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Phys 101
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15-1 Simple Harmonic Motion

Learning Objectives

15.01 Distinguish simple harmonic motion from other types of periodic motion.

15.02 For a simple harmonic oscillator, apply the relationship between position x and time t to calculate either if given a value for the other.

15.03 Relate period T, frequency f, and angular frequency ω.

15.04 Identify (displacement) amplitude x_m, phase constant (or phase angle) ϕ, and phase $\omega t + \phi$.

15.05 Sketch a graph of the oscillator's position x versus time t, identifying amplitude x_m and period T.

15.06 From a graph of position versus time, velocity versus time, or acceleration versus time, determine the amplitude of the plot and the value of the phase constant ϕ.

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15.07 On a graph of position x versus time t describe the effects of changing period T, frequency f, amplitude $x_{m,}$ or phase constant ϕ .

15.08 Identify the phase constant ϕ that corresponds to the starting time (t=0) being set when a particle in SHM is at an extreme point or passing through the center point.

15.09 Given an oscillator's position $x(t)$ as a function of time, find its velocity $v(t)$ as a function of time, identify the velocity amplitude v_m in the result, and calculate the velocity at any given time.

15.10 Sketch a graph of an oscillator's velocity v versus time t, identifying the velocity amplitude $v_{m.}$

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15.11 Apply the relationship between velocity amplitude v_m, angular frequency ω, and (displacement) x_m.

15.12 Given an oscillator's velocity $v(t)$ as a function of time, calculate its acceleration $a(t)$ as a function of time, identify the acceleration amplitude a_m in the result, and calculate the acceleration at any given time.

15.13 Sketch a graph of an oscillator's acceleration a versus time t, identifying the acceleration amplitude a_m.

15-1 Simple Harmonic Motion

Learning Objectives continued

15.14 Identify that for a simple harmonic oscillator the acceleration a at any instant is *always* given by the product of a negative constant and the displacement x just then.

15.15 For any given instant in an oscillation, apply the relationship between acceleration a, angular frequency ω, and displacement x.

15.16 Given data about the position x and velocity v at one instant, determine the phase $\omega t + \phi$ and phase constant ϕ.

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Learning Objectives Continued

15.17 For a spring-block oscillator, apply the relationships between spring constant k and mass m and either period T or angular frequency ω.

15.18 Apply Hooke's law to relate the force F on a simple harmonic oscillator at any instant to the displacement x of the oscillator at that instant.

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15-1 Simple Harmonic Motion

- The **frequency** of an oscillation is the number of times per second that it completes a full oscillation (cycle)
- Unit of hertz: 1 Hz = 1 oscillation per second
- The time in seconds for one full cycle is the **period**

$$T = \frac{1}{f}.$$

Eq. (15-2)

- Any motion that repeats regularly is called periodic
- **Simple harmonic motion** is periodic motion that is a sinusoidal function of time

$$x(t) = x_m \cos(\omega t + \phi)$$

Eq. (15-3)

