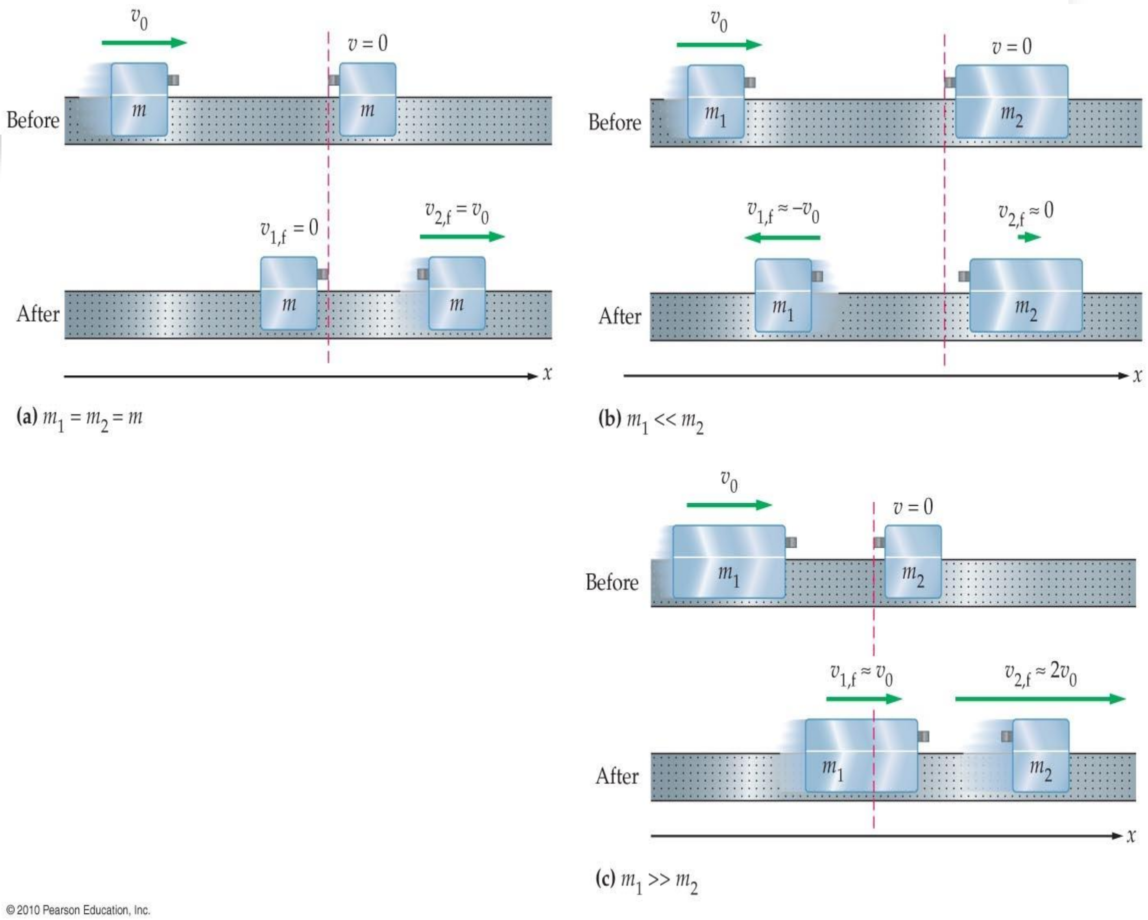
Elastic Collisions in One Dimension

- Note: For the same previous special case ($v_{1,i} = v_0$ and $v_{2,i} = 0$)
- If the two masses were equal:

after setting $m_1 = m_2$ in the previous equations we get:

$$v_{1,f} = 0$$
$$v_{2,f} = v_0$$

That is, the first mass (the moving mass) completely stops after collision, while the second mass (at rest) starts to move with the same initial velocity of the first mass.



Example

At an amusement park, a 96 kg bumper car moving with a speed of 1.24 m/s bounces elastically off a 135 kg bumper car at rest.

Find the final velocities of the cars.

Answers : $V_{f1} = -0.209$ m/s, $V_{f2} = 1.03$ m/s

Impulse

Example: A 10 kg cart collides with a wall and changes its direction, as shown. What is its change in x-momentum Δp_x?

