Chapter 9

Center of Mass and Linear Momentum

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9-1 Center of Mass

Learning Objectives

9.01 Given the positions of several particles along an axis or a plane, determine the location of their center of mass.

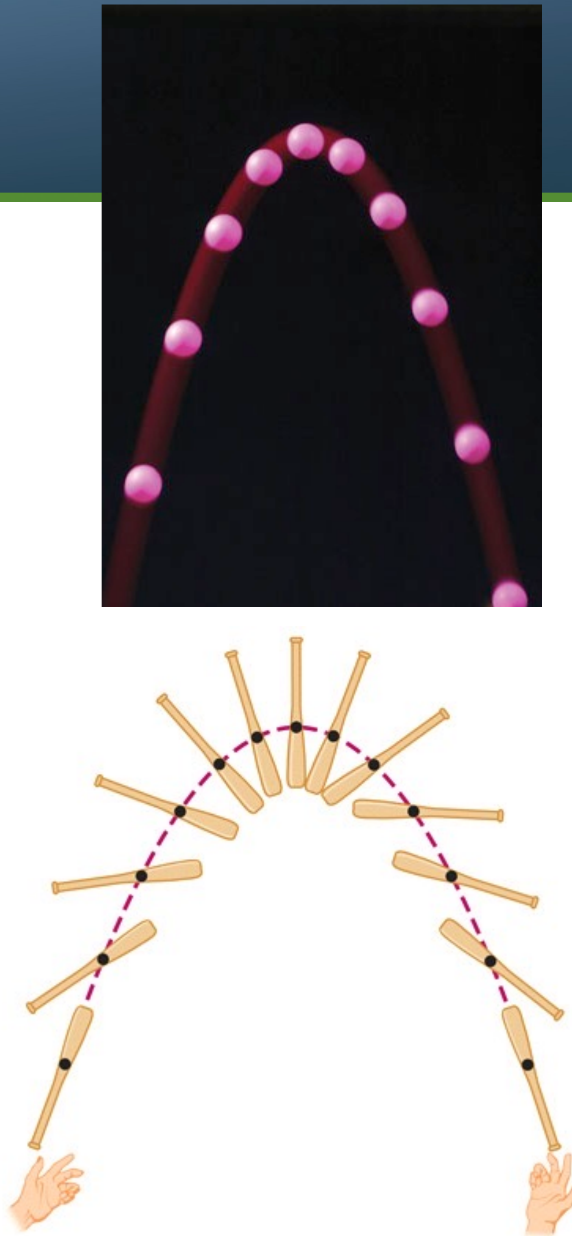
9.02 Locate the center of mass of an extended, symmetric object by using the symmetry.

9.03 For a two-dimensional or three-dimensional extended object with a uniform distribution of mass, determine the center of mass by (a) mentally dividing the object into simple geometric figures, each of which can be replaced by a particle at its center and (b) finding the center of mass of those particles.

9-1 Center of Mass

- The motion of rotating objects can be complicated (imagine flipping a baseball bat into the air)
- But there is a special point on the object for which the motion is simple

- The center of mass of the bat traces out a parabola, just as a tossed ball does
- All other points rotate around this point



(b)

Figure 9-1

9-1 Center of Mass

- The **center of mass** (com) of a system of particles:

> The center of mass of a system of particles is the point that moves as though (1) all of the system's mass were concentrated there and (2) all external forces were applied there.

- For two particles separated by a distance *d*, where the origin is chosen at the position of particle 1:

$$x_{com} = \frac{m_2}{m_1 + m_2} d. \qquad \textbf{Eq. (9-1)}$$

- For two particles, for an arbitrary choice of origin:

$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}. \qquad \textbf{Eq. (9-2)}$$

9-1 Center of Mass

- The center of mass is in the same location regardless of the coordinate system used
- It is a property of the particles, not the coordinates

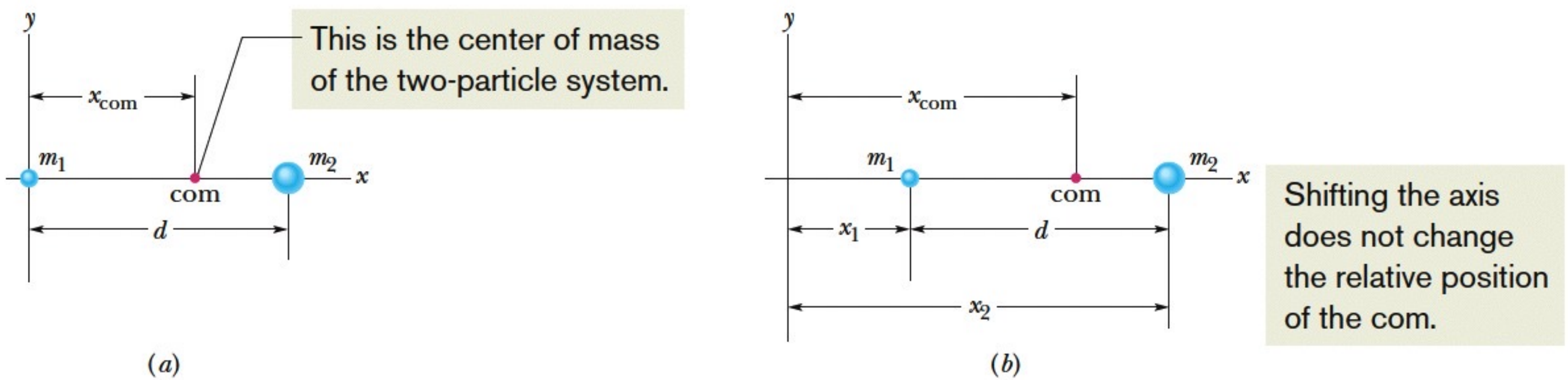


Figure 9-2

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9-1 Center of Mass

- For many particles, we can generalize the equation, where $M = m_1 + m_2 + \ldots + m_n$:

$$x_{com} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots + m_nx_n}{M} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i .$$

Eq. (9-4)

- In three dimensions, we find the center of mass along each axis separately:

$$x_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i, \qquad y_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i, \qquad z_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i .$$

Eq. (9-5)