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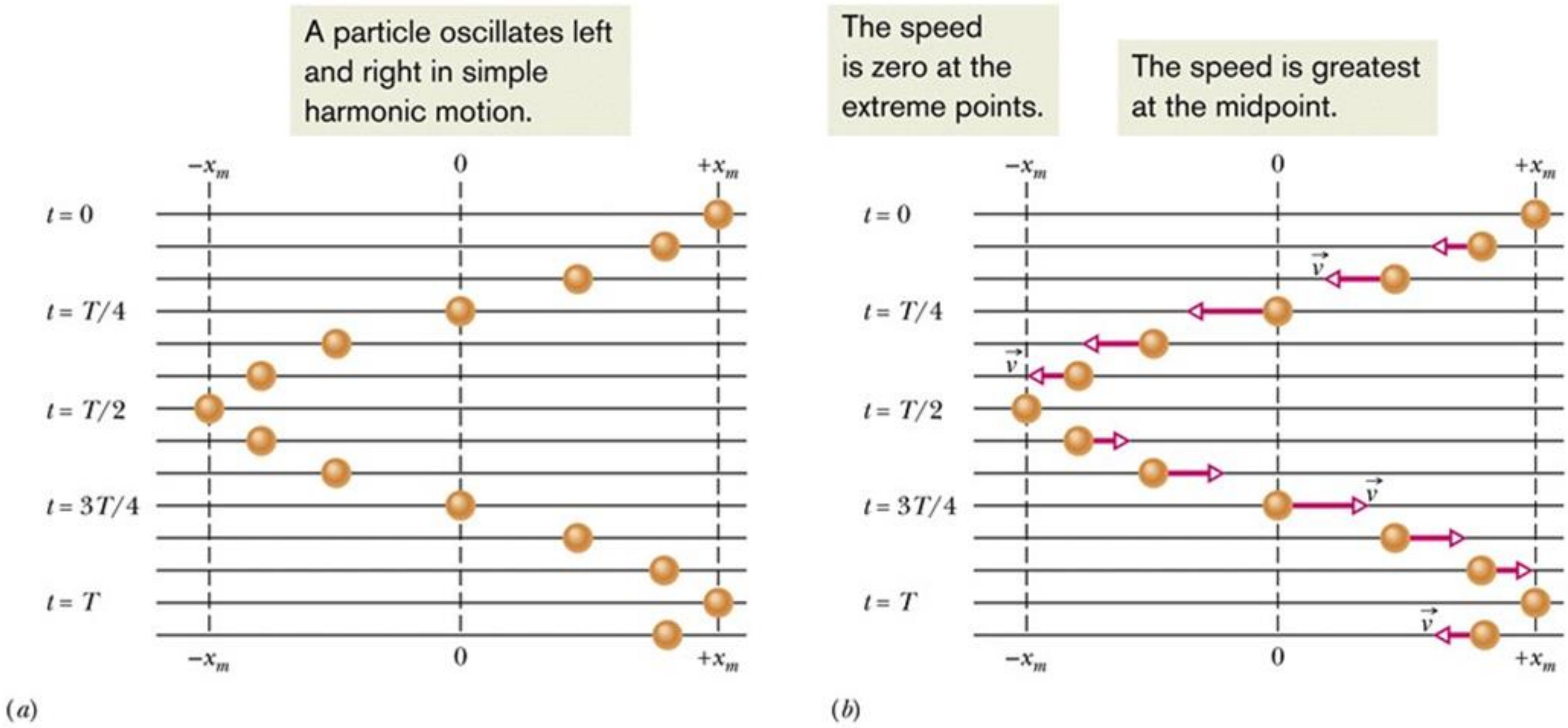
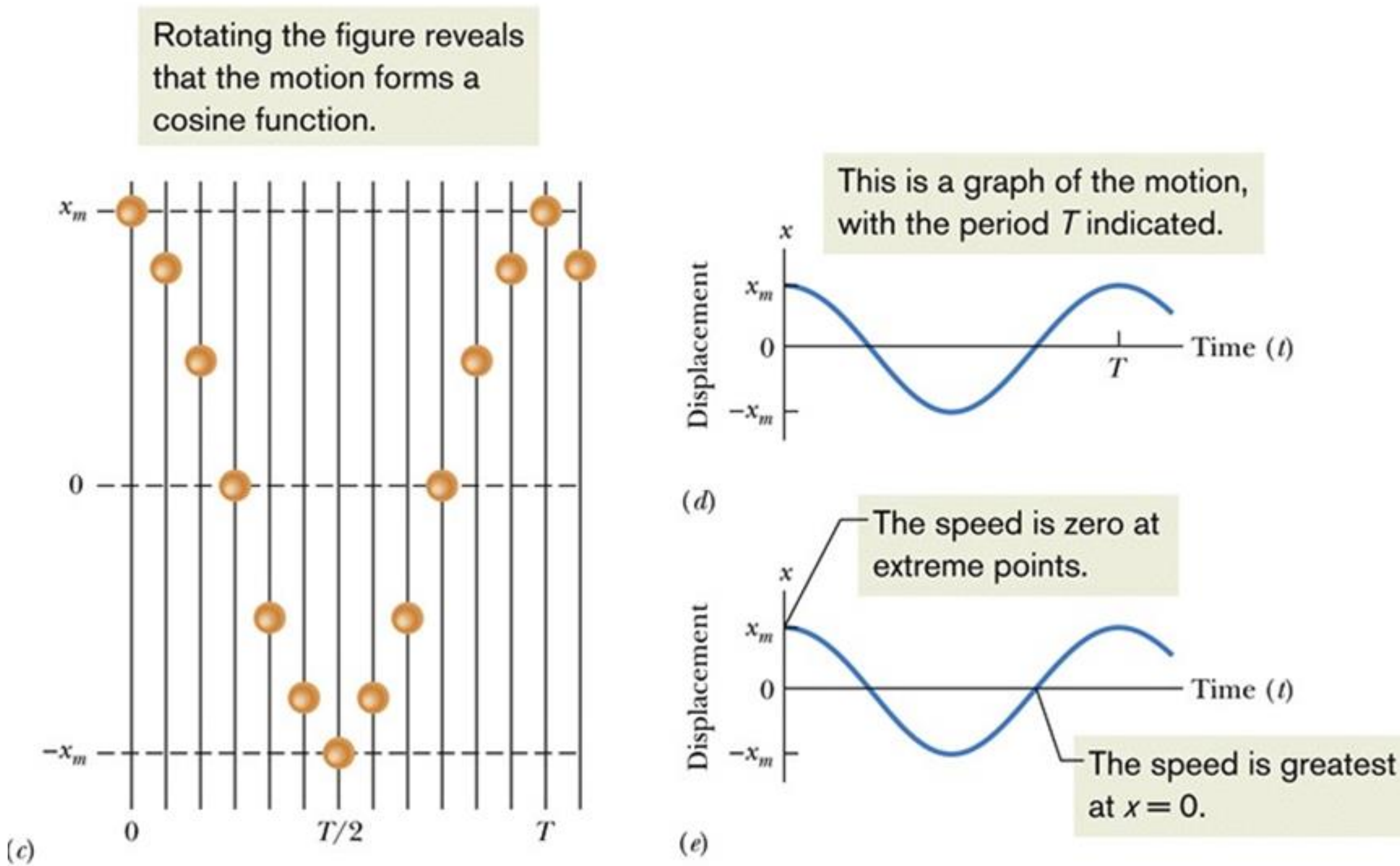


