Chapter 14

Fluids

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14-1 Fluid Density and Pressure

Learning Objectives

14.01 Distinguish fluids from solids.

14.02 When mass is uniformly distributed, relate density to mass and volume.

14.03 Apply the relationship between hydrostatic pressure, force, and the surface area over which that force acts.

14-1 Fluid Density and Pressure

- Physics of fluids is the basis of hydraulic engineering
- A **fluid** is a substance that can flow, like water or air, and conform to a container
- This occurs because fluids cannot sustain a shearing force (tangential to the fluid surface)
- They can however apply a force perpendicular to the fluid surface
- Some materials (pitch) take a long time to conform to a container, but are still fluids
- The essential identifier is that fluids do not have a crystalline structure

14-1 Fluid Density and Pressure

- The **density**, ρ, is defined as:

$$\rho = \frac{\Delta m}{\Delta V}.$$

Eq. (14-1)

- In theory the density at a point is the limit for an infinitesimal volume, but we assume a fluid sample is large relative to atomic dimensions and has uniform density. Then

$$\rho = \frac{m}{V}$$

Eq. (14-2)

- Density is a scalar quantity
- Units kg/m^3

14-1 Fluid Density and Pressure

- The **pressure**, force acting on an area, is defined as:

$$p = \frac{\Delta F}{\Delta A}.$$

Eq. (14-3)

- We could take the limit of this for infinitesimal area, but if the force is uniform over a flat area *A* we write

$$p = \frac{F}{A}$$

Eq. (14-4)

- We can measure pressure with a sensor

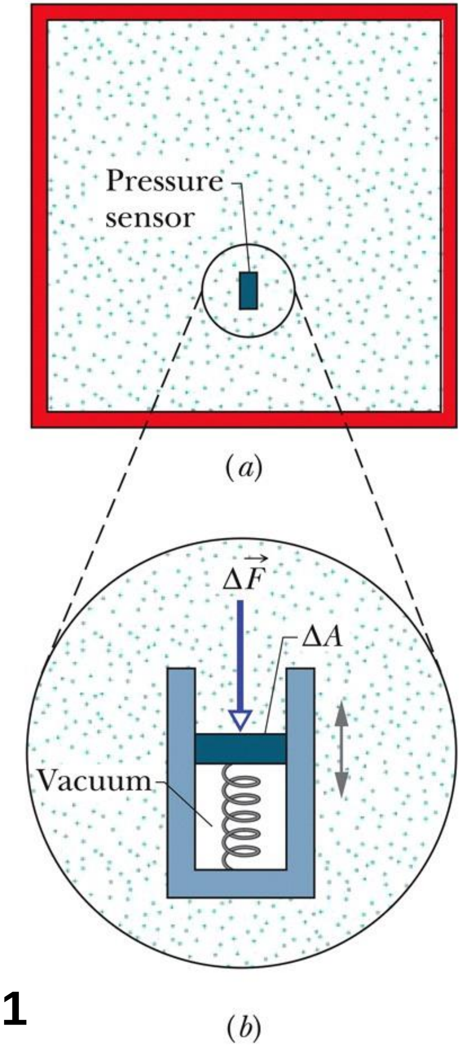


Figure 14-1

14-1 Fluid Density and Pressure

- We find by experiment that for a fluid at rest, pressure has the same value at a point regardless of sensor orientation
- Therefore static pressure is scalar, even though force is not
- Only the magnitude of the force is involved
- Units: the pascal (1 Pa = 1 N/m^2)
  the atmosphere (atm)
  the torr (1 torr = 1 mm Hg)
  the pound per square inch (psi)

$$1 \text{ atm} = 1.01 \times 10^5 \text{ Pa} = 760 \text{ torr} = 14.7 \text{ lb/in.}^2.$$

14-2 Fluids at Rest

Learning Objectives

14.04 Apply the relationship between the hydrostatic pressure, fluid density, and the height above or below a reference level.

14.05 Distinguish between total pressure (absolute pressure) and gauge pressure.

14-2 Fluids at Rest

- *Hydrostatic* pressures are those caused by fluids at rest (air in the atmosphere, water in a tank)