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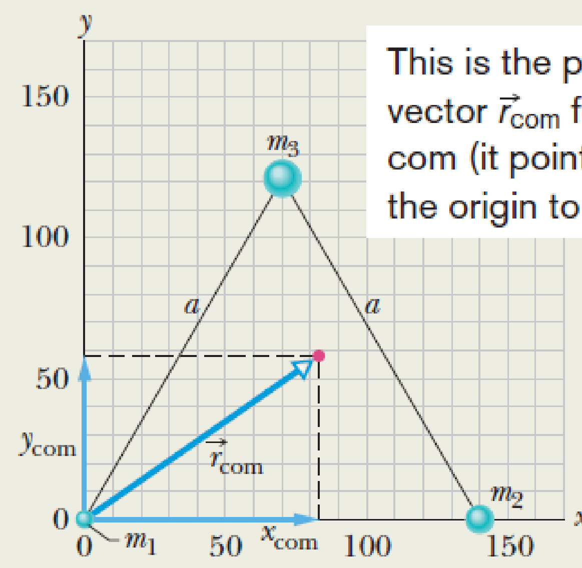
Sample Problem 9.01 com of three particles

Three particles of masses $m_1 = 1.2$ kg, $m_2 = 2.5$ kg, and $m_3 = 3.4$ kg form an equilateral triangle of edge length $a = 140$ cm. Where is the center of mass of this system?



This is the position vector $\vec{r}_{com}$ for the com (it points from the origin to the com).

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$$x_{com} = \frac{1}{M}\sum_{i=1}^{3} m_i x_i = \frac{m_1x_1 + m_2x_2 + m_3x_3}{M}$$

$$= \frac{(1.2\text{ kg})(0) + (2.5\text{ kg})(140\text{ cm}) + (3.4\text{ kg})(70\text{ cm})}{7.1\text{ kg}}$$

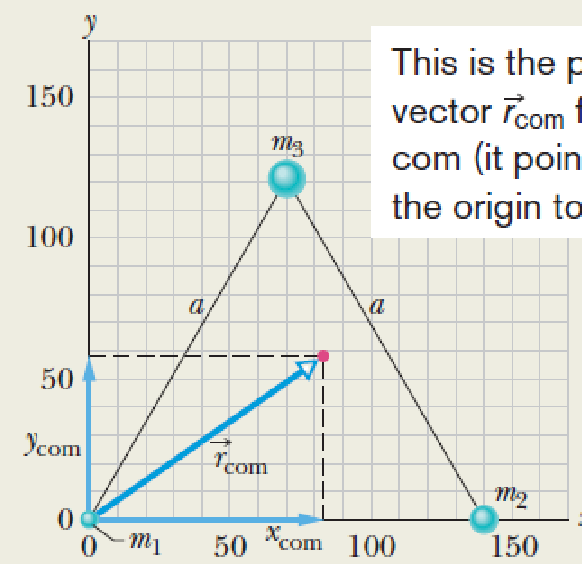
$$= 83\text{ cm} \qquad \text{(Answer)}$$

and

$$y_{com} = \frac{1}{M}\sum_{i=1}^{3} m_i y_i = \frac{m_1y_1 + m_2y_2 + m_3y_3}{M}$$

$$= \frac{(1.2\text{ kg})(0) + (2.5\text{ kg})(0) + (3.4\text{ kg})(120\text{ cm})}{7.1\text{ kg}}$$

$$= 58\text{ cm}. \qquad \text{(Answer)}$$



This is the position vector $\vec{r}_{com}$ for the com (it points from the origin to the com).

9-1 Center of Mass

- More concisely, we can write in terms of vectors:

$$\vec{r}_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i\vec{r}_i, \qquad \textbf{Eq. (9-8)}$$

- For solid bodies, we take the limit of an infinite sum of infinitely small particles → integration!
- Coordinate-by-coordinate, we write:

$$x_{com} = \frac{1}{M}\int x\,dm, \qquad y_{com} = \frac{1}{M}\int y\,dm, \qquad z_{com} = \frac{1}{M}\int z\,dm,$$

Eq. (9-9)

- Here *M* is the mass of the object

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9-1 Center of Mass

- We limit ourselves to objects of uniform density, ρ, for the sake of simplicity

$$\rho = \frac{dm}{dV} = \frac{M}{V},$$ Eq. (9-10)

- Substituting, we find the center of mass simplifies:

$$x_{\text{com}} = \frac{1}{V}\int x\,dV, \qquad y_{\text{com}} = \frac{1}{V}\int y\,dV, \qquad z_{\text{com}} = \frac{1}{V}\int z\,dV.$$

Eq. (9-11)

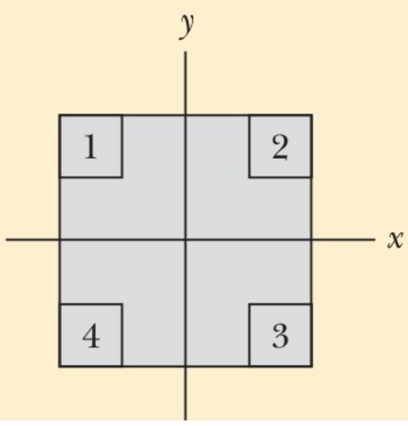
- You can bypass one or more of these integrals if the object has symmetry

9-1 Center of Mass

- The center of mass lies at a point of symmetry (if there is one)
- It lies on the line or plane of symmetry (if there is one)
- It need not be on the object (consider a doughnut)

Checkpoint 1

The figure shows a uniform square plate from which four identical squares at the corners will be removed. (a) Where is the center of mass of the plate originally? Where is it after the removal of (b) square 1; (c) squares 1 and 2; (d) squares 1 and 3; (e) squares 1, 2, and 3; (f) all four squares? Answer in terms of quadrants, axes, or points (without calculation, of course).



Answer: (a) at the origin (b) in Q4, along y=-x (c) along the -y axis

(d) at the origin (e) in Q3, along y=x (f) at the origin

9-2 Newton's Second Law for a System of Particles

Learning Objectives

9.04 Apply Newton's second law to a system of particles by relating the net force (of the forces acting on the particles) to the acceleration of the system's center of mass.

9.05 Apply the constant-acceleration equations to the motion of the individual particles in a system and to the motion of the system's center of mass.

9.06 Given the mass and velocity of the particles in a system, calculate the velocity of the system's center of mass.

9.07 Given the mass and acceleration of the particles in a system, calculate the acceleration of the system's center of mass.

9.08 Given the position of a system's center of mass as a function of time, determine the velocity of the center of mass.