A particle oscillates left and right in simple harmonic motion.
The speed is zero at the extreme points.
The speed is greatest at the midpoint.
$-x_m$
0
$+x_m$
$t = 0$
$t = T/4$
$t = T/2$
$t = 3T/4$
$t = T$
$\vec{v}$
(a)
(b)



Rotating the figure reveals that the motion forms a cosine function.
x_m
0
$-x_m$
0
T/2
T
(c)
This is a graph of the motion, with the period T indicated.
x
Displacement
x_m
0
$-x_m$
Time (t)
T
(d)
The speed is zero at extreme points.
x
Displacement
x_m
0
$-x_m$
Time (t)
The speed is greatest at x = 0.
(e)

15-1 Simple Harmonic Motion

- The value written x_m is how far the particle moves in either direction: the **amplitude**
- The argument of the cosine is the **phase**
- The constant φ is called the **phase angle** or phase constant
- It adjusts for the initial conditions of motion at $t = 0$
- The **angular frequency** is written ω

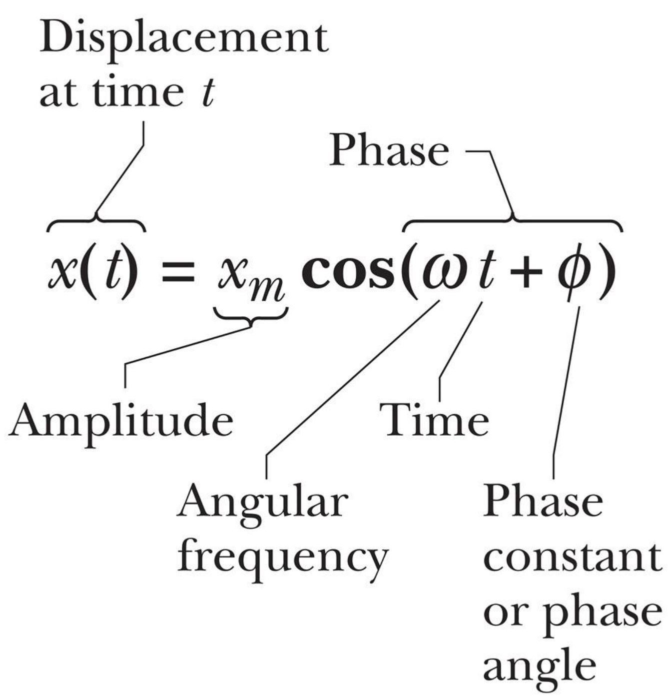


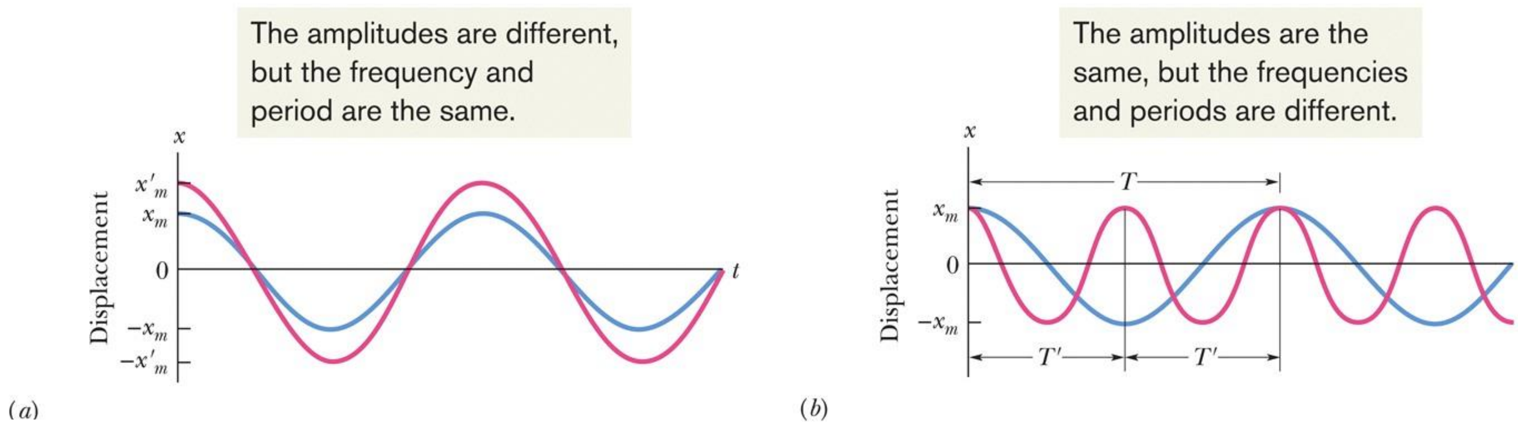
Figure 15-3

15-1 Simple Harmonic Motion

- The angular frequency has the value:

$$\omega = \frac{2\pi}{T} = 2\pi f.$$

Eq. (15-5)



$$x_m \cos \omega t = x_m \cos \omega(t + T).$$

$$\omega(t + T) = \omega t + 2\pi$$

$$\omega T = 2\pi.$$

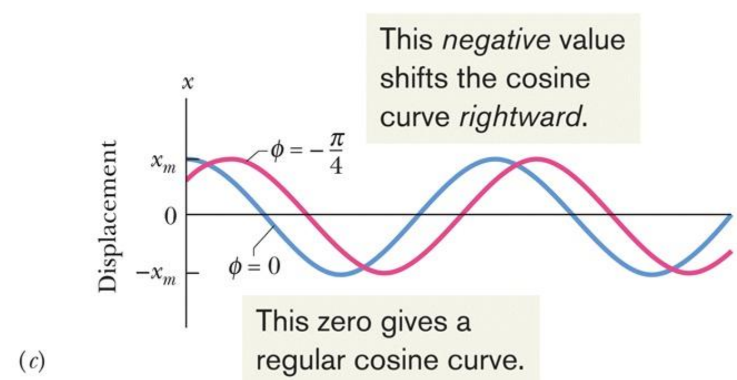


Figure 15-5

15-1 Simple Harmonic Motion

Checkpoint 1

A particle undergoing simple harmonic oscillation of period T (like that in Fig. 15-2) is at $-x_m$ at time $t = 0$. Is it at $-x_m$, at $+x_m$, at 0, between $-x_m$ and 0, or between 0 and $+x_m$ when (a) $t = 2.00T$, (b) $t = 3.50T$, and (c) $t = 5.25T$?

Answer: (a) at $-x_m$ (b) at x_m (c) at 0

- The velocity can be found by the time derivative of the position function:

$$v(t) = -\omega x_m \sin(\omega t + \phi)$$ **Eq. (15-6)**

- The value ωx_m is the **velocity amplitude** v_m

15-1 Simple Harmonic Motion

- The acceleration can be found by the time derivative of the velocity function, or 2nd derivative of position:

$$a(t) = -\omega^2 x_m \cos(\omega t + \phi)$$

Eq. (15-7)

- The value $\omega^2 x_m$ is the **acceleration amplitude** a_m
- Acceleration related to position:

$$a(t) = -\omega^2 x(t).$$

Eq. (15-8)

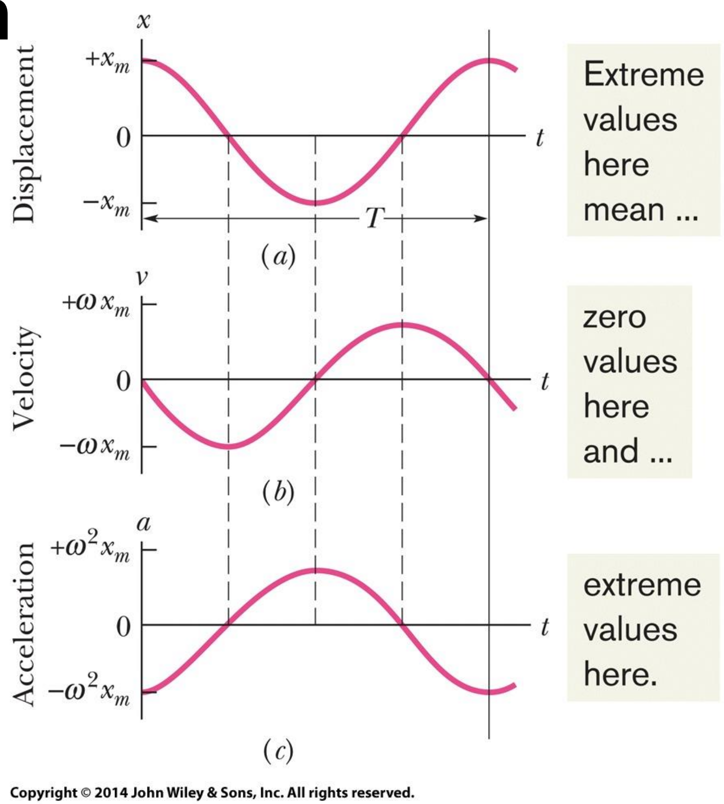


Figure 15-6

15-1 Simple Harmonic Motion

In SHM, the acceleration a is proportional to the displacement x but opposite in sign, and the two quantities are related by the square of the angular frequency ω.

Checkpoint 2

Which of the following relationships between a particle's acceleration a and its position x indicates simple harmonic oscillation: (a) $a = 3x^2$, (b) $a = 5x$, (c) $a = -4x$, (d) $a = -2/x$? For the SHM, what is the angular frequency (assume the unit of rad/s)?

