Chapter 8

Potential Energy and Conservation of Energy

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8-1 Potential Energy

Learning Objectives

8.01 Distinguish a conservative force force from a nonconservative force.

8.02 For a particle moving between two points, identify that the work done by a conservative force does not depend on which path the particle takes.

8.03 Calculate the gravitational potential energy of a particle (or, more properly, a particle-Earth system).

8.04 Calculate the elastic potential energy of a block-spring system.

8-1 Potential Energy

- **Potential energy** *U* is energy that can be associated with the configuration of a system of objects that exert forces on one another
- A system of objects may be:
  - Earth and a bungee jumper
  - **Gravitational potential energy** accounts for kinetic energy increase during the fall
  - **Elastic potential energy** accounts for deceleration by the bungee cord
- Physics determines how potential energy is calculated, to account for stored energy

8-1 Potential Energy

- For an object being raised or lowered:

$$\Delta U = -W. \qquad \text{Eq. (8-1)}$$

- The change in gravitational potential energy is the negative of the work done
- This also applies to an elastic block-spring system

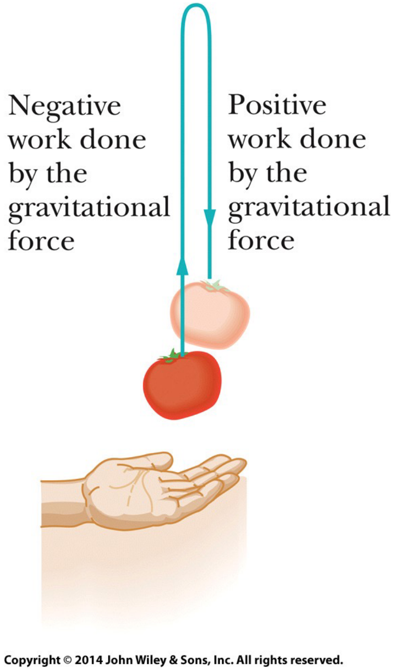

Figure 8-2

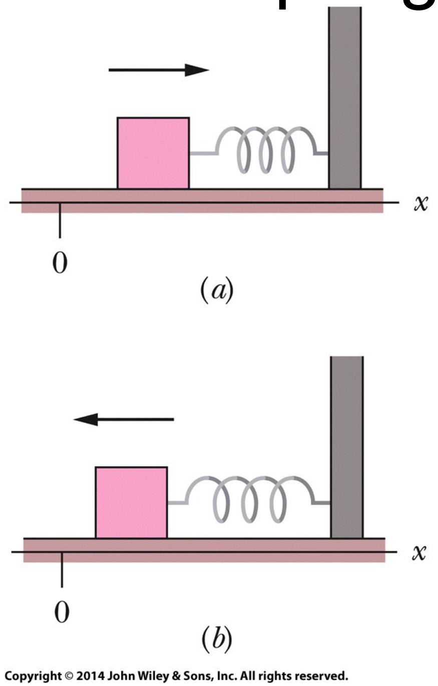

Figure 8-3

8-1 Potential Energy

- Key points:
  1. The *system* consists of two or more objects
  2. A *force* acts between a particle (tomato/block) and the rest of the system
  3. When the configuration changes, the force does *work* W_1, changing kinetic energy to another form
  4. When the configuration change is reversed, the force reverses the energy transfer, doing work W_2
- Thus the kinetic energy of the tomato/block becomes potential energy, and then kinetic energy again

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8-1 Potential Energy

- **Conservative forces** are forces for which $W_1 = -W_2$ is always true
  - Examples: gravitational force, spring force
  - Otherwise we could not speak of their potential energies
- **Nonconservative forces** are those for which it is false
  - Examples: kinetic friction force,
  - Kinetic energy of a moving particle is transferred to heat by friction
  - Thermal energy cannot be recovered back into kinetic energy of the object via the friction force
  - Therefore the force is not conservative, thermal energy is not a potential energy

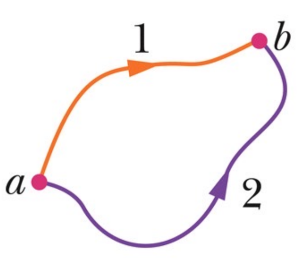
8-1 Potential Energy

- When only conservative forces act on a particle, we find many problems can be simplified:

> The net work done by a conservative force on a particle moving around any closed path is zero.

