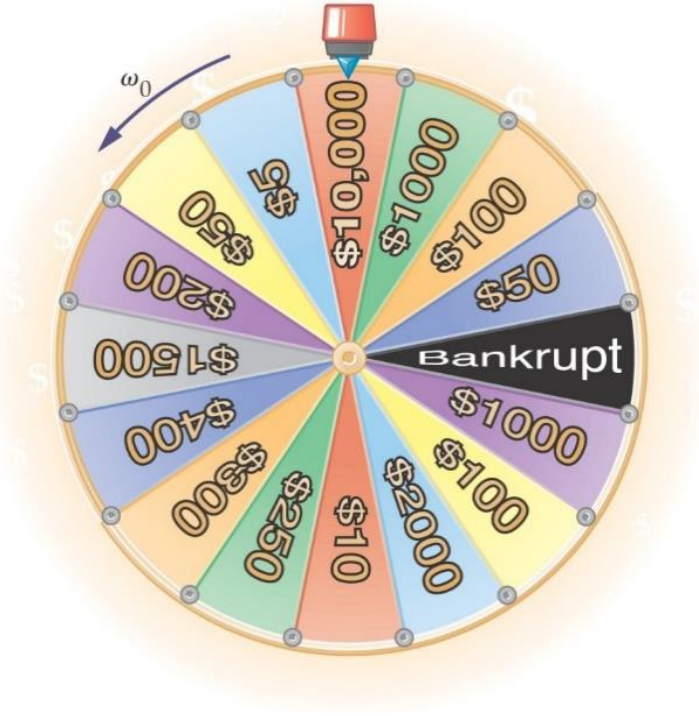
Chapter 10

Rotational Kinematics and Energy

- **Units of Chapter 10**

10-1 Angular Position, Velocity, & Acceleration

10-3 Relationships Between Linear & Rotational Quantities

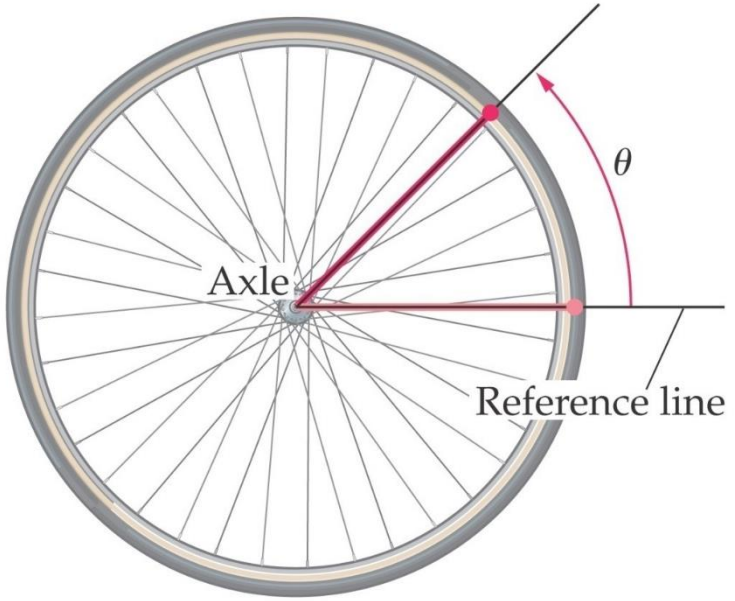


10-1 Angular Position, Velocity, & Acceleration

Definition of Angular Position, θ

θ = angle measured from reference line

SI unit: radian (rad), which is dimensionless



Sign Convention for Angular Position

By convention:

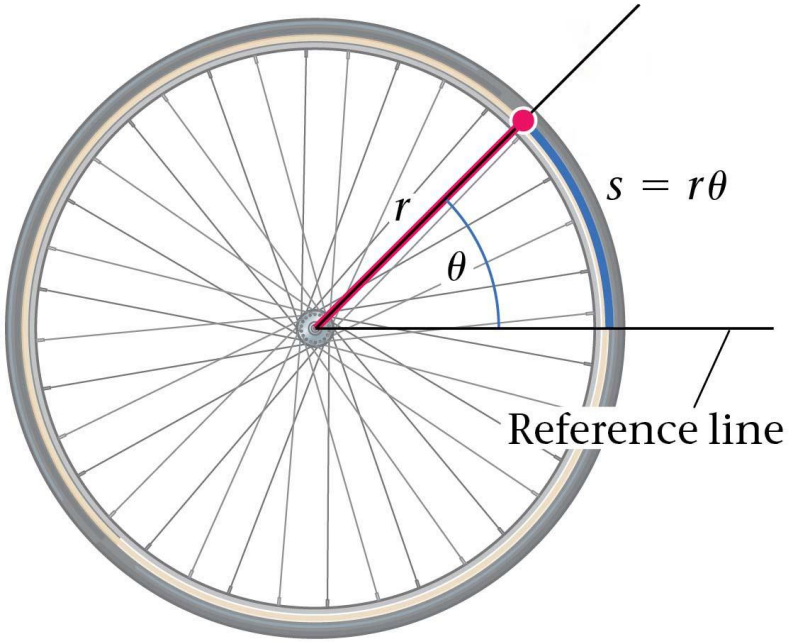
$\theta > 0$ counterclockwise rotation from reference line

$\theta < 0$ clockwise rotation from reference line

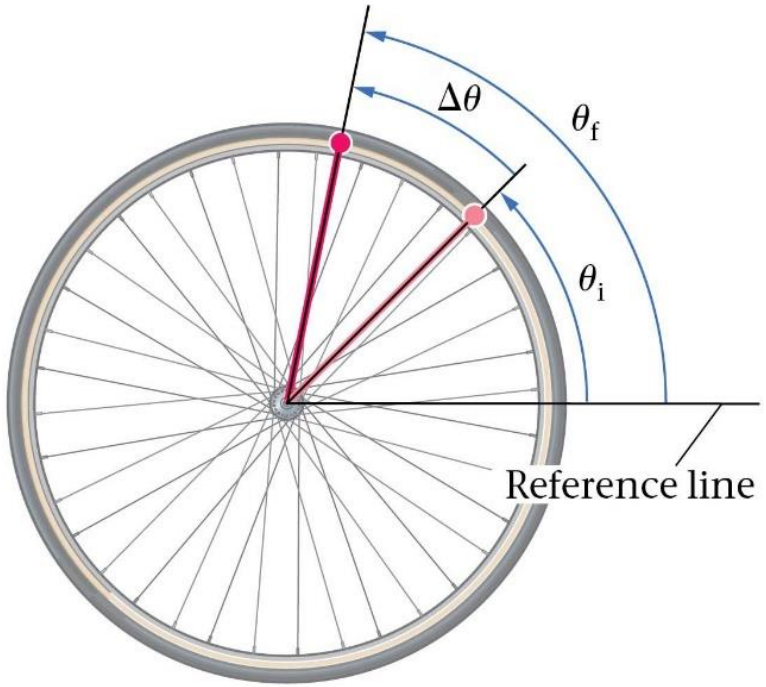
Degrees and revolutions: $1 \text{ rev} = 360°$

Angular Position

Arc length *s* : measured in radians



Angular Velocity



Definition of Average Angular Velocity, ω_{av}

$$\omega_{av} = \frac{\Delta\theta}{\Delta t}$$

SI unit: radian per second (rad/s) $= s^{-1}$

Period & Angular Acceleration

Definition of Period, T

$$T = \frac{2\pi}{\omega}$$

SI unit: second, s

Definition of Average Angular Acceleration, α_{av}

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t}$$

SI unit: radian per second per second $(\text{rad/s}^2) = \text{s}^{-2}$

Exercises

(1) An old phonograph record rotates clockwise at 33.3 rpm (revolutions per minute) what is its angular velocity in rad/s?

Answer: $-3.49\ rad/s$

(2) if a CD rotates at 22.0 rad/s, what is its angular speed in rpm?

Answer: 210 rpm

(3) Find the period of a record that is rotating at 45 rpm?