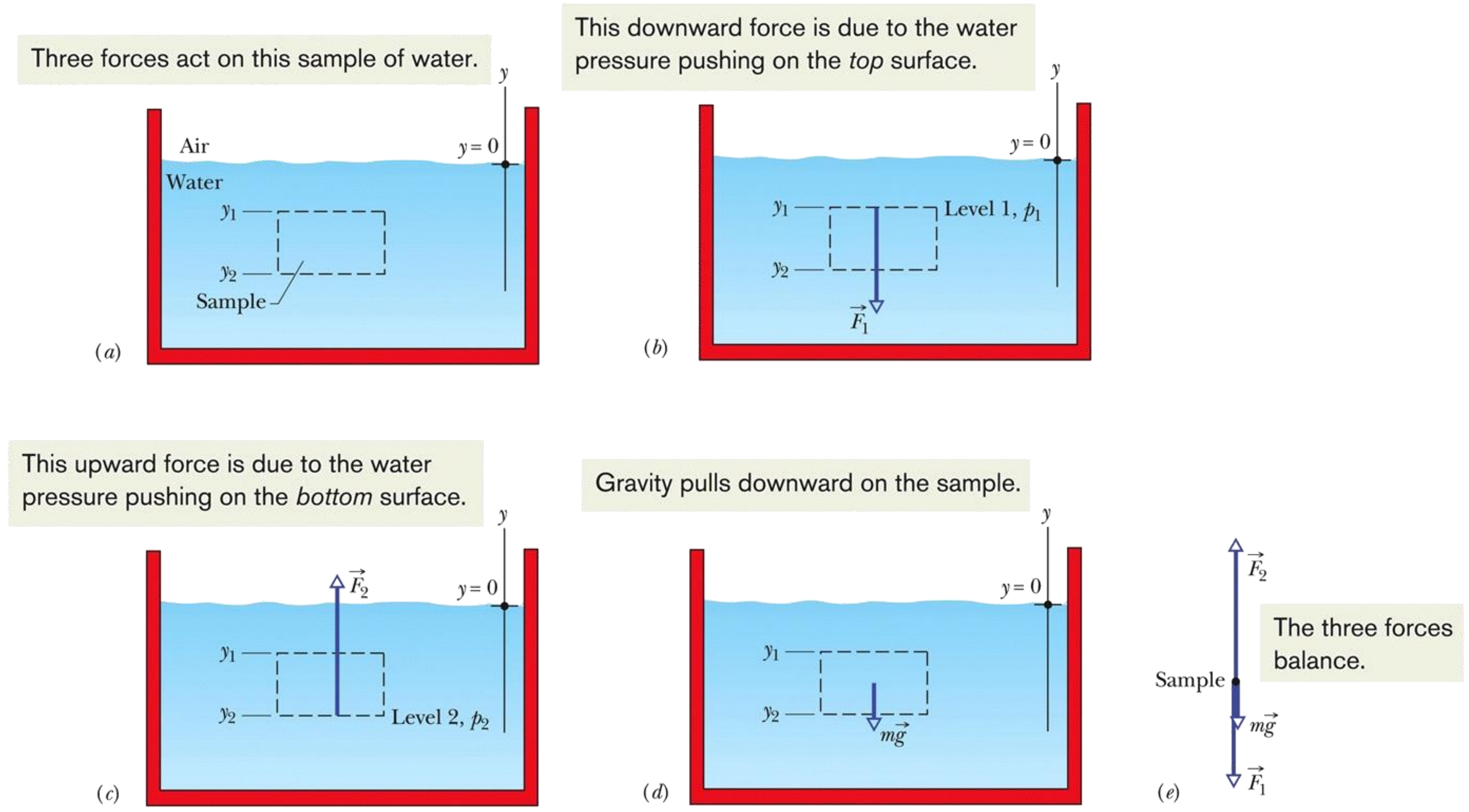


Figure 14-2

14-2 Fluids at Rest

- Write the balance of forces:

$$F_2 = F_1 + mg.$$ Eq. (14-5)

- Rewrite: forces with pressures, mass with density

$$p_2 A = p_1 A + \rho A g(y_1 - y_2)$$

$$p_2 = p_1 + \rho g(y_1 - y_2).$$ Eq. (14-7)

- For a depth *h* below the surface in a liquid this becomes:

$$p = p_0 + \rho g h$$ Eq. (14-8)

The pressure at a point in a fluid in static equilibrium depends on the depth of that point but not on any horizontal dimension of the fluid or its container.

14-2 Fluids at Rest

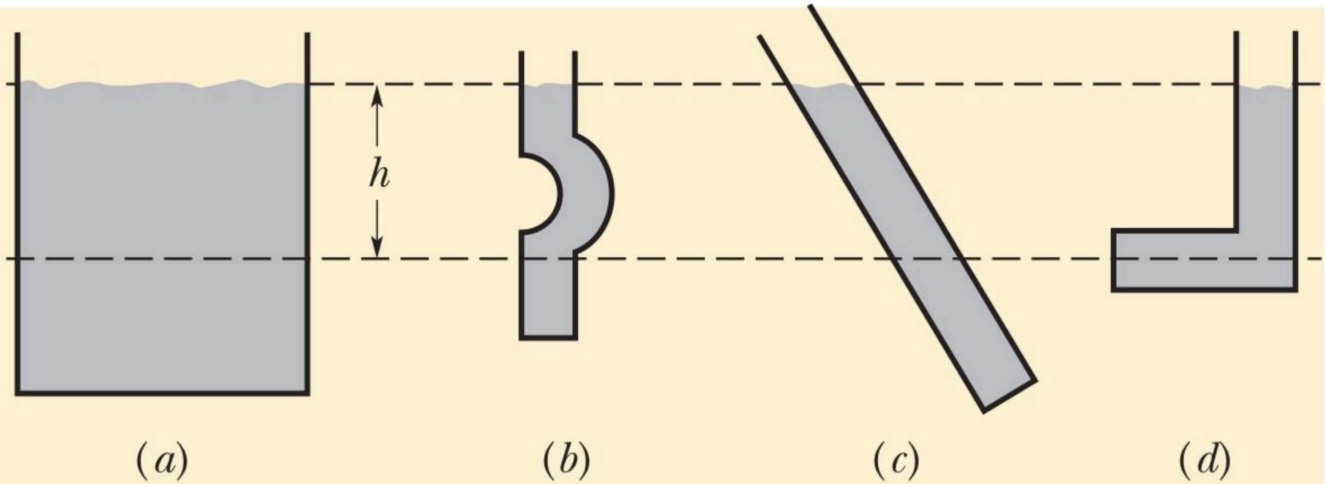
- The pressure in 14-8 is the absolute pressure
- Consists of p_0, the pressure due to the atmosphere, and the additional pressure from the fluid
- The difference between absolute pressure and atmospheric pressure is called the **gauge pressure** because we use a gauge to measure this pressure difference
- The equation can be turned around to calculate the atmospheric pressure at a given height above ground:

$$p = p_0 - \rho_{\text{air}} g d.$$

14-2 Fluids at Rest

Checkpoint 1

The figure shows four containers of olive oil. Rank them according to the pressure at depth *h*, greatest first.