Answer: (c) where the angular frequency is 2

15-1 Simple Harmonic Motion

- We can apply Newton's second law

$$F = ma = m(-\omega^2 x) = -(m\omega^2)x.$$ **Eq. (15-9)**

- Relating this to Hooke's law we see the similarity

Simple harmonic motion is the motion of a particle when the force acting on it is proportional to the particle's displacement but in the opposite direction.

- **Linear simple harmonic oscillation** (*F* is proportional to *x* to the first power) gives:

$$\omega = \sqrt{\frac{k}{m}} \quad \text{(angular frequency).}$$ **Eq. (15-12)**

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{(period).}$$ **Eq. (15-13)**

15-1 Simple Harmonic Motion

Checkpoint 3

Which of the following relationships between the force F on a particle and the particle's position x gives SHM: (a) $F = -5x$, (b) $F = -400x^2$, (c) $F = 10x$, (d) $F = 3x^2$?

Answer: only (a) is simple harmonic motion

(note that b is harmonic motion, but nonlinear and not SHM)

15-2 Energy in Simple Harmonic Motion

Learning Objectives

15.19 For a spring-block oscillator, calculate the kinetic energy and elastic potential energy at any given time.

15.20 Apply the conservation of energy to relate the total energy of a spring-block oscillator at one instant to the total energy at another instant.

15.21 Sketch a graph of the kinetic energy, potential energy, and total energy of a spring-block oscillator, first as a function of time and then as a function of the oscillator's position.

15.22 For a spring-block oscillator, determine the block's position when the total energy is entirely kinetic energy and when it is entirely potential energy.

15-2 Energy in Simple Harmonic Motion

- Write the functions for kinetic and potential energy:

$$U(t) = \frac{1}{2}kx^2 = \frac{1}{2}kx_m^2 \cos^2(\omega t + \phi).$$

Eq. (15-18)

$$K(t) = \frac{1}{2}mv^2 = \frac{1}{2}kx_m^2 \sin^2(\omega t + \phi).$$

Eq. (15-20)

- Their sum is defined by:

$$E = U + K = \frac{1}{2}kx_m^2.$$

Eq. (15-21)