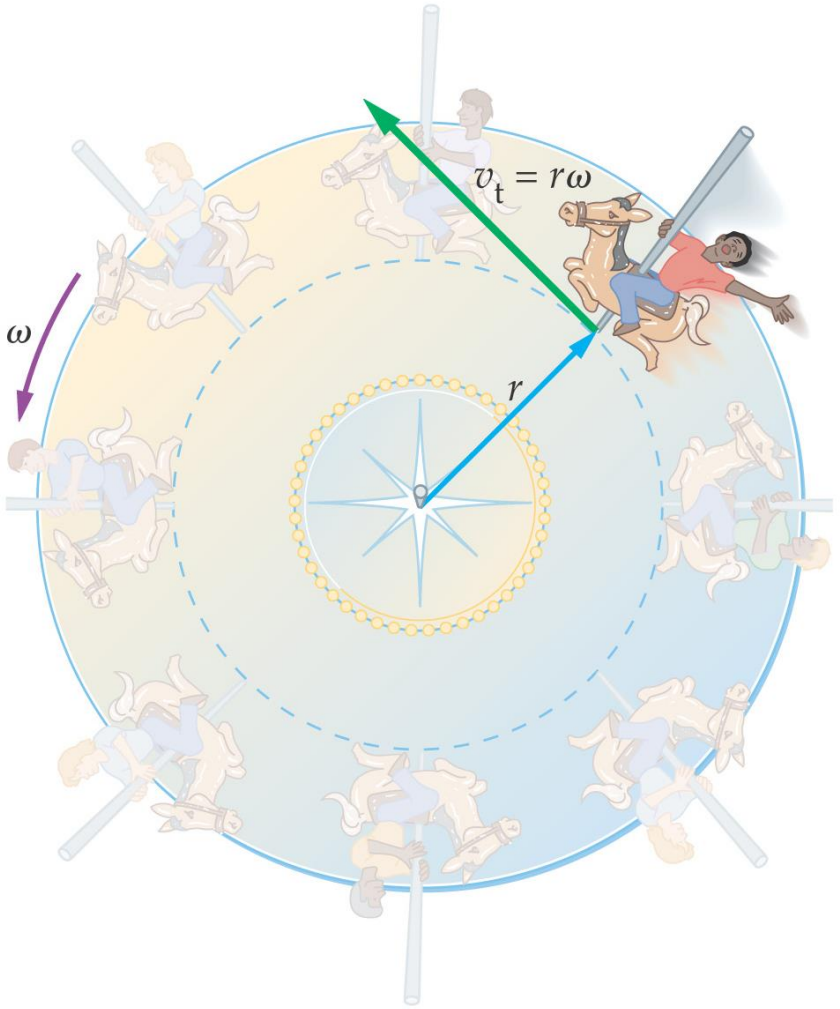
Answer: 1.3s

Relationship Between Linear and Rotational Velocities

Tangential speed of a Rotating Object

$$v_t = r\omega$$

SI unit: m/s



Exercise 10.5

Find the angular speed a CD must have to give a linear speed of 1.25 m/s when the laser beam shines on the disk