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9.09 Given the velocity of a system's center of mass as a function of time, determine the acceleration of the center of mass.

9.10 Calculate the change in the velocity of a com by integrating the com's acceleration function with respect to time.

9.11 Calculate a com's displacement by integrating the com's velocity function with respect to time.

9.12 When the particles in a two-particle system move without the system's com moving, relate the displacements of the particles and the velocities of the particles.

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9-2 Newton's Second Law for a System of Particles

- Center of mass motion continues unaffected by forces internal to a system (collisions between billiard balls)
- Motion of a system's center of mass:

$$\vec{F}_{\text{net}} = M\vec{a}_{\text{com}} \quad \text{(system of particles).}$$ **Eq. (9-14)**

$$F_{\text{net},x} = Ma_{\text{com},x} \qquad F_{\text{net},y} = Ma_{\text{com},y} \qquad F_{\text{net},z} = Ma_{\text{com},z}.$$ **Eq. (9-15)**

- Reminders:
  1. F_{net} is the sum of all *external* forces
  2. *M* is the total, constant, mass of the **closed** system
  3. a_{com} is the *center of mass* acceleration

6-2 Newton's Second Law for a System of Particles

Examples Using the <u>c</u>enter <u>o</u>f <u>m</u>ass motion equation:

- Billiard collision: forces are only internal, $F = 0$ so $a = 0$
- Baseball bat: $a = g$, so com follows gravitational trajectory
- Exploding rocket: explosion forces are internal, so only the gravitational force acts on the system, and the com follows a gravitational trajectory

  as long as air resistance can

  be ignored for the fragments.

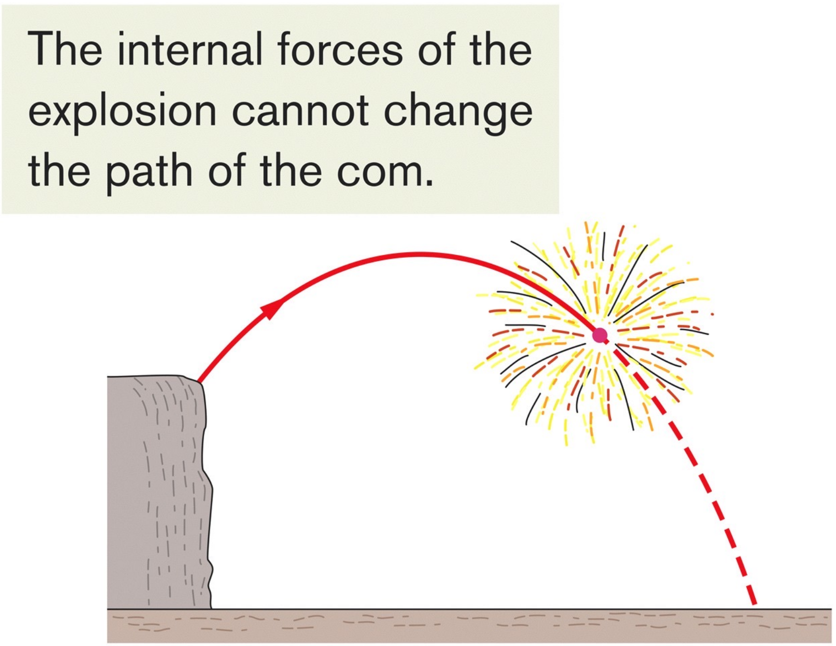


Figure 9-5

9-2 Newton's Second Law for a System of Particles

Checkpoint 2

Two skaters on frictionless ice hold opposite ends of a pole of negligible mass. An axis runs along it, with the origin at the center of mass of the two-skater system. One skater, Fred, weighs twice as much as the other skater, Ethel. Where do the skaters meet if (a) Fred pulls hand over hand along the pole so as to draw himself to Ethel, (b) Ethel pulls hand over hand to draw herself to Fred, and (c) both skaters pull hand over hand?

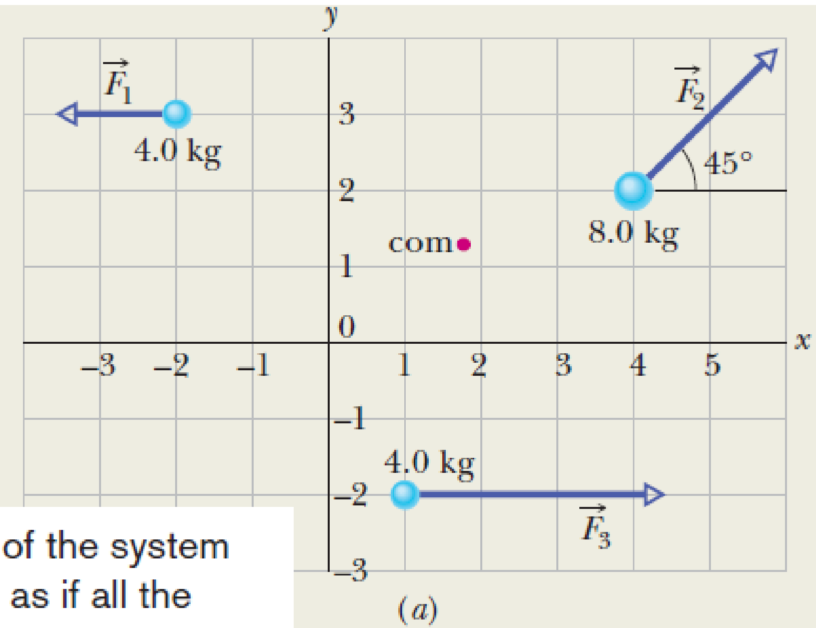
Two skaters om fri

Answer: The system consists of Fred, Ethel and the pole. All forces are internal. Therefore the com will remain in the same place. Since the origin is the com, they will meet at the origin in all three cases! (Of course the origin where the com is located is closer to Fred than to Ethel.)