Answer: All the pressures will be the same. All that matters is the distance *h*, from the surface to the location of interest, and *h* is the same in all cases.

14-3 Measuring Pressure

Learning Objectives

14.06 Describe how a barometer can measure atmospheric pressure.

14.07 Describe how an open-tube manometer can measure the gauge pressure of a gas.

14-3 Measuring Pressure

- Figure 14-5 shows *mercury barometers*
- The height difference between the air-mercury interface and the mercury level is *h*:

$$p_0 = \rho g h,$$

Eq. (14-9)

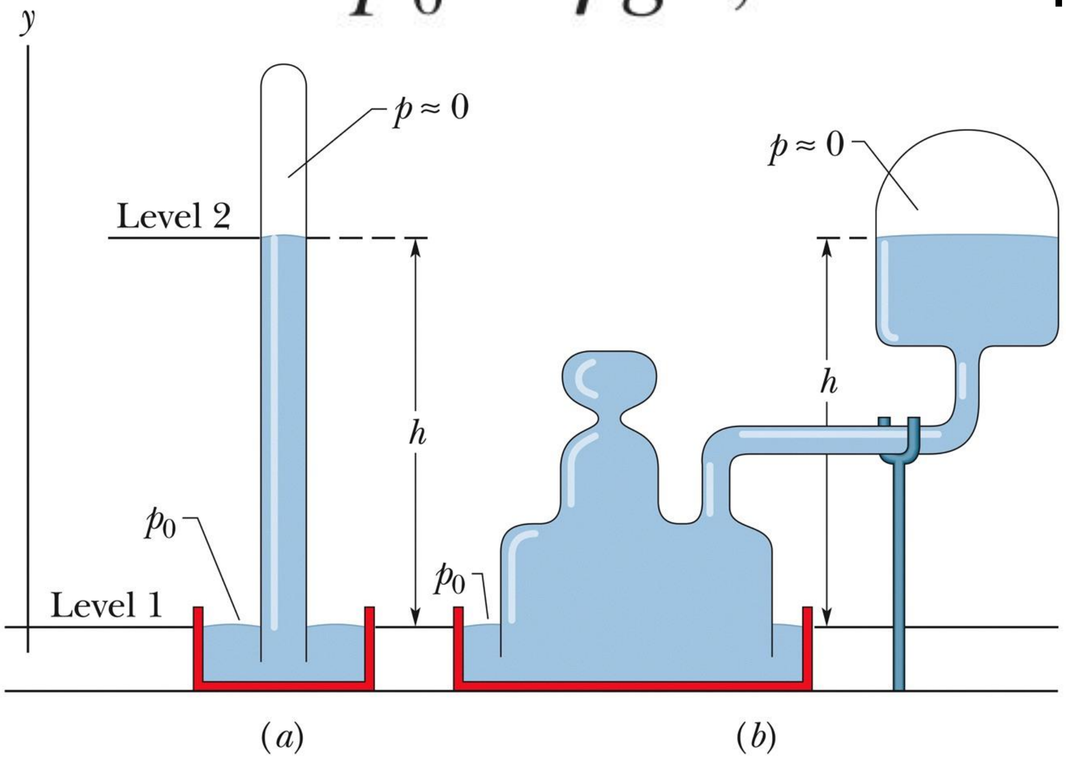


Figure 14-5

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14-3 Measuring Pressure

- Only the height matters, not the cross-sectional area
- Height of mercury column is numerically equal to torr pressure only if:
  - Barometer is at a place where g has its standard value
  - Temperature of mercury is 0°C

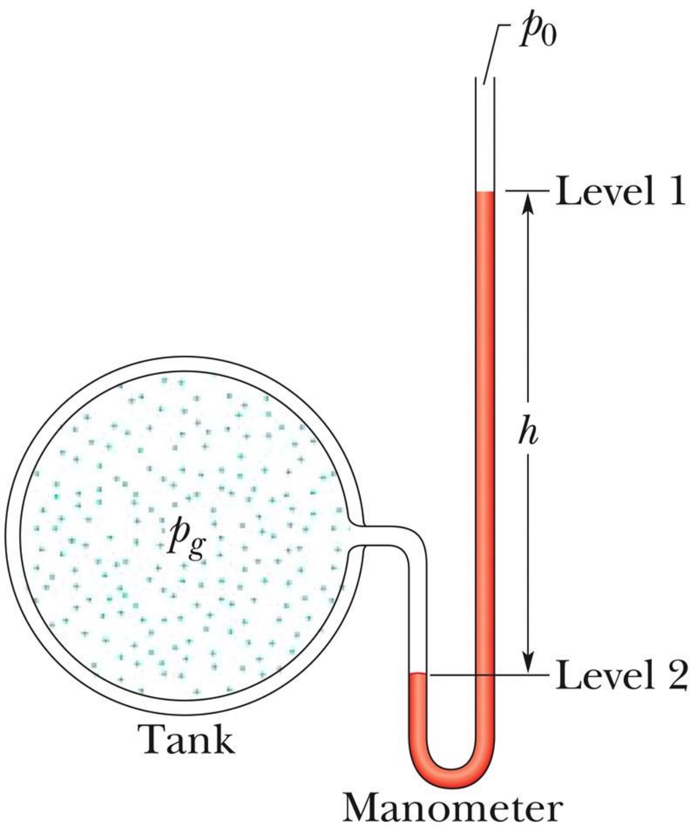
14-3 Measuring Pressure

Figure 14-6 shows an *open-tube manometer*

- The height difference between
- the two interfaces, *h*, is related
- to the gauge pressure:

$$p_g = p - p_0 = \rho g h,$$

Eq. (14-10)



- The gauge pressure can be positive or negative, depending on whether the pressure being measured is greater or less than atmospheric pressure

11-4 Pascal's Principle

Learning Objectives

14.08 Identify Pascal's principle.

14.09 For a hydraulic lift, apply the relationship between the input area and displacement and the output area and displacement.