a) 2.5 cm from its center? Answer : 50 rad/s

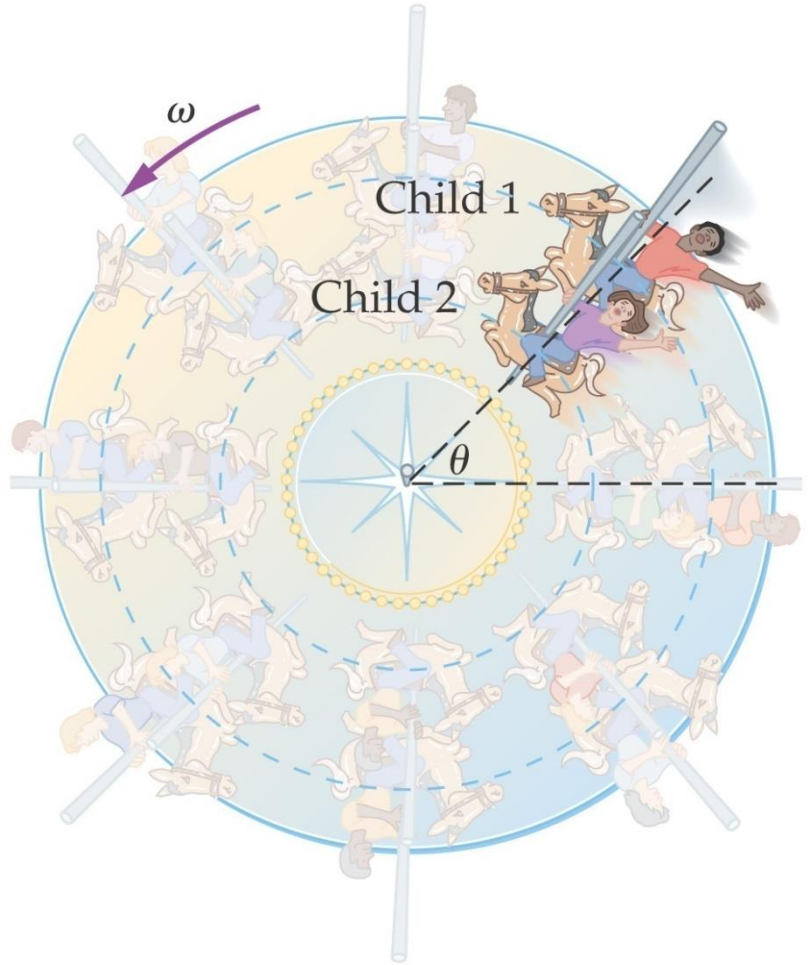
b) 6.00 cm from its center? Answer : 20.8 rad/s

Relationship Between Linear and Rotational Accelerations

Centripetal Acceleration of Rotating Object

$$a_{cp} = r\omega^2$$

SI unit: m/s^2



Relationship Between Linear and Rotational Accelerations

Tangential Acceleration of Rotating Object

$$a_t = r\alpha$$

SI unit: m/s^2

Tangential Versus Centripetal Accelerations

- $a_t = r\alpha$ Due to changing angular speed
- $a_{cp} = r\omega^2$ Due to changing direction of motion

Relationship Between Linear and Rotational Accelerations

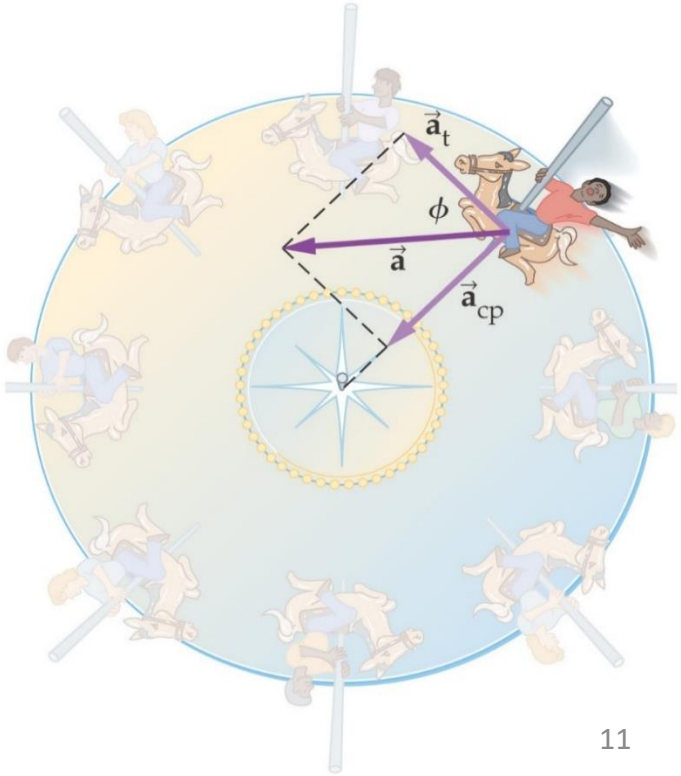
This merry-go-round has both tangential and centripetal accelerations:

The total acceleration is the vector sum of the two:

$$a = \sqrt{a_t^2 + a_{cp}^2}$$

The direction of the total acceleration:

$$\varphi = \tan^{-1}\left(\frac{a_{cp}}{a_t}\right)$$

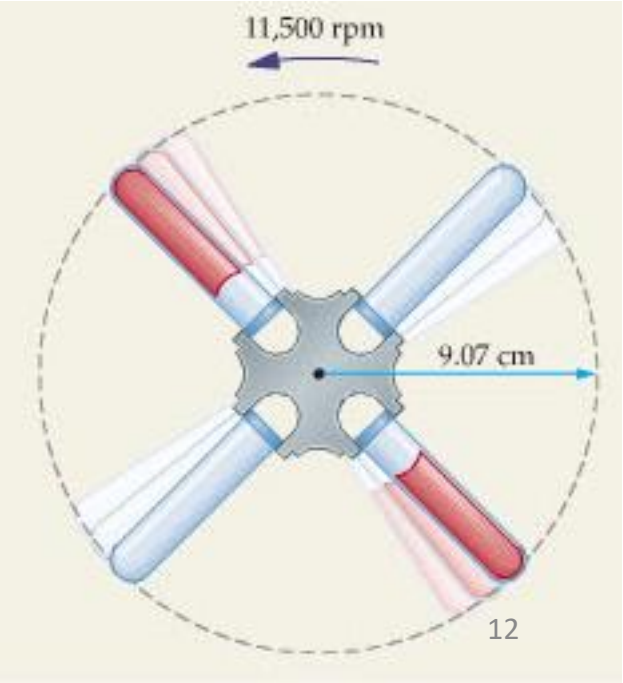


Example 10.3

In a microhematocrit centrifuge, small samples of blood are placed in heparinized capillary tubes. The tubes are rotated at 11,500 rpm, with the bottoms of the tubes 9.07 cm from the axis of rotation.

a) Find the linear speed of the bottom of the tubes?

b) What is the centripetal acceleration at the bottom of the tubes?



Answers : $\omega = 1.2 \times 10^3 rad/s$

$v = 109\, m/s$

$a_{cp} = 131{,}000\, m/s^2$

Summary of Chapter 10

- Describing rotational motion requires analogs to position, velocity, and acceleration

$$\theta\,(\text{in radians}) = \text{arc length/radius} = s/r$$

- Average angular velocity:

$$\omega_{\text{av}} = \frac{\Delta\theta}{\Delta t}$$

- Average angular acceleration:

$$\alpha_{\text{av}} = \frac{\Delta\omega}{\Delta t}$$

- Period: $T = \dfrac{2\pi}{\omega}$
- Counterclockwise rotations are positive, clockwise negative
- Linear and angular quantities:

Linear Quantity	Angular Quantity
x	θ
v	ω
a	α

Tangential speed: $v_t = r\omega$

Centripetal acceleration: $a_{cp} = r\omega^2$

Tangential acceleration: $a_t = r\alpha$

Examples

1) A wheel rotates with a constant angular acceleration of 3.50 rad/s^2. If the angular speed of the wheel is 2.00 rad/s at $t = 0$,

What is the angular speed of the wheel at $t = 2.00$ s?

2) A race car accelerates uniformly from a speed of 40.0 m/s to a speed of 60.0 m/s in 5.00 s while traveling counterclockwise around a circular track of radius 4.00×10^2 m. When the car reaches a speed of 50.0 m/s, find

(a) the magnitude of the car's centripetal acceleration,

(b) the angular speed,

(c) the magnitude of the tangential acceleration, and

(d) the magnitude of the total acceleration.

(Answers: (a) 6.25 m/s^2, (b) 0.125 rad/s, (c) 4 m/s^2, (d) 7.42 m/s^2)

Suppose the centrifuge in Ex 10.3, is starting up with a constant angular acceleration of 95.0 rad/s^2.

a) What are the magnitudes of the centripetal, tangential, and total acceleration of the bottom of a tube when the angular speed is 8.00 rad/s?

b) What angle does the total acceleration make with the direction of motion?