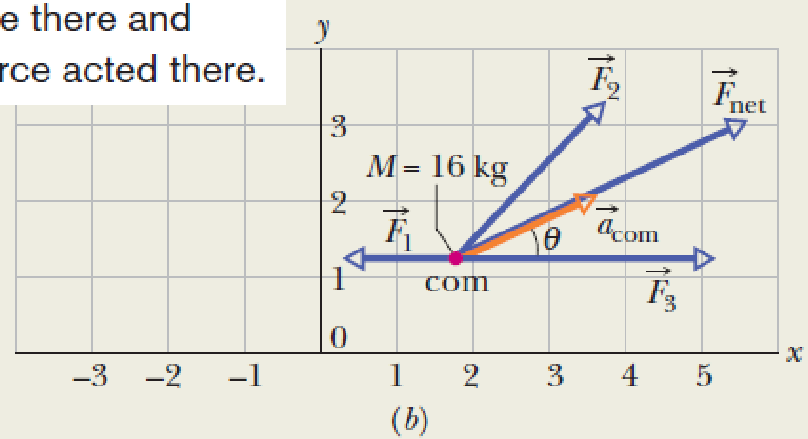
Sample Problem 9.03 Motion of the com of three particles

If the particles in a system all move together, the com moves with them—no trouble there. But what happens when they move in different directions with different accelerations? Here is an example.

The three particles in Fig. 9-7*a* are initially at rest. Each experiences an *external* force due to bodies outside the three-particle system. The directions are indicated, and the magnitudes are $F_1 = 6.0$ N, $F_2 = 12$ N, and $F_3 = 14$ N. What is the acceleration of the center of mass of the system, and in what direction does it move?



The com of the system will move as if all the mass were there and the net force acted there.



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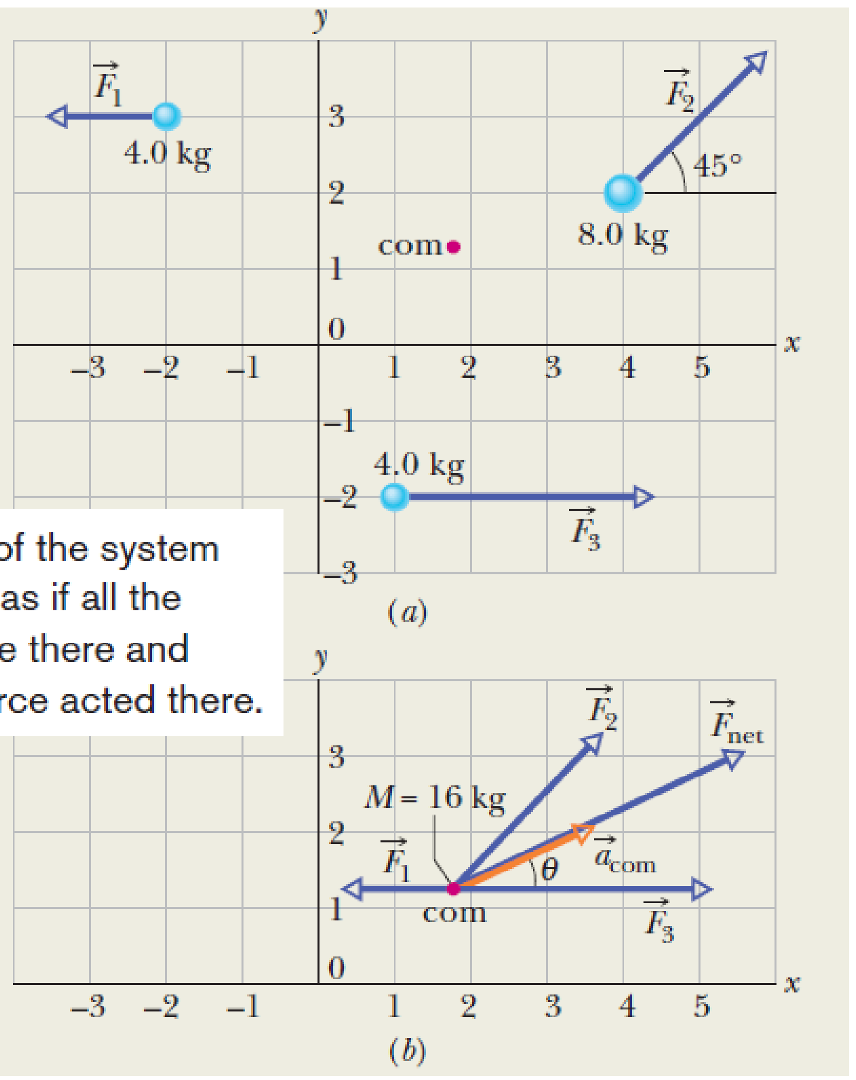
$$\vec{F}_{net} = M\vec{a}_{com}$$

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = M\vec{a}_{com}$$

$$\vec{a}_{com} = \frac{\vec{F}_1 + \vec{F}_2 + \vec{F}_3}{M}.$$



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$$a_{\text{com},x} = \frac{F_{1x} + F_{2x} + F_{3x}}{M}$$
$$= \frac{-6.0\text{ N} + (12\text{ N})\cos 45° + 14\text{ N}}{16\text{ kg}} = 1.03\text{ m/s}^2.$$

Along the y axis, we have

$$a_{\text{com},y} = \frac{F_{1y} + F_{2y} + F_{3y}}{M}$$
$$= \frac{0 + (12\text{ N})\sin 45° + 0}{16\text{ kg}} = 0.530\text{ m/s}^2.$$

From these components, we find that $\vec{a}_{\text{com}}$ has the magnitude

$$a_{\text{com}} = \sqrt{(a_{\text{com},x})^2 + (a_{\text{com},y})^2}$$
$$= 1.16\text{ m/s}^2 \approx 1.2\text{ m/s}^2 \qquad \text{(Answer)}$$

and the angle (from the positive direction of the x axis)

$$\theta = \tan^{-1}\frac{a_{\text{com},y}}{a_{\text{com},x}} = 27°. \qquad \text{(Answer)}$$

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9-3 Linear Momentum

Learning Objectives

9.13 Identify that momentum is a vector quantity and thus has both magnitude and direction and also components.

9.14 Calculate the (linear) momentum of a particle as the product of the particle's mass and velocity.

9.15 Calculate the change in momentum (magnitude and direction) when a particle changes its speed and direction of travel.

9.16 Apply the relationship between a particle's momentum and the (net) force acting on the particle.

9.17 Calculate the momentum of a system of particles as the product of the system's total mass and its center-of-mass velocity.

9.18 Apply the relationship between a system's center-of-mass momentum and the net force acting on the system.

9-3 Linear Momentum

- The **linear momentum** is defined as:

$$\vec{p} = m\vec{v}$$ Eq. (9-22)

- The momentum:
  - Points in the same direction as the velocity
  - Can only be changed by a net external force

The time rate of change of the momentum of a particle is equal to the net force acting on the particle and is in the direction of that force.