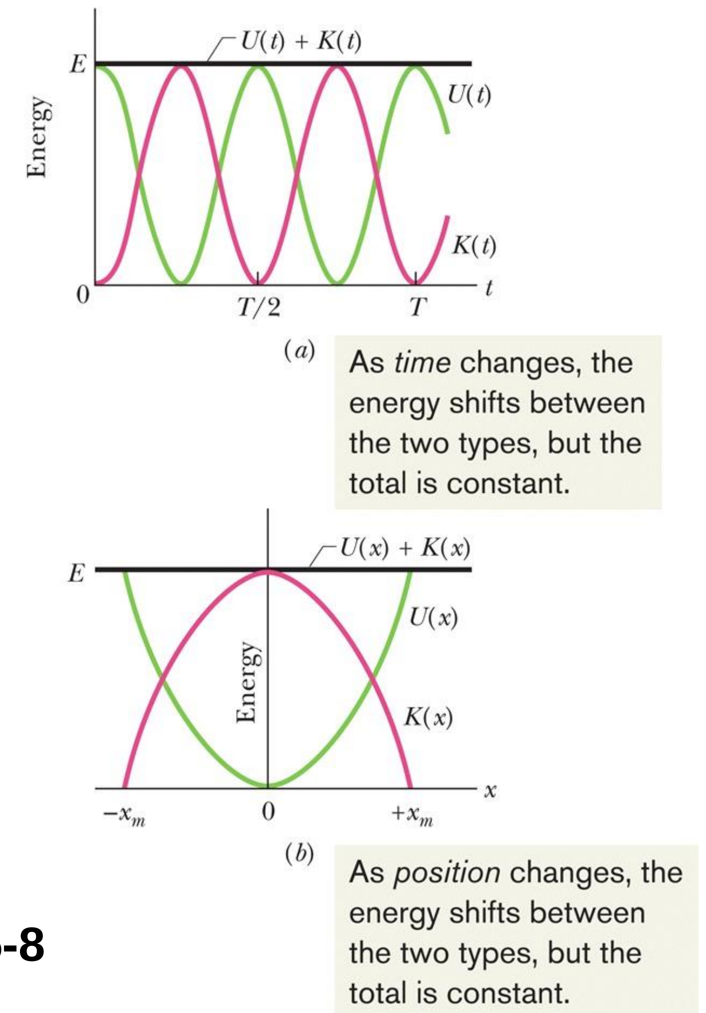


Figure 15-8

15-2 Energy in Simple Harmonic Motion

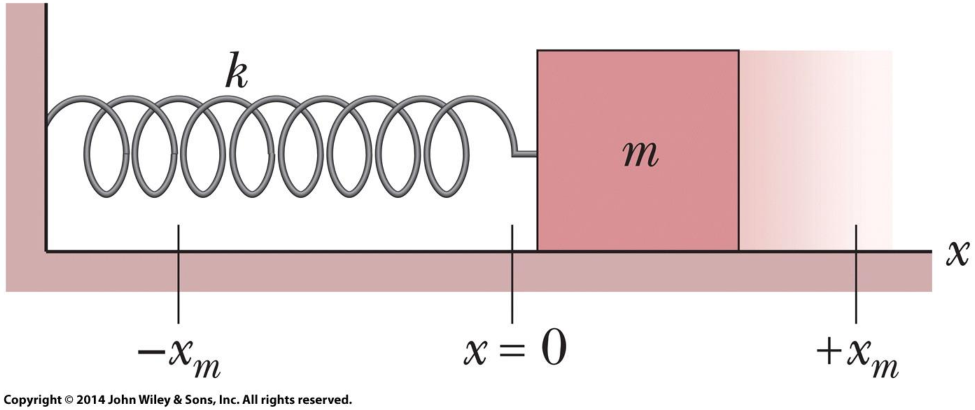


Figure 15-7

Checkpoint 4

In Fig. 15-7, the block has a kinetic energy of 3 J and the spring has an elastic potential energy of 2 J when the block is at $x = +2.0$ cm. (a) What is the kinetic energy when the block is at $x = 0$? What is the elastic potential energy when the block is at (b) $x = -2.0$ cm and (c) $x = -x_m$?

Answer: (a) 5 J (b) 2 J (c) 5 J

15-4 Pendulums, Circular Motion

Learning Objectives

15.27 Describe the motion of an oscillating simple pendulum.

15.28 Draw a free-body diagram.

15.29-31 Distinguish between a simple and physical pendulum, and relate their variables.

15.32 Find angular frequency from torque and angular displacement or acceleration and displacement.

15.33 Distinguish angular frequency from $d\theta/dt$.

15.34 Determine phase and amplitude.

15.35 Describe how free-fall acceleration can be measured with a pendulum.

15.36 For a physical pendulum, find the center of the oscillation.

15.37 Relate SHM to uniform circular motion.

15-4 Pendulums, Circular Motion

- A **simple pendulum**: a *bob* of mass *m* suspended from an unstretchable, massless string
- Bob feels a restoring torque:

$$\tau = -L(F_g \sin\theta),$$ **Eq. (15-24)**

- Relating this to moment of inertia:

$$\alpha = -\frac{mgL}{I}\theta.$$ **Eq. (15-26)**

- Angular acceleration proportional to position but opposite in sign

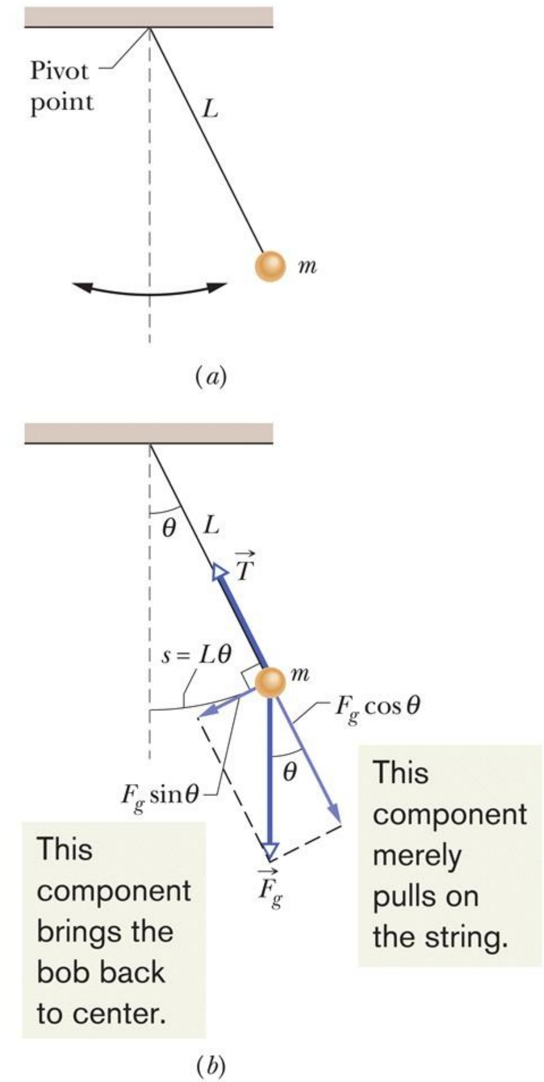


Figure 15-11

15-4 Pendulums, Circular Motion

- **Angular amplitude** θ_m of the motion must be small
- The angular frequency is:

$$\omega = \sqrt{\frac{mgL}{I}}.$$

- The period is (for simple pendulum, $I = mL^2$):

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Eq. (15-28)