14-4 Pascal's Principle

- **Pascal's principle** governs the transmission of pressure through an incompressible fluid:

> A change in the pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of its container.

- Consider a cylinder of fluid (Figure 14-7)
- Increase p_{ext}, and p at any point must change

$$\Delta p = \Delta p_{\text{ext}}.$$ **Eq. (14-12)**

- Independent of h

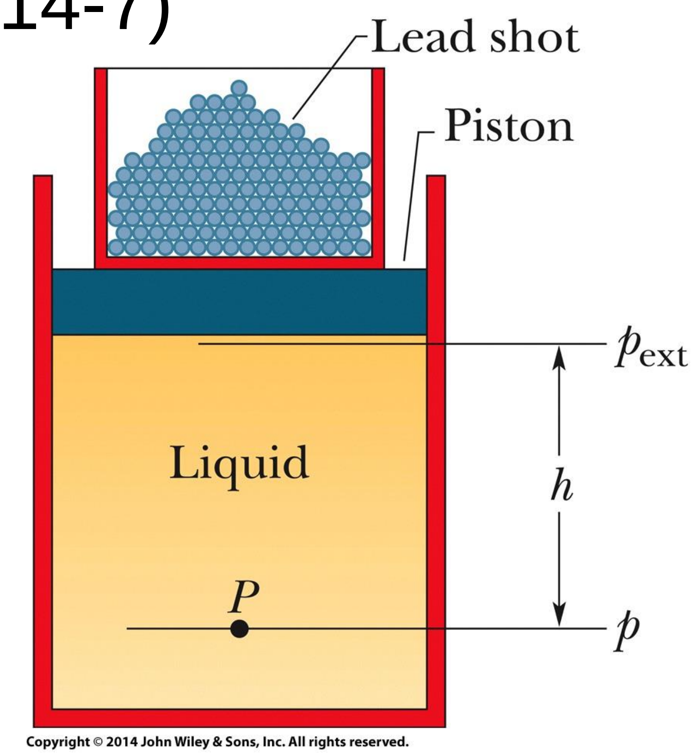


Figure 14-7

14-4 Pascal's Principle

- Describes the basis for a hydraulic lever
- Input and output forces related by:

$$F_o = F_i \frac{A_o}{A_i}.$$ **Eq. (14-13)**

- The distances of movement are related by:

$$d_o = d_i \frac{A_i}{A_o}.$$ **Eq. (14-14)**

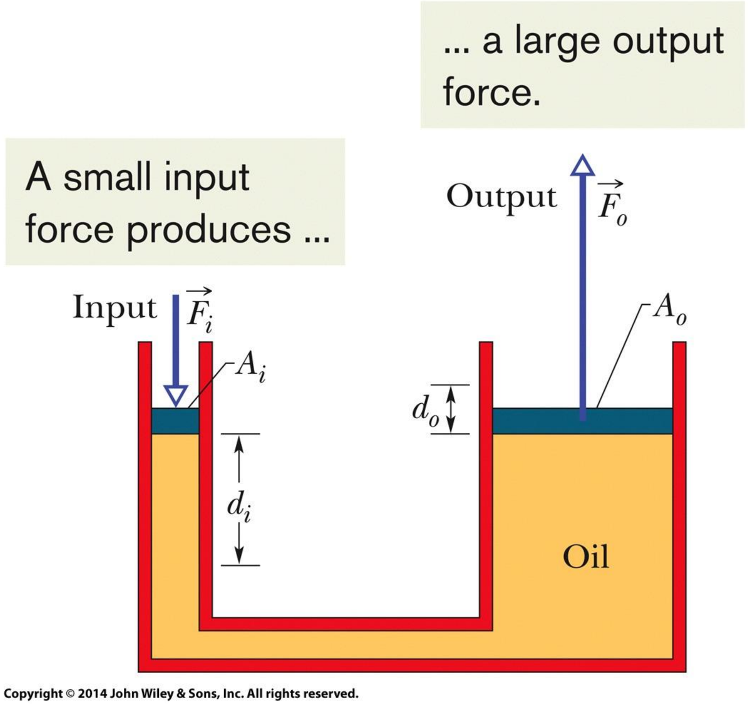


Figure 14-8

14-4 Pascal's Principle

- So the work done on the input piston equals the work output

$$W = F_o d_o = \left(F_i \frac{A_o}{A_i}\right)\left(d_i \frac{A_i}{A_o}\right) = F_i d_i, \quad \textbf{Eq. (14-15)}$$

- The advantage of the hydraulic lever is that:

> With a hydraulic lever, a given force applied over a given distance can be transformed to a greater force applied over a smaller distance.