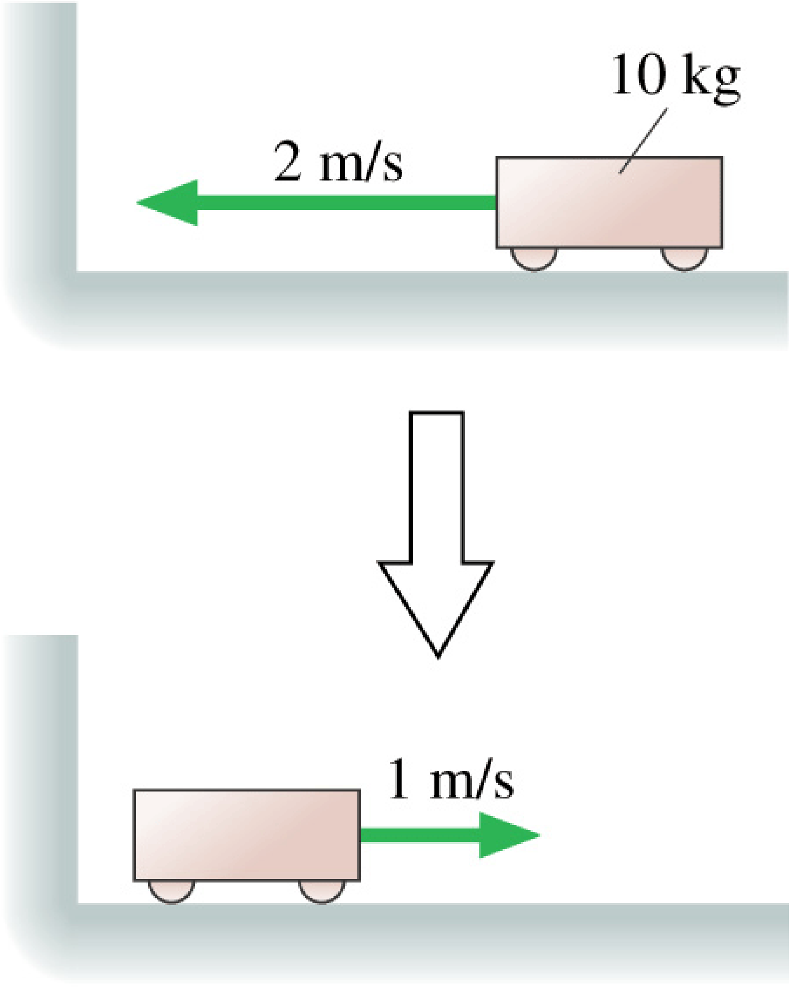
a) 30 kg.m/s left

b) 10 kg.m/s left

c) 10 kg.m/s right

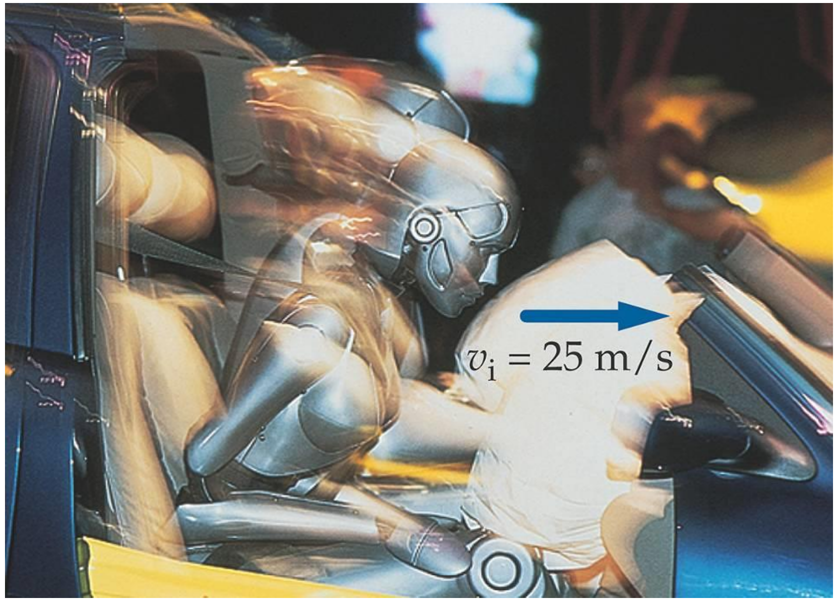
d) 20 kg.m/s right

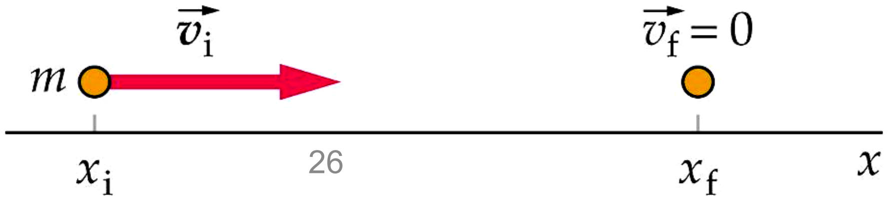
e) 30 kg.m/s right



Impulse

A car equipped with an 80 kg crash dummy drives into a massive concrete wall at 25 m/s and the dummy stops in 0.08 s. Estimate the average force exerted on the dummy.