- A **physical pendulum** has a complicated mass distribution

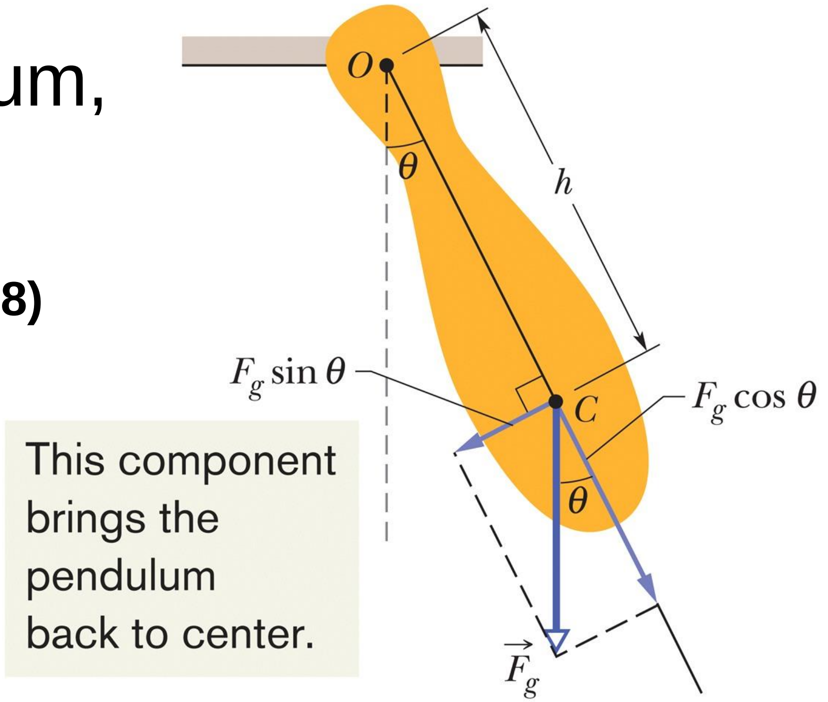


Figure 15-12

15-4 Pendulums, Circular Motion

- An analysis is the same except rather than length *L* we have distance *h* to the com, and *I* will be particular to the mass distribution
- The period is:

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

Eq. (15-29)

- A physical pendulum will not show SHM if pivoted about its com
- The *center of oscillation* of a physical pendulum is the length L_0 of a simple pendulum with the same period

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15-4 Pendulums, Circular Motion

- A physical pendulum can be used to determine free-fall acceleration *g*
- Assuming the pendulum is a uniform rod of length L:

$$I = I_{\text{com}} + mh^2 = \tfrac{1}{12}mL^2 + m(\tfrac{1}{2}L)^2 = \tfrac{1}{3}mL^2.$$

Eq. (15-30)

- Then solve Eq. 15-29 for *g*:

$$g = \frac{8\pi^2 L}{3T^2}.$$

Eq. (15-31)

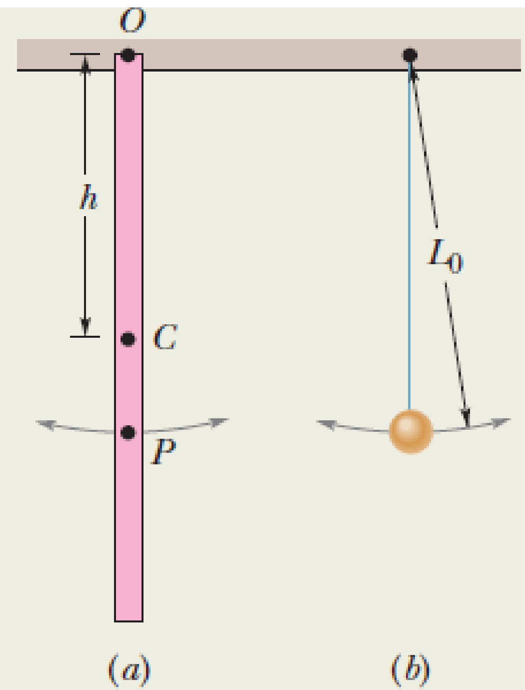
Checkpoint 5

Three physical pendulums, of masses m_0, $2m_0$, and $3m_0$, have the same shape and size and are suspended at the same point. Rank the masses according to the periods of the pendulums, greatest first.

Answer: All the same: mass does not affect the period of a pendulum

Sample Problem 15.05 Physical pendulum, period and length

In Fig. 15-13*a*, a meter stick swings about a pivot point at one end, at distance h from the stick's center of mass.



(a) What is the period of oscillation T?

Calculations: The period for a physical pendulum is given by Eq. 15-29, for which we need the rotational inertia I of the stick about the pivot point. We can treat the stick as a uniform rod of length L and mass m. Then Eq. 15-30 tells us that $I = \frac{1}{3}mL^2$, and the distance h in Eq. 15-29 is $\frac{1}{2}L$. Substituting these quantities into Eq. 15-29, we find

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{\frac{1}{3}mL^2}{mg(\frac{1}{2}L)}} \qquad (15\text{-}32)$$

$$= 2\pi\sqrt{\frac{2L}{3g}} \qquad (15\text{-}33)$$

$$= 2\pi\sqrt{\frac{(2)(1.00\text{ m})}{(3)(9.8\text{ m/s}^2)}} = 1.64\text{ s.} \qquad \text{(Answer)}$$

Note the result is independent of the pendulum's mass m.