14-5 Archimedes' Principle

Learning Objectives

14.10 Describe Archimedes' principle.

14.11 Apply the relationship between the buoyant force on a body and the mass of the fluid displaced by the body.

14.12 For a floating body, relate the buoyant force to the gravitational force.

14.13 For a floating body, relate the gravitational force to the mass of the fluid displaced by the body.

14.14 Distinguish between apparent weight and actual weight.

14.15 Calculate the apparent weight of a body that is fully or partially submerged.

14-5 Archimedes' Principle

- The **buoyant force** is the net upward force **on** a submerged object **by** the fluid in which it is submerged
- This force opposes the weight of the object

It comes from the increase in pressure with depth

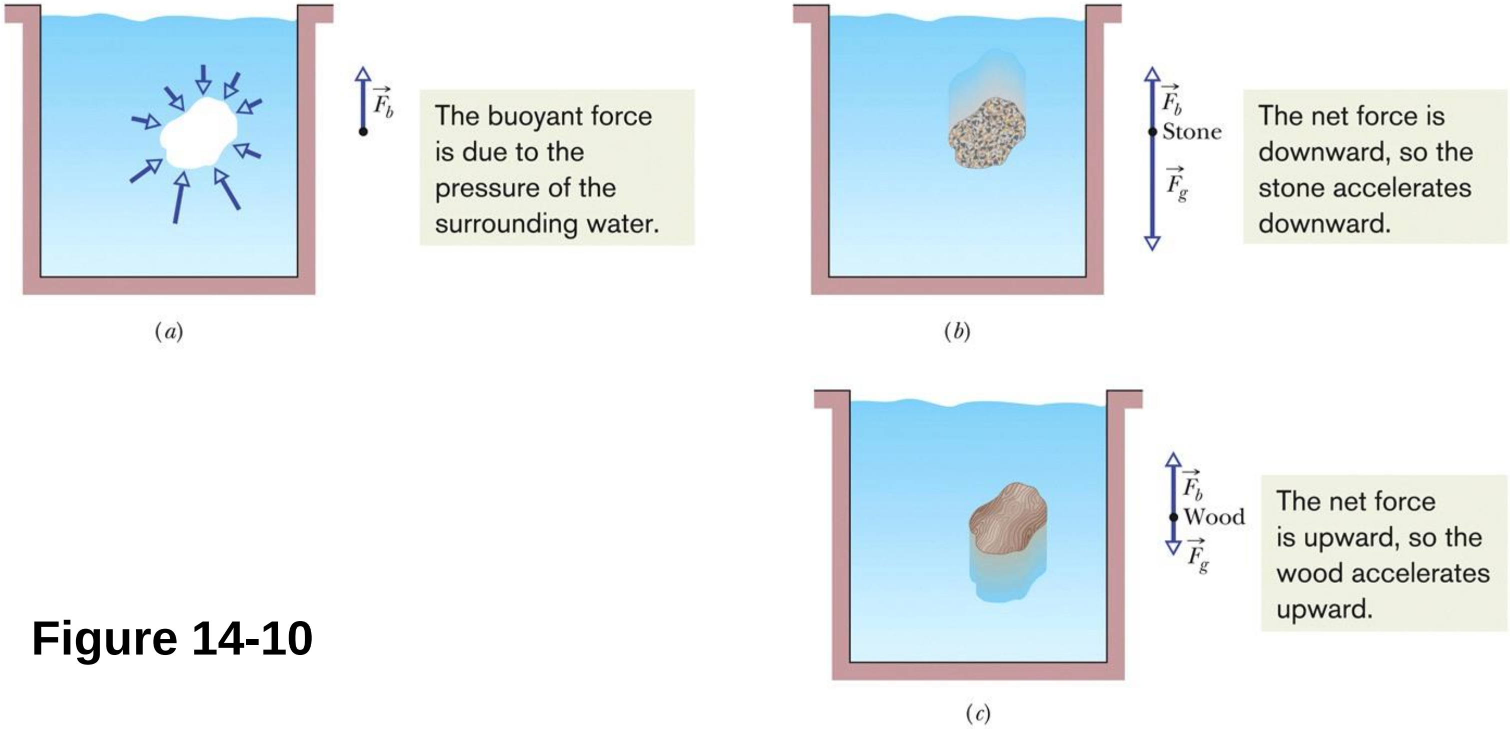


Figure 14-10

14-5 Archimedes' Principle

- The stone and piece of wood *displace* the water that would otherwise occupy that space
- **Archimedes' Principle** states that:

> When a body is fully or partially submerged in a fluid, a buoyant force $\vec{F}_b$ from the surrounding fluid acts on the body. The force is directed upward and has a magnitude equal to the weight $m_f g$ of the fluid that has been displaced by the body.

- The buoyant force has magnitude

$$F_b = m_f g$$

Eq. (14-16)

- Where m_f is the mass of displaced fluid

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14-5 Archimedes' Principle

- A block of wood in static equilibrium is *floating*:

When a body floats in a fluid, the magnitude F_b of the buoyant force on the body is equal to the magnitude F_g of the gravitational force on the body.

- This is expressed: $$F_b = F_g \quad \text{(floating).}$$ **Eq. (14-17)**

- Because of Eq. 14-16 we know:

When a body floats in a fluid, the magnitude F_g of the gravitational force on the body is equal to the weight $m_f g$ of the fluid that has been displaced by the body.