Answers

$$\vec{I} = \vec{F}_{av}\Delta t = \Delta\vec{p} = m\Delta\vec{v}$$
$$= (80\text{ kg})(-25\text{ m/s }\hat{x}) = -2000\text{ N s }\hat{x}$$

$$\vec{F}_{av} = I/\Delta t = (-2000\text{ N s }\hat{x})/(0.080\text{ s}) = -25{,}000\text{ N }\hat{x}$$

Problems

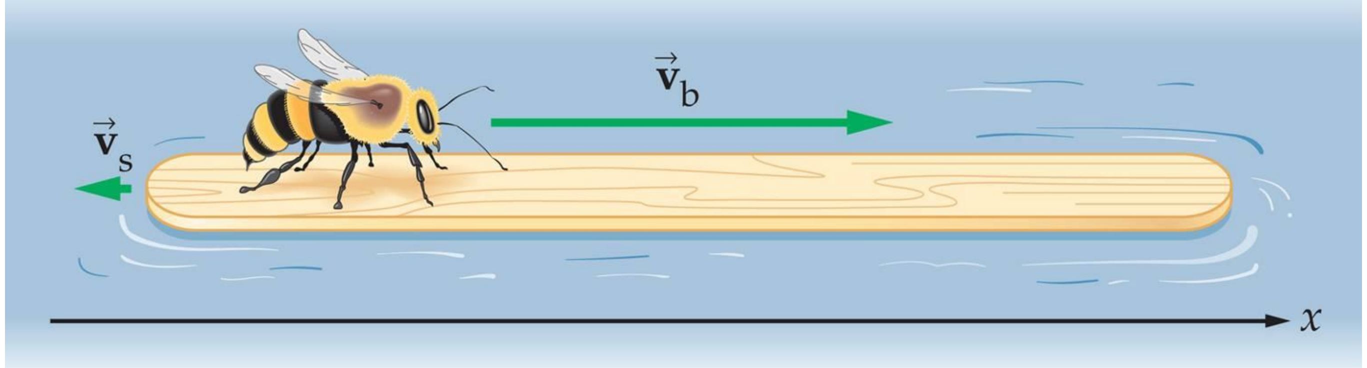
Problem 13: Find the magnitude of the impulse delivered to a soccer ball when a player kicks it with a force of 1250 N. Assume that the player's foot is in contact with the ball for 5.95x 10^{-3} s.

Answer: 7.44 kg.m/s

Problem 14: In a typical golf swing, the club is in contact with the ball for about 0.0010 s. If the 45 g ball acquires a speed of 67 m/s, estimate the magnitude of the force exerted by the club on the ball.

Problem 15: A 0.50 kg croquet ball is initially at rest on the grass. When the ball is struck by a mallet, the average force on it is 230 N. If the ball's speed after being struck is 3.2 m/s, how long was the mallet in contact with the ball?

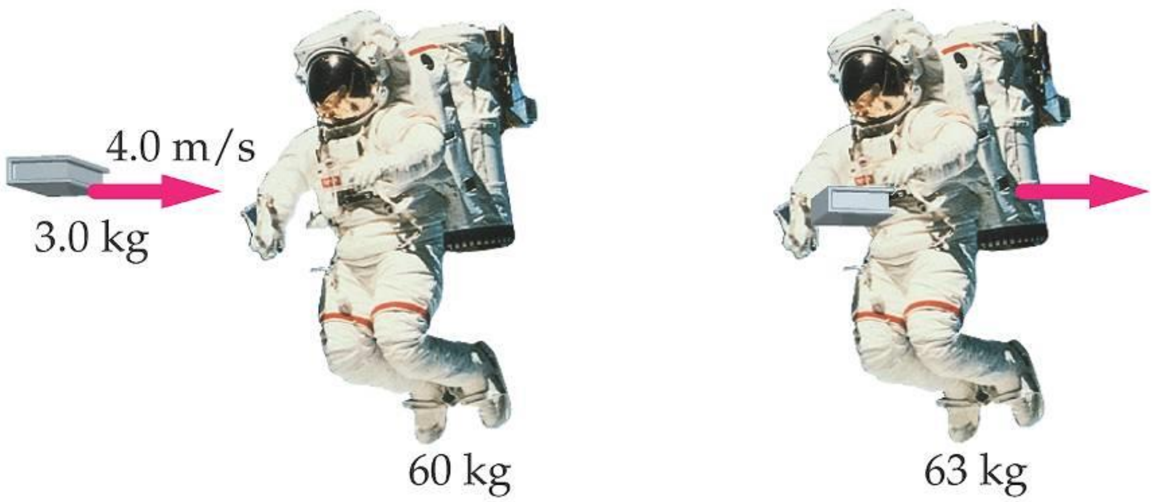
Conservation of Linear Momentum



Example: A honeybee with a mass of 0.150 g lands on one end of a floating 4.75 g popsicle stick. After sitting at rest for a moment, it runs to the other end of the stick with a velocity $\vec{v}_b$ relative to still water. The stick moves in the opposite direction with a velocity of 0.120 cm/s. Find the velocity $\vec{v}_b$ of the bee.

Answer: 3.8 cm/s

Inelastic Collisions in One Dimension



An astronaut of mass 60 kg is on a spacewalk to repair a communications satellite, when he realizes that he needs to consult the repair manual. You throw it to him at a speed of 4.0 m/s relative to the spacecraft. He is at rest when he catches the 3.0 kg book

(a) Find his velocity just after he catches the book.

(b) Find the initial and final kinetic energies of the book-astronaut system.

(c) Find the impulse exerted by the book on the astronaut.

Answers:

$v_f = 0.19\ \mathrm{m/s}$ $K_i = 24\ \mathrm{J},\ K_f = 1.1\ \mathrm{J}$ $I = 11.4\ \mathrm{N.s}$