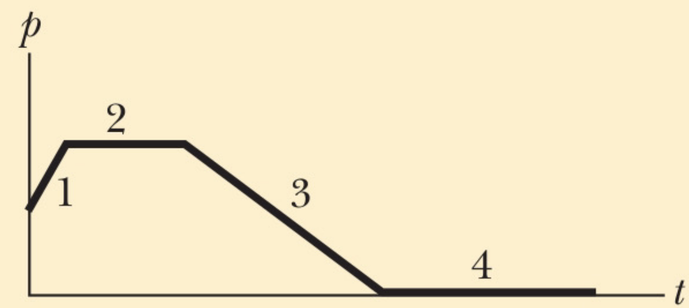
- We can write Newton's second law thus:

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}.$$ Eq. (9-23)

9-3 Linear Momentum

Checkpoint 3

The figure gives the magnitude p of the linear momentum versus time t for a particle moving along an axis. A force directed along the axis acts on the particle. (a) Rank the four regions indicated according to the magnitude of the force, greatest first. (b) In which region is the particle slowing?



Answer: (a) 1, 3, 2 & 4   (b) region 3

- We can sum momenta for a system of particles to find:

$$\vec{P} = M\vec{v}_{com} \quad \text{(linear momentum, system of particles)},$$

Eq. (9-25)

9-3 Linear Momentum

The linear momentum of a system of particles is equal to the product of the total mass M of the system and the velocity of the center of mass.

- Taking the time derivative we can write Newton's second law for a system of particles as:

$$\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt} \quad \text{(system of particles)}, \qquad \textbf{Eq. (9-27)}$$

- The net external force on a system changes linear momentum
- Without a net external force, the *total* linear momentum of a system of particles cannot change

9-4 Collision and Impulse

Learning Objectives

9.19 Identify that impulse is a vector quantity and thus has both magnitude and direction and components.

9.20 Apply the relationship between impulse and momentum change.

9.21 Apply the relationship between impulse, average force, and the time interval taken by the impulse.

9.22 Apply the constant-acceleration equations to relate impulse to force.

9.23 Given force as a function of time, calculate the impulse (and thus also the momentum change) by integrating the function.

9.24 Given a graph of force versus time, calculate the impulse (and thus also the momentum change) by graphical integration.

9.25 In a continuous series of collisions by projectiles, calculate average force on the target by relating it to the mass collision rate and the velocity change of each projectile.

9-4 Collision and Impulse

- In a collision, momentum of a particle can change
- We define the **impulse *J*** acting during a collision:

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\, dt$$ **Eq. (9-30)**

- This means that the applied impulse is equal to the change in momentum of the object during the collision:

$$\Delta\vec{p} = \vec{J} \qquad \text{(linear momentum–impulse theorem).}$$ **Eq. (9-31)**

- This equation can be rewritten component-by-component, like other vector equations

9-4 Collision and Impulse

- Given F_{avg} and duration:

$$J = F_{avg}\,\Delta t. \qquad \text{Eq. (9-35)}$$

- We are integrating: we only need to know the area under the force curve

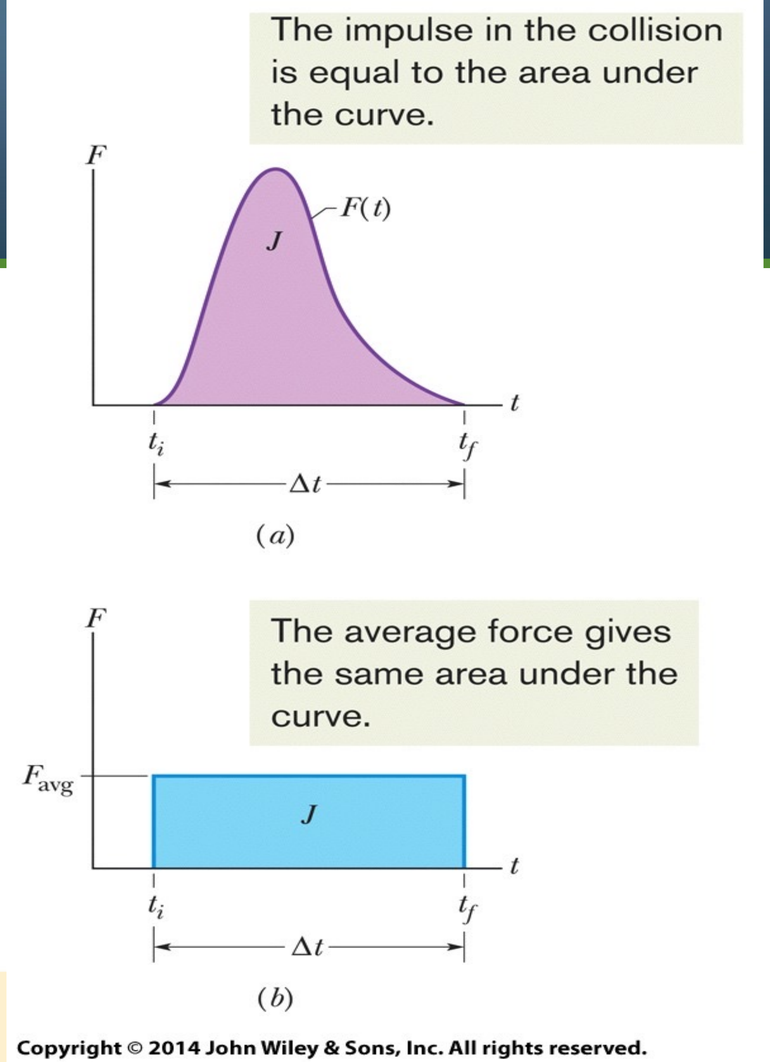


Figure 9-9

Checkpoint 4

A paratrooper whose chute fails to open lands in snow; he is hurt slightly. Had he landed on bare ground, the stopping time would have been 10 times shorter and the collision lethal. Does the presence of the snow increase, decrease, or leave unchanged the values of (a) the paratrooper's change in momentum, (b) the impulse stopping the paratrooper, and (c) the force stopping the paratrooper?