- Which means: $$F_g = m_f g \quad \text{(floating).}$$ **Eq. (14-18)**

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14-5 Archimedes' Principle

- The apparent weight of a body in a fluid is related to the actual weight of the body by:

(*apparent weight*) = (*actual weight*) – (*buoyant force*)

- We write this as:

$$\text{weight}_{\text{app}} = \text{weight} - F_b \quad \text{(apparent weight).}$$

Eq. (14-19)

Checkpoint 2

A penguin floats first in a fluid of density ρ_0, then in a fluid of density $0.95\rho_0$, and then in a fluid of density $1.1\rho_0$. (a) Rank the densities according to the magnitude of the buoyant force on the penguin, greatest first. (b) Rank the densities according to the amount of fluid displaced by the penguin, greatest first.

Answer: (a) all the same (b) $0.95\rho_0$, $1\rho_0$, $1.1\rho_0$

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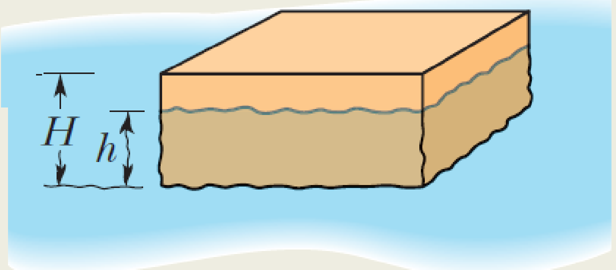
Sample Problem 14.04 Floating, buoyancy, and density

In Fig. 14-11, a block of density $\rho = 800\ \text{kg/m}^3$ floats face down in a fluid of density $\rho_f = 1200\ \text{kg/m}^3$. The block has height $H = 6.0$ cm.

(a) By what depth h is the block submerged?



$$F_b = m_f g = \rho_f V_f g = \rho_f LWhg.$$

$$F_g = mg = \rho V g = \rho_f LWHg.$$

$$F_b - F_g = m(0)$$

$$\rho_f LWhg - \rho LWHg = 0$$

$$h = \frac{\rho}{\rho_f} H = \frac{800\ \text{kg/m}^3}{1200\ \text{kg/m}^3}(6.0\ \text{cm})$$

(b) If the block is held fully submerged and then released what is the magnitude of its acceleration?

$$F_b - F_g = ma$$

$$\rho_f LWHg - \rho LWHg = \rho LWHa$$

$$a = \left(\frac{\rho_f}{\rho} - 1\right)g$$

14-6 The Equation of Continuity

Learning Objectives

14.16 Describe steady flow, incompressible flow, nonviscous flow, and irrotational flow.

14.17 Explain the term streamline.

14.18 Apply the equation of continuity to relate the cross-sectional area and flow speed at one point in a tube to those quantities at a different point.

14.19 Identify and calculate volume flow rate.

14.20 Identify and calculate mass flow rate.

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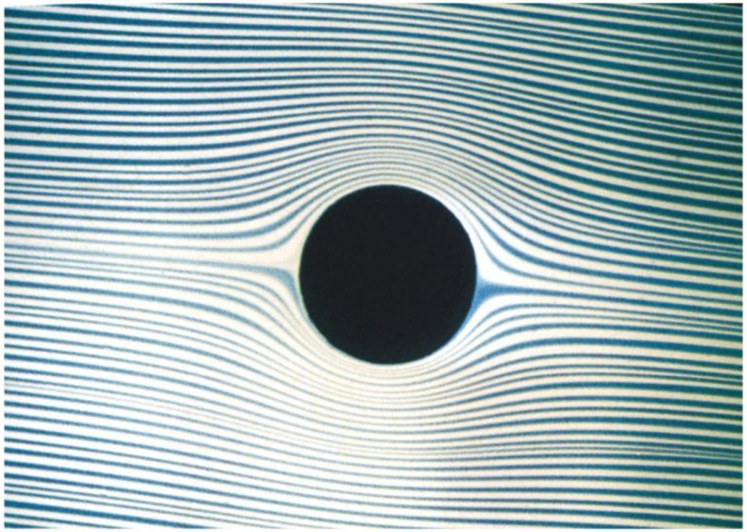
14-6 The Equation of Continuity

- Motion of *real fluids* is complicated and poorly understood (e.g., turbulence)
- We discuss motion of an **ideal fluid**

1. **Steady flow**: Laminar flow, the velocity of the moving fluid at any fixed point does not change with time
2. **Incompressible flow**: The ideal fluid density has a constant, uniform value
3. **Nonviscous flow**: Viscosity is, roughly, resistance to flow, fluid analog of friction. No resistive force here
4. **Irrotational flow**: May flow in a circle, but a dust grain suspended in the fluid will not rotate about com

14-6 The Equation of Continuity

- Visualize fluid flow by adding a *tracer*
- Each bit of tracer (see figure 14-13) follows a *streamline*
- A streamline is the path a tiny element of fluid follows
- Velocity is tangent to streamlines, so they can never intersect (then 1 point would experience 2 velocities)