$$T = 2\pi\sqrt{\frac{L_0}{g}} = 2\pi\sqrt{\frac{2L}{3g}}.$$

You can see by inspection that

$$L_0 = \tfrac{2}{3}L$$
$$= (\tfrac{2}{3})(100 \text{ cm}) = 66.7 \text{ cm}.$$

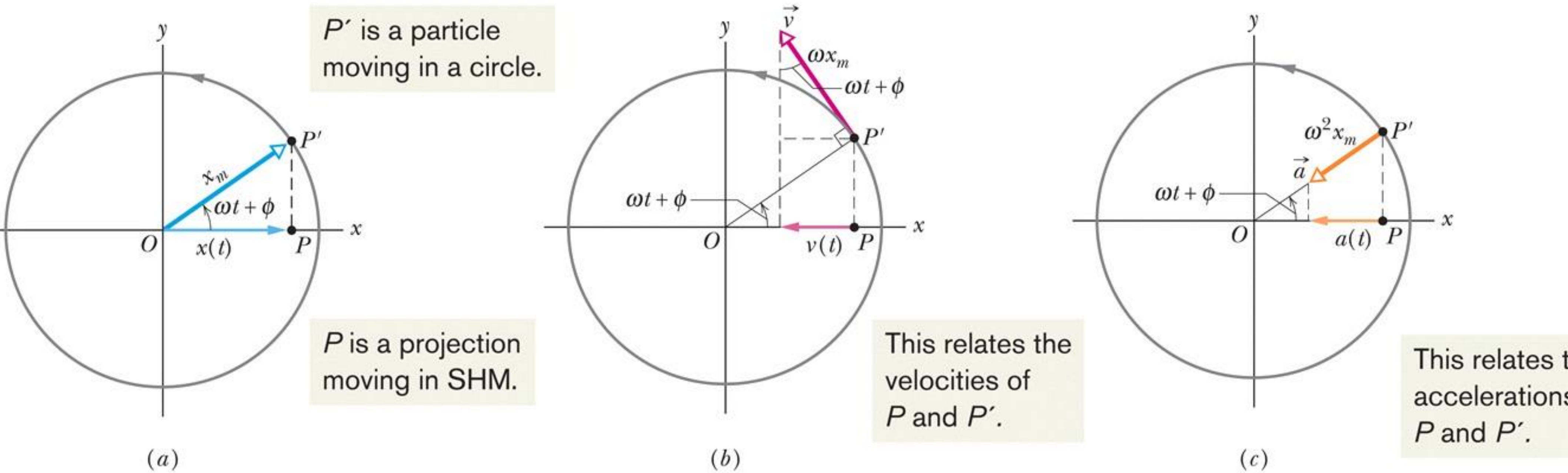
15-4 Pendulums, Circular Motion

- Simple harmonic motion is circular motion viewed edge-on

> Simple harmonic motion is the projection of uniform circular motion on a diameter of the circle in which the circular motion occurs.

- Figure 15-15 shows a reference particle moving in uniform circular motion
- Its angular position at any time is $\omega t + \varphi$

Figure 15-15


P′ is a particle moving in a circle.
P is a projection moving in SHM.
This relates the velocities of P and P′.
This relates t accelerations P and P′.
y
x
O
P′
P
x_m
$\omega t + \phi$
$x(t)$
$\vec{v}$
ωx_m
$v(t)$
$\omega^2 x_m$
$\vec{a}$
$a(t)$
(a)
(b)
(c)

15-4 Pendulums, Circular Motion

- Projecting its position onto *x*:

$$x(t) = x_m \cos(\omega t + \phi), \qquad \text{Eq. (15-36)}$$

- Similarly with velocity and acceleration:

$$v(t) = -\omega x_m \sin(\omega t + \phi), \qquad \text{Eq. (15-37)}$$

$$a(t) = -\omega^2 x_m \cos(\omega t + \phi), \qquad \text{Eq. (15-38)}$$

- We indeed find this projection is simple harmonic motion

15 Summary

Frequency

- 1 Hz = 1 cycle per second

Period

$$T = \frac{1}{f}.$$ Eq. (15-2)

The Linear Oscillator

$$\omega = \sqrt{\frac{k}{m}}$$ Eq. (15-12)

$$T = 2\pi\sqrt{\frac{m}{k}}$$ Eq. (15-13)

Simple Harmonic Motion

- Find *v* and *a* by differentiation

$$x(t) = x_m \cos(\omega t + \phi)$$ Eq. (15-3)

$$\omega = \frac{2\pi}{T} = 2\pi f.$$ Eq. (15-5)

Energy

- Mechanical energy remains constant as *K* and *U* change
- $K = \frac{1}{2} mv^2$, $U = \frac{1}{2} kx^2$

15 Summary

Pendulums

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$ **Eq. (15-23)**

$$T = 2\pi\sqrt{\frac{L}{g}}$$ **Eq. (15-28)**

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$ **Eq. (15-29)**

Simple Harmonic Motion and Uniform Circular Motion

- SHM is the projection of UCM onto the diameter of the circle in which the UCM occurs