Answer: (a) unchanged (b) unchanged (c) decreased

9-4 Collision and Impulse

- For a steady stream of *n* projectiles, each undergoes a momentum change *Δp*

$$J = -n\,\Delta p,$$ **Eq. (9-36)**

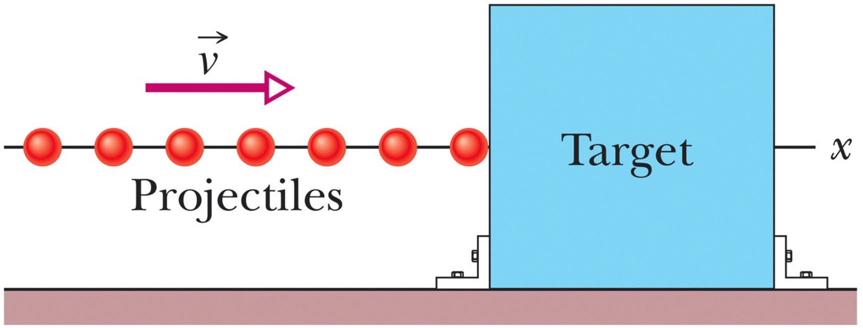


Figure 9-10

- The average force is: $$F_{avg} = \frac{J}{\Delta t} = -\frac{n}{\Delta t}\Delta p = -\frac{n}{\Delta t} m\,\Delta v.$$ **Eq. (9-37)**
- If the particles stop:

$$\Delta v = v_f - v_i = 0 - v = -v,$$ **Eq. (9-38)**

- If the particles bounce back with equal speed:

$$\Delta v = v_f - v_i = -v - v = -2v.$$ **Eq. (9-39)**

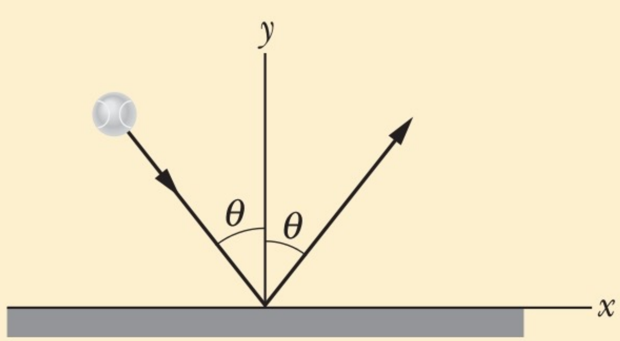
9-4 Collision and Impulse

- The product *nm* is the total mass for *n* collisions so we can write:

$$F_{avg} = -\frac{\Delta m}{\Delta t}\Delta v. \qquad \text{Eq. (9-40)}$$

Checkpoint 5

The figure shows an overhead view of a ball bouncing from a vertical wall without any change in its speed. Consider the change $\Delta\vec{p}$ in the ball's linear momentum. (a) Is Δp_x positive, negative, or zero? (b) Is Δp_y positive, negative, or zero? (c) What is the direction of $\Delta\vec{p}$?



Answer: (a) zero (b) positive (c) along the positive *y*-axis (normal force)

9-5 Conservation of Linear Momentum

Learning Objectives

9.26 For an isolated system of particles, apply the conservation of linear momenta to relate the initial momenta of the particles to their momenta at a later instant.

9.27 Identify that the conservation of linear momentum can be done along an individual axis by using components along that axis, *provided* there is no net external force component along that axis.

9-5 Conservation of Linear Momentum

- For an impulse of zero we find:

$$\vec{P} = \text{constant} \quad \text{(closed, isolated system).}$$

Eq. (9-42)

- Which says that:

> If no net external force acts on a system of particles, the total linear momentum $\vec{P}$ of the system cannot change.

- This is called the **law of conservation of linear momentum**
- Check the components of the net external force to know if you should apply this

> If the component of the net *external* force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.

9-5 Conservation of Linear Momentum

- Internal forces can change momenta of parts of the system, but cannot change the linear momentum of the entire system
- Do not confuse momentum and energy

Checkpoint 6

An initially stationary device lying on a frictionless floor explodes into two pieces, which then slide across the floor, one of them in the positive x direction. (a) What is the sum of the momenta of the two pieces after the explosion? (b) Can the second piece move at an angle to the x axis? (c) What is the direction of the momentum of the second piece?

Answer: (a) zero (b) no (c) the negative x direction

9-6 Momentum and Kinetic Energy in Collisions

Learning Objectives

9.28 Distinguish between elastic collisions, inelastic collisions, and completely inelastic collisions.

9.29 Identify a one-dimensional collision as one where the objects move along a single axis, both before and after the collision.

9.30 Apply the conservation of momentum for an isolated one-dimensional collision to relate the initial momenta of the objects to their momenta after the collision.

9.31 Identify that in an isolated system, the momentum and velocity of the center of mass are not changed even if the objects collide.

9-6 Momentum and Kinetic Energy in Collisions

- Types of collisions:
- **Elastic collisions**:
  - Total kinetic energy is unchanged (conserved)
  - A useful approximation for common situations
  - In real collisions, some energy is always transferred
- **Inelastic collisions**: some energy is transferred
- **Completely inelastic collisions**:
  - The objects stick together
  - Greatest loss of kinetic energy

9-6 Momentum and Kinetic Energy in Collisions

- For one dimension:
- Inelastic collision

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}.$$ **Eq. (9-51)**

- Completely inelastic collision, for target at rest:

$$m_1 v_{1i} = (m_1 + m_2)V$$ **Eq. (9-52)**

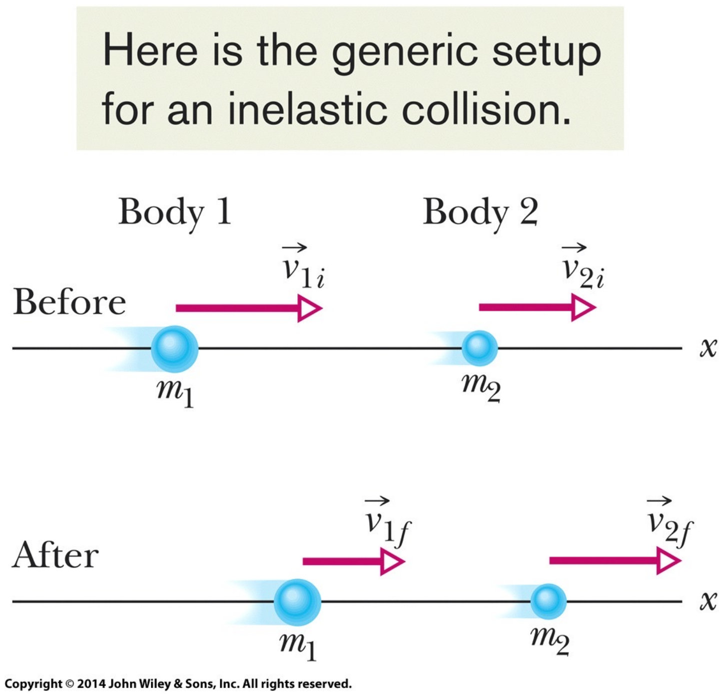


Figure 9-14

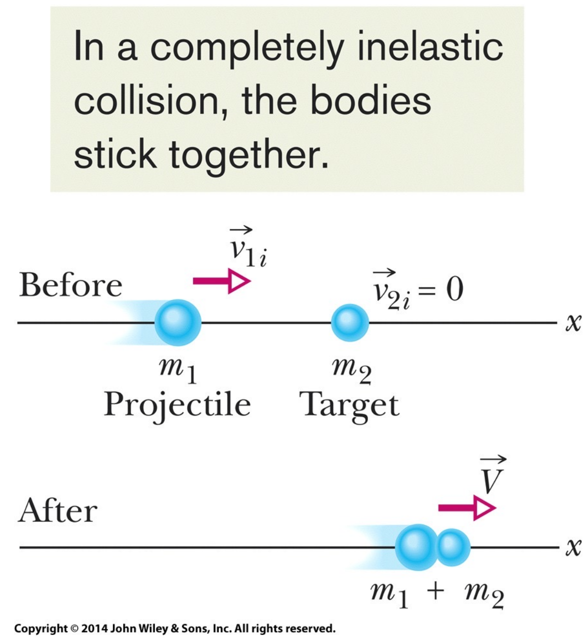


Figure 9-15

9-6 Momentum and Kinetic Energy in Collisions

- The center of mass velocity remains unchanged:

$$\vec{v}_{com} = \frac{\vec{P}}{m_1 + m_2} = \frac{\vec{p}_{1i} + \vec{p}_{2i}}{m_1 + m_2}.$$

Eq. (9-56)

- Figure 9-16 shows freeze frames of a completely inelastic collision, showing center of mass velocity

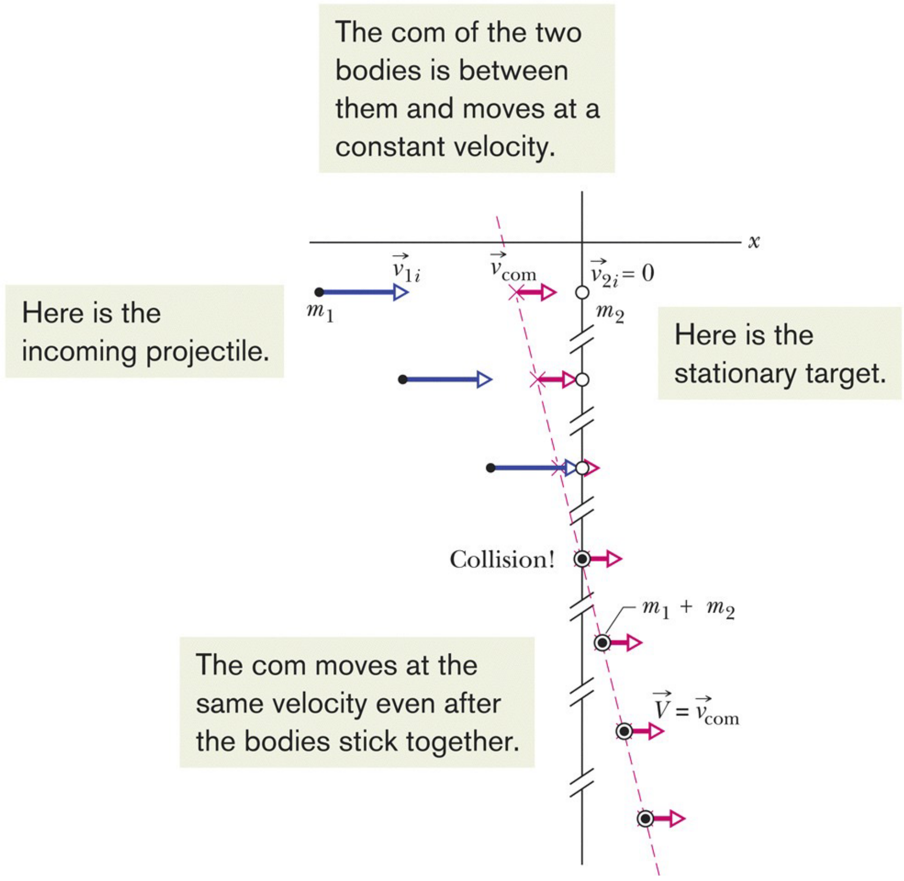


Figure 9-16

9-6 Momentum and Kinetic Energy in Collisions

Checkpoint 7

Body 1 and body 2 are in a completely inelastic one-dimensional collision. What is their final momentum if their initial momenta are, respectively, (a) 10 kg · m/s and 0; (b) 10 kg · m/s and 4 kg · m/s; (c) 10 kg · m/s and −4 kg · m/s?

Answer: (a) 10 kg m/s (b) 14 kg m/s (c) 6 kg m/s

9-7 Elastic Collisions in One Dimension

Learning Objectives

9.32 For isolated elastic collisions in one dimension, apply the conservation laws for both the total energy and the net momentum of the colliding bodies to relate the initial values to the values after the collision.

9.33 For a projectile hitting a stationary target, identify the resulting motion for the three general cases: equal masses, target more massive than projectile, projectile more massive than target.

9-7 Elastic Collisions in One Dimension

- Total kinetic energy is conserved in elastic collisions

> In an elastic collision, the kinetic energy of each colliding body may change, but the total kinetic energy of the system does not change.

- For a stationary target, conservation laws give:

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \quad \text{(linear momentum).}$$ **Eq. (9-63)**

$$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2 \quad \text{(kinetic energy).}$$ **Eq. (9-64)**

Here is the generic setup for an elastic collision with a stationary target.