Courtesy D. H. Peregrine, University of Bristol

Figure 14-13

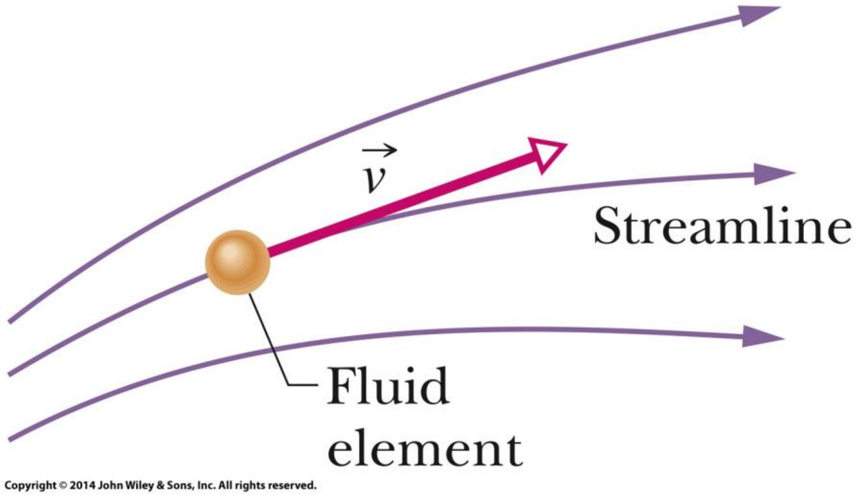


Figure 14-14

14-6 The Equation of Continuity

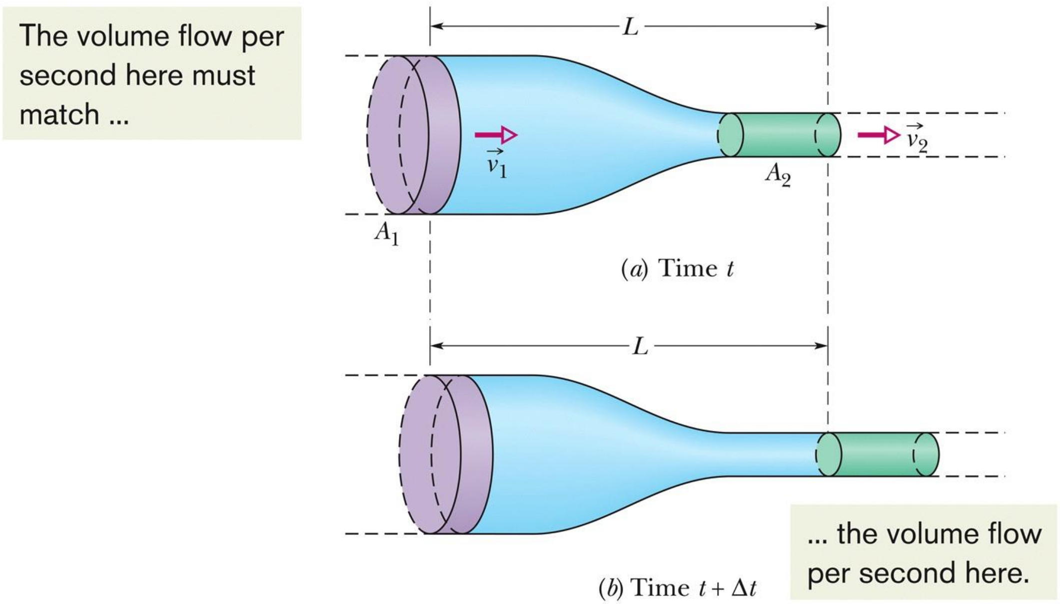
- Fluid speed depends on cross-sectional area
- Because of incompressibility, the volume flow rate through any cross-section must be constant
- We write the **equation of continuity**:

$$A_1 v_1 = A_2 v_2$$

Eq. (14-23)

- Holds for any *tube of flow* whose boundaries consist of streamlines
- Fluid elements cannot cross streamlines

Figure 14-15



14-6 The Equation of Continuity

- We can rewrite the equation as:

$$R_V = Av = \text{a constant}$$ Eq. (14-24)

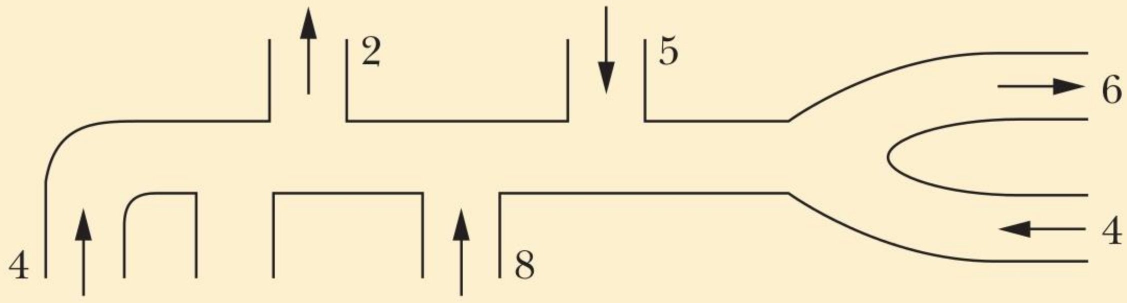
- Where RV is the **volume flow rate** of the fluid (volume passing a point per unit time)
- If the fluid density is uniform, we can multiply by the density to get the **mass flow rate**:

$$R_m = \rho R_V = \rho A v = \text{a constant}$$ Eq. (14-25)

14-6 The Equation of Continuity

Checkpoint 3

The figure shows a pipe and gives the volume flow rate (in cm^3/s) and the direction of flow for all but one section. What are the volume flow rate and the direction of flow for that section?



Answer: 13, out

14-7 Bernoulli's Equation

Learning Objectives

14.21 Calculate the kinetic energy density in terms of a fluid's density and flow speed.

14.22 Identify the fluid pressure as being a type of energy density.

14.23 Calculate the gravitational potential energy density.

14.24 Apply Bernoulli's equation to relate the total energy density at one point on a streamline to the value at another point.