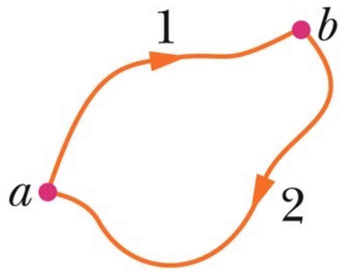
- A result of this is that:

> The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.



(a)

The force is conservative. Any choice of path between the points gives the same amount of work.



(b)

And a round trip gives a total work of zero.

Figure 8-4

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The main lesson of this sample problem is this: It is perfectly all right to choose an easy path instead of a hard path. Figure 8-5*a* shows a 2.0 kg block of slippery cheese that slides along a frictionless track from point *a* to point *b*. The cheese travels through a total distance of 2.0 m along the track, and a net vertical distance of 0.80 m. How much work is done on the cheese by the gravitational force during the slide?

$W_{ab,2}$, **Eq. (8-2)**

- This result allows you to substitute a simpler path for a more complex one if only conservative forces are involved

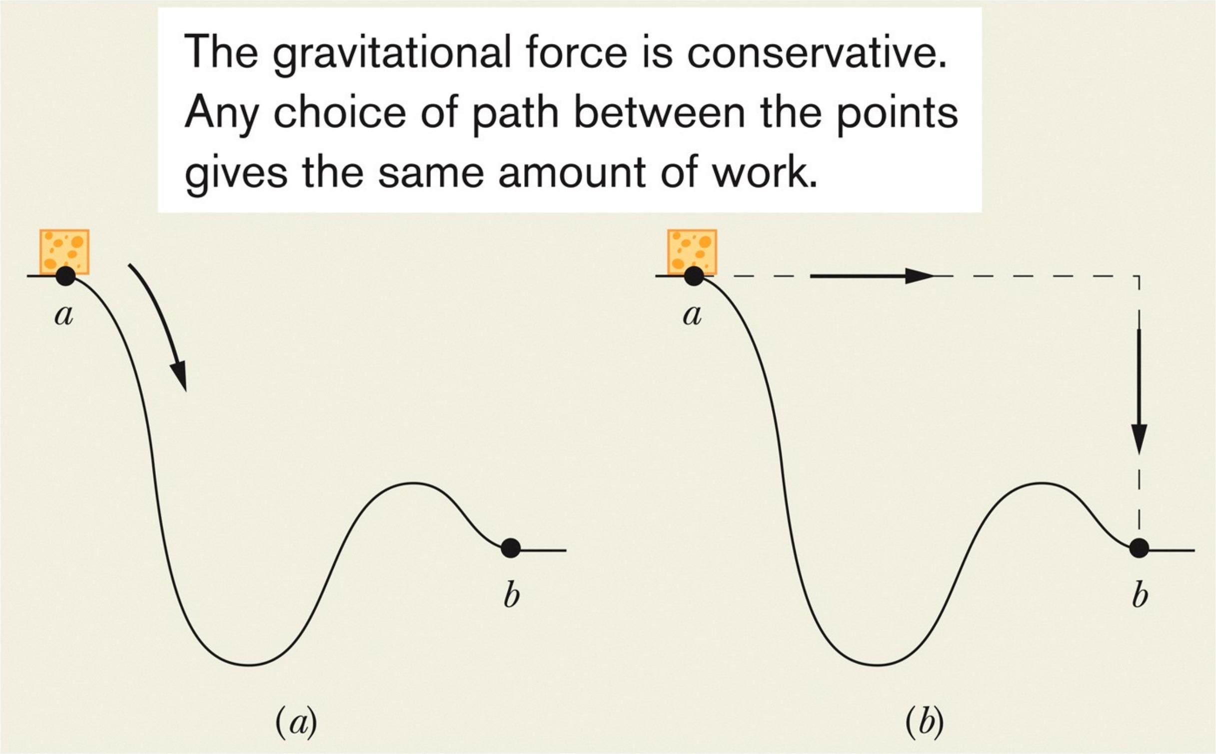


Figure 8-5

8-1 Potential Energy

- Mathematically:

$$W_{ab,1} = W_{ab,2}, \quad \text{Eq. (8-2)}$$

- This result allows you to substitute a simpler path for a more complex one if only conservative forces are involved

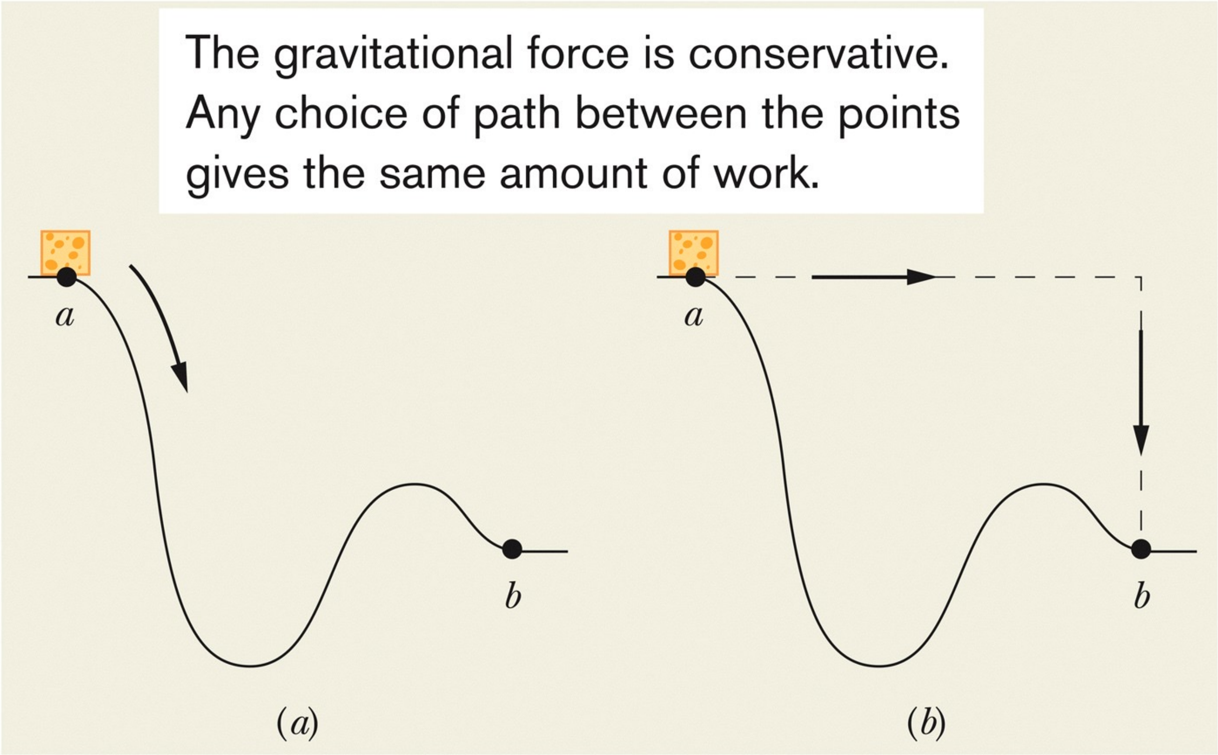
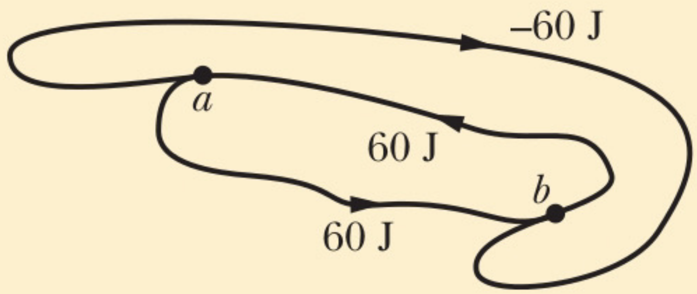