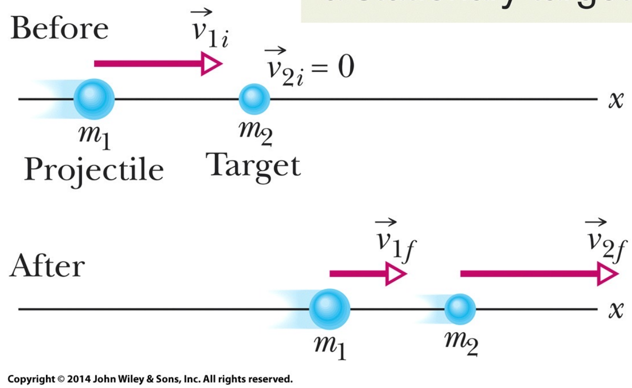


Figure 9-18

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9-7 Elastic Collisions in One Dimension

- With some algebra we get:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$ **Eq. (9-67)**

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}.$$ **Eq. (9-68)**

- Results
  - Equal masses: $v_{1f} = 0$, $v_{2f} = v_{1i}$: the first object stops
  - Massive target, $m_2 >> m_1$: the first object just bounces back, speed mostly unchanged
  - Massive projectile: $v_{1f} \approx v_{1i}$, $v_{2f} \approx 2v_{1i}$: the first object keeps going, the target flies forward at about twice its speed

9-7 Elastic Collisions in One Dimension

- For a target that is also moving, we get:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} \quad \textbf{Eq. (9-75)}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}. \quad \textbf{Eq. (9-76)}$$

Here is the generic setup for an elastic collision with a moving target.

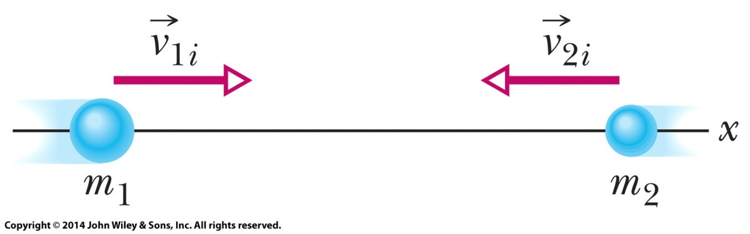

Figure 9-19

Checkpoint 8

What is the final linear momentum of the target in Fig. 9-18 if the initial linear momentum of the projectile is 6 kg·m/s and the final linear momentum of the projectile is (a) 2 kg·m/s and (b) −2 kg·m/s? (c) What is the final kinetic energy of the target if the initial and final kinetic energies of the projectile are, respectively, 5 J and 2 J?

Answer: (a) 4 kg m/s (b) 8 kg m/s (c) 3 J

9-8 Collisions in Two Dimensions

Learning Objectives

9.34 For an isolated system in which a two-dimensional collision occurs, apply the conservation of momentum along each axis of a coordinate system to relate the momentum components along an axis before the collision to the momentum components *along the same axis* after the collision.

9.35 For an isolated system in which a two-dimensional *elastic* collision occurs, (a) apply the conservation of momentum along each axis to relate the momentum components along an axis before the collision to the momentum components *along the same axis* after the collision and (b) apply the conservation of total kinetic energy to relate the kinetic energies before and after the collision.

9-8 Collisions in Two Dimensions

- Apply the conservation of momentum along each axis
- Apply conservation of energy for elastic collisions

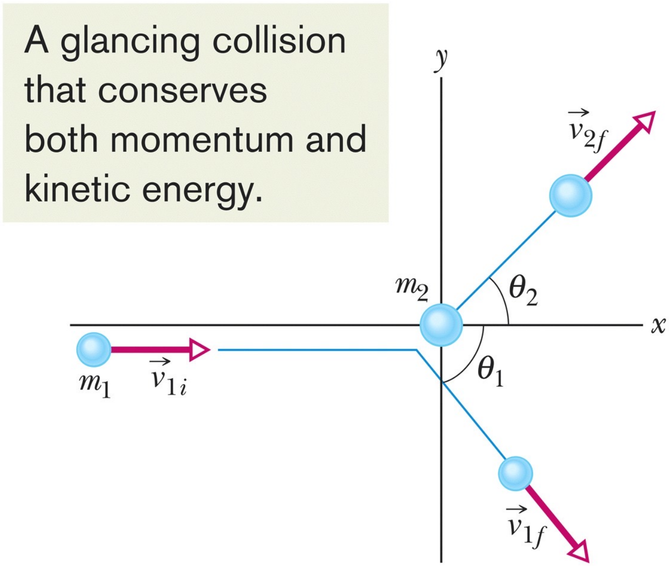

Figure 9-21

Example For Figure 9-21 for a stationary target:

- Along x: $$m_1 v_{1i} = m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2,$$ **Eq. (9-79)**
- Along y: $$0 = -m_1 v_{1f} \sin\theta_1 + m_2 v_{2f} \sin\theta_2.$$ **Eq. (9-80)**
- Energy: $$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$ **Eq. (9-81)**

9-8 Collisions in Two Dimensions

- These 3 equations for a stationary target have 7 unknowns (since $v_{2i} = 0$) : if we know 4 of them we can solve for the remaining ones.

Checkpoint 9

In Fig. 9-21, suppose that the projectile has an initial momentum of 6 kg · m/s, a final *x* component of momentum of 4 kg · m/s, and a final *y* component of momentum of −3 kg · m/s. For the target, what then are (a) the final *x* component of momentum and (b) the final *y* component of momentum?

Answer: (a) 2 kg m/s   (b) 3 kg m/s

9 Summary

Linear Momentum & Newton's 2nd Law

- Linear momentum defined as:

$$\vec{P} = M\vec{v}_{com}$$ **Eq. (9-25)**

- Write Newton's 2nd law:

$$\vec{F}_{net} = \frac{d\vec{P}}{dt}$$ **Eq. (9-27)**

Conservation of Linear Momentum

$$\vec{P} = \text{constant} \quad \text{(closed, isolated system)}.$$

Eq. (9-42)

Collision and Impulse

- Defined as:

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\, dt$$ **Eq. (9-30)**

- Impulse causes changes in linear momentum

Inelastic Collision in 1D

- Momentum conserved along that dimension

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}.$$

Eq. (9-51)

9 Summary

Motion of the Center of Mass

- Unaffected by collisions/internal forces

Elastic Collisions in One Dimension

- *K* is also conserved

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$ **Eq. (9-67)**

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}.$$ **Eq. (9-68)**

Collisions in Two Dimensions

- Apply conservation of momentum along each axis individually
- Conserve *K* if elastic