14.25 Identify that Bernoulli's equation is a statement of the conservation of energy.

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14-7 Bernoulli's Equation

- Figure 14-19 represents a tube through which an ideal fluid flows
- Applying the conservation of energy to the equal volumes of input and output fluid:

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2.$$

Eq. (14-28)

- The $\frac{1}{2}\rho v^2$ term is called the fluid's **kinetic energy density**

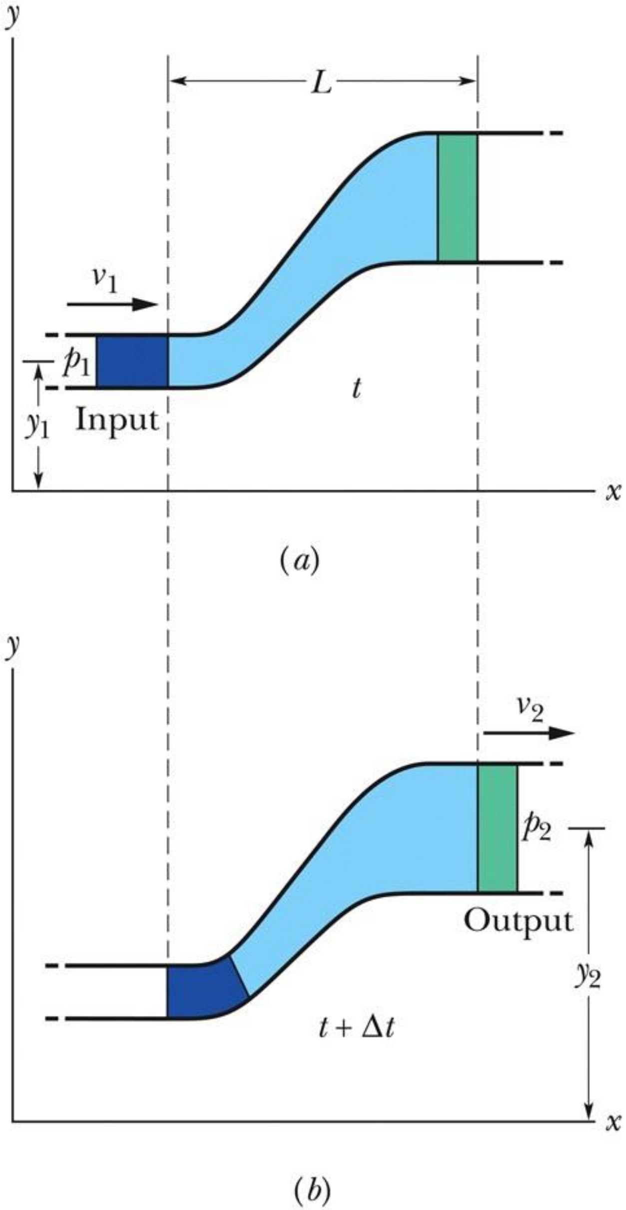


Figure 14-19

14-7 Bernoulli's Equation

- Equivalent to Eq. 14-28, we can write:

$$p + \tfrac{1}{2}\rho v^2 + \rho g y = \text{a constant}$$ **Eq. (14-29)**

- These are both forms of **Bernoulli's Equation**
- Applying this for a fluid at rest we find Eq. 14-7
- Applying this for flow through a horizontal pipe:

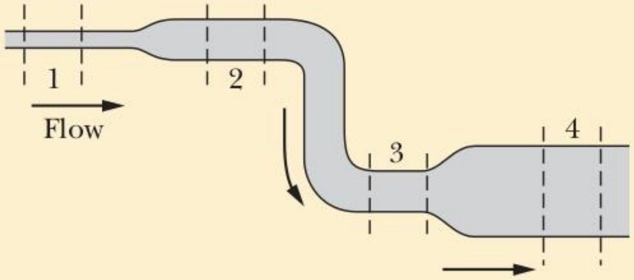
$$p_1 + \tfrac{1}{2}\rho v_1^2 = p_2 + \tfrac{1}{2}\rho v_2^2,$$ **Eq. (14-30)**

If the speed of a fluid element increases as the element travels along a horizontal streamline, the pressure of the fluid must decrease, and conversely.

14-7 Bernoulli's Equation

Checkpoint 4

Water flows smoothly through the pipe shown in the figure, descending in the process. Rank the four numbered sections of pipe according to (a) the volume flow rate R_V through them, (b) the flow speed v through them, and (c) the water pressure p within them, greatest first.



Answer: (a) all the same volume flow rate
(b) 1, 2 & 3, 4
(c) 4, 3, 2, 1

14 Summary

Density

$$\rho = \frac{m}{V}$$

Eq. (14-2)

Fluid Pressure

- A substance that can flow
- Can exert a force perpendicular to its surface

$$p = \frac{F}{A}$$

Eq. (14-4)

Pressure Variation with Height and Depth

$$p = p_0 + \rho g h$$

Eq. (14-8)

Pascal's Principle

- A change in pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and to the walls of the containing vessel

14 Summary

Archimedes' Principle

$$F_b = m_f g \quad \text{(buoyant force)},$$

Eq. (14-16)

$$\text{weight}_{\text{app}} = \text{weight} - F_b$$

Eq. (14-19)

Bernoulli's Equation

$$p + \tfrac{1}{2}\rho v^2 + \rho g y = \text{a constant}$$

Eq. (14-29)

Flow of Ideal Fluids

$$R_V = Av = \text{a constant}$$

Eq. (14-24)

$$R_m = \rho R_V = \rho A v = \text{a constant}$$

Eq. (14-25)