Figure 8-5

8-1 Potential Energy

Checkpoint 1

The figure shows three paths connecting points a and b. A single force $\vec{F}$ does the indicated work on a particle moving along each path in the indicated direction. On the basis of this information, is force $\vec{F}$ conservative?



Answer: No. The paths from $a \rightarrow b$ have different signs. One pair of paths allows the formation of a zero-work loop. The other does not.

8-1 Potential Energy

- For the general case, we calculate work as:

$$W = \int_{x_i}^{x_f} F(x)\, dx. \qquad \textbf{Eq. (8-5)}$$

- So we calculate potential energy as:

$$\Delta U = -\int_{x_i}^{x_f} F(x)\, dx. \qquad \textbf{Eq. (8-6)}$$

- Using this to calculate gravitational PE, relative to a **reference configuration** with **reference point** $y_i = 0$:

$$U(y) = mgy \qquad \textbf{Eq. (8-9)}$$

The gravitational potential energy associated with a particle–Earth system depends only on the vertical position y (or height) of the particle relative to the reference position $y = 0$, not on the horizontal position.

8-1 Potential Energy

- Use the same process to calculate spring PE:

$$\Delta U = -\int_{x_i}^{x_f} (-kx)\, dx = k \int_{x_i}^{x_f} x\, dx = \tfrac{1}{2}k \Big[x^2 \Big]_{x_i}^{x_f}, \qquad \textbf{Eq. (8-10)}$$

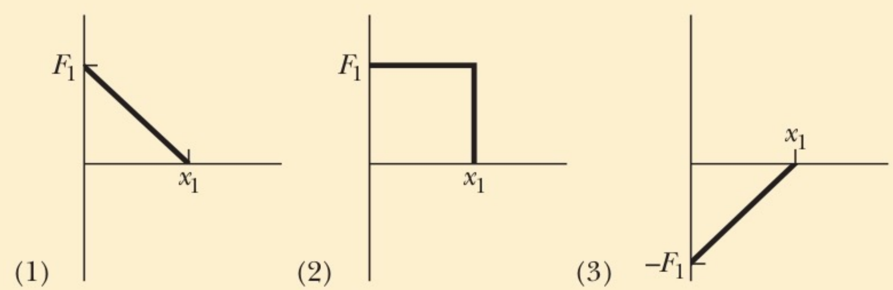
$$\Delta U = \tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2.$$

- With reference point x_i = 0 for a relaxed spring:

$$\boxed{U(x) = \tfrac{1}{2}kx^2} \qquad \textbf{Eq. (8-11)}$$

Checkpoint 2

A particle is to move along an x axis from $x = 0$ to x_1 while a conservative force, directed along the x axis, acts on the particle. The figure shows three situations in which the x component of that force varies with x. The force has the same maximum magnitude F_1 in all three situations. Rank the situations according to the change in the associated potential energy during the particle's motion, most positive first.



Answer: (3), (1), (2); a positive force does positive work, decreasing the PE; a negative force (e.g., 3) does negative work, increasing the PE

8-2 Conservation of Mechanical Energy

Learning Objectives

8.05 After first clearly defining which objects form a system, identify that the mechanical energy of the system is the sum of the kinetic energies and potential energies of those objects.

8.06 For an isolated system in which only conservative forces act, apply the conservation of mechanical energy to relate the initial potential and kinetic energies to the potential and kinetic energies at a later instant.

8-2 Conservation of Mechanical Energy

- The mechanical energy of a system is the sum of its potential energy *U* and kinetic energy *K*:

$$E_{\text{mec}} = K + U$$ **Eq. (8-12)**

- Work done by conservative forces increases *K* and decreases *U* by that amount, so:

$$\Delta K = -\Delta U.$$ **Eq. (8-15)**

- Using subscripts to refer to different instants of time:

$$K_2 + U_2 = K_1 + U_1$$ **Eq. (8-17)**

In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.

8-2 Conservation of Mechanical Energy

- This is the principle of the **conservation of mechanical energy**:

$$\Delta E_{\text{mec}} = \Delta K + \Delta U = 0. \quad \text{Eq. (8-18)}$$

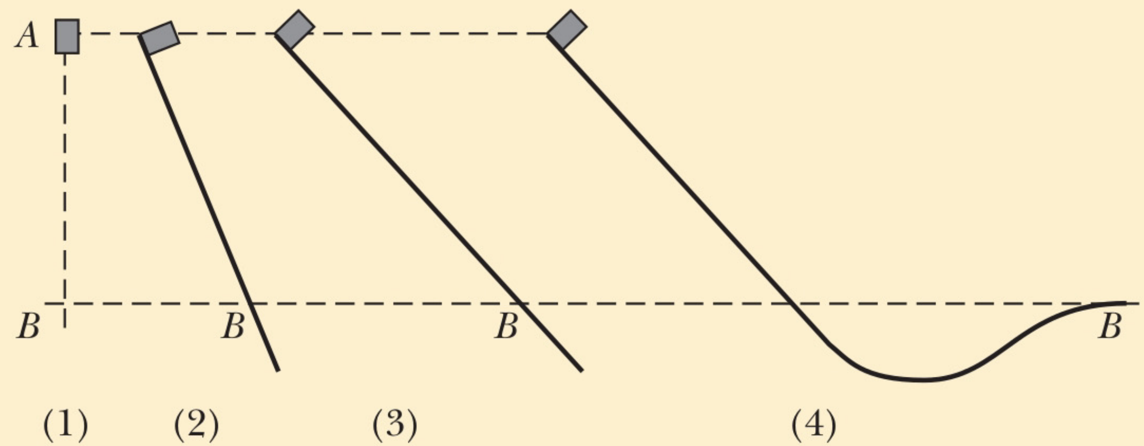
- This is very powerful tool:

> When the mechanical energy of a system is conserved, we can relate the sum of kinetic energy and potential energy at one instant to that at another instant *without considering the intermediate motion* and *without finding the work done by the forces involved.*

- One application:
  - Choose the lowest point in the system as $U = 0$
  - Then at the highest point U = max, and K = min

8-2 Conservation of Mechanical Energy

Checkpoint 3



The figure shows four situations—one in which an initially stationary block is dropped and three in which the block is allowed to slide down frictionless ramps. (a) Rank the situations according to the kinetic energy of the block at point *B*, greatest first. (b) Rank them according to the speed of the block at point *B*, greatest first.

Answer: Since there are no nonconservative forces, all of the difference in potential energy must go to kinetic energy. Therefore all are equal in (a). Because of this fact, they are also all equal in (b).

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8 Summary

Conservative Forces

- Net work on a particle over a closed path is 0

Gravitational Potential Energy

- Energy associated with Earth + a nearby particle

$$U(y) = mgy$$ **Eq. (8-9)**

Potential Energy

- Energy associated with the configuration of a system and a conservative force

$$\Delta U = -\int_{x_i}^{x_f} F(x)\, dx.$$ **Eq. (8-6)**

Elastic Potential Energy

- Energy associated with compression or extension of a spring

$$U(x) = \tfrac{1}{2}kx^2$$ **Eq. (8-11)**

8 Summary

Mechanical Energy

$$E_{\text{mec}} = K + U$$ **Eq. (8-12)**

- For only conservative forces within an isolated system, mechanical energy is conserved

Eq. (8-35)