Chapter 13

Gravitation

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13-1 Newton's Law of Gravitation

Learning Objectives

13.01 Apply Newton's law of gravitation to relate the gravitational force between two particles to their masses and their separation.

13.02 Identify that a uniform spherical shell of matter attracts a particle that is outside the shell as if all the shell's mass were concentrated as a particle at its center.

13.03 Draw a free-body diagram to indicate the gravitational force on a particle due to another particle or a uniform, spherical distribution of matter.

13-1 Newton's Law of Gravitation

- The gravitational force
  - Holds us to the Earth
  - Holds Earth in orbit around the Sun
  - Holds the Sun together with the stars in our galaxy
  - Reaches out across intergalactic space to hold together the Local Group of galaxies
  - Holds together the Local Supercluster of galaxies
  - Pulls the supercluster toward the Great Attractor
  - Attempts to slow the expansion of the universe
  - Is responsible for black holes
- Gravity is far-reaching and very important!

13-1 Newton's Law of Gravitation

- Gravitational attraction depends on the amount of “stuff” an object is made of
- Earth has lots of “stuff” and produces a large attraction
- The force is always attractive, never repulsive
- **Gravitation** is the tendency for bodies to attract each other
- Newton realized this attraction was responsible for maintaining the orbits of celestial bodies
- Newton's **law of gravitation** defines the strength of this attractive force between particles
- For apple & Earth: 0.8 N; for 2 people: $< 1\ \mu N$

13-1 Newton's Law of Gravitation

- The magnitude of the force is given by:

$$F = G\frac{m_1 m_2}{r^2} \quad \text{(Newton's law of gravitation).}$$ **Eq. (13-1)**

- Where G is the **gravitational constant**:

$$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$$ **Eq. (13-2)**

- The force always points from one particle to the other, so this equation can be written in vector form:

$$\vec{F} = G\frac{m_1 m_2}{r^2}\hat{\mathrm{r}}.$$ **Eq. (13-3)**

13-1 Newton's Law of Gravitation

A uniform spherical shell of matter attracts a particle that is outside the shell as if all the shell's mass were concentrated at its center.

- The *shell theorem* describes gravitational attraction for objects
- Earth is a nesting of shells, so we feel Earth's mass as if it were all located at its center
- Gravitational force forms third-law force pairs
- E.g., Earth-apple and apple-Earth forces are both 0.8 N

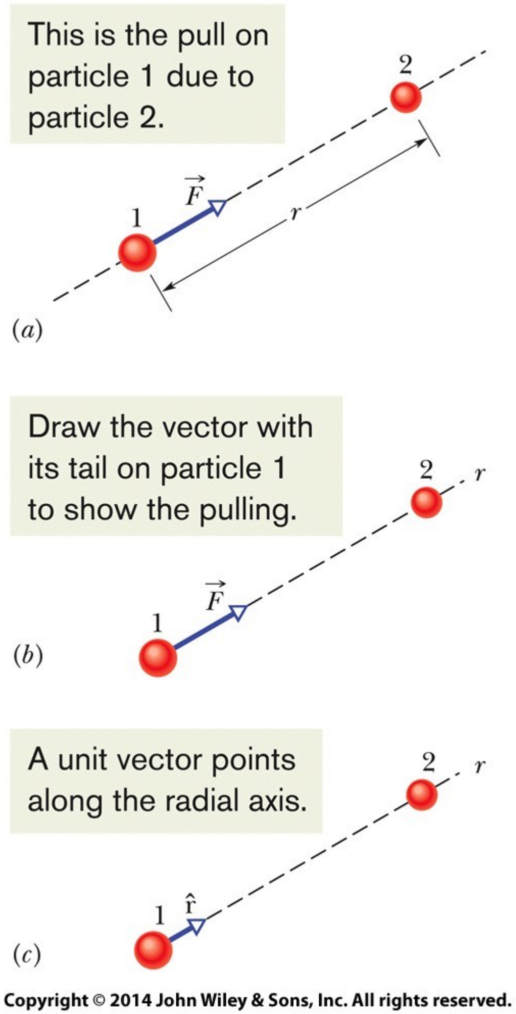

Figure 13-2

13-1 Newton's Law of Gravitation

- The difference in mass causes the difference in the apple:Earth accelerations:

  $\sim 10\ m/s^2$ *vs.* $\sim 10^{-25}\ m/s^2$

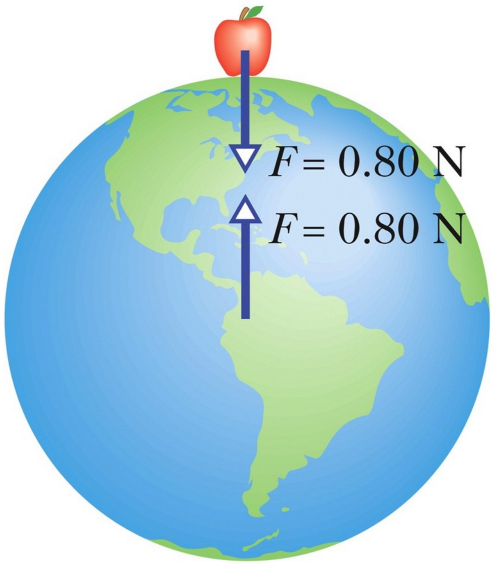

Figure 13-3

Checkpoint 1

A particle is to be placed, in turn, outside four objects, each of mass *m*: (1) a large uniform solid sphere, (2) a large uniform spherical shell, (3) a small uniform solid sphere, and (4) a small uniform shell. In each situation, the distance between the particle and the center of the object is *d*. Rank the objects according to the magnitude of the gravitational force they exert on the particle, greatest first.

Answer: All exert equal forces on the particle

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13-2 Gravitation and the Principle of Superposition

Learning Objectives

13.04 If more than one gravitational force acts on a particle, draw a free-body diagram showing those forces, with the tails of the force vectors anchored on the particle.

13.05 If more than one gravitational force acts on a particle, find the net force by adding the individual forces as vectors.

13-2 Gravitation and the Principle of Superposition

- Find the net gravitational force by the **principle of superposition**: the net is the sum of individual effects
- Add the individual forces as vectors:

$$\vec{F}_{1,\text{net}} = \sum_{i=2}^{n} \vec{F}_{1i}.$$

Eq. (13-5)

- For a real (extended) object, this becomes an integral:

$$\vec{F}_1 = \int d\vec{F},$$

Eq. (13-6)

- If the object is a uniform sphere or shell we can treat its mass as being at its center instead

13-2 Gravitation and the Principle of Superposition

Example Summing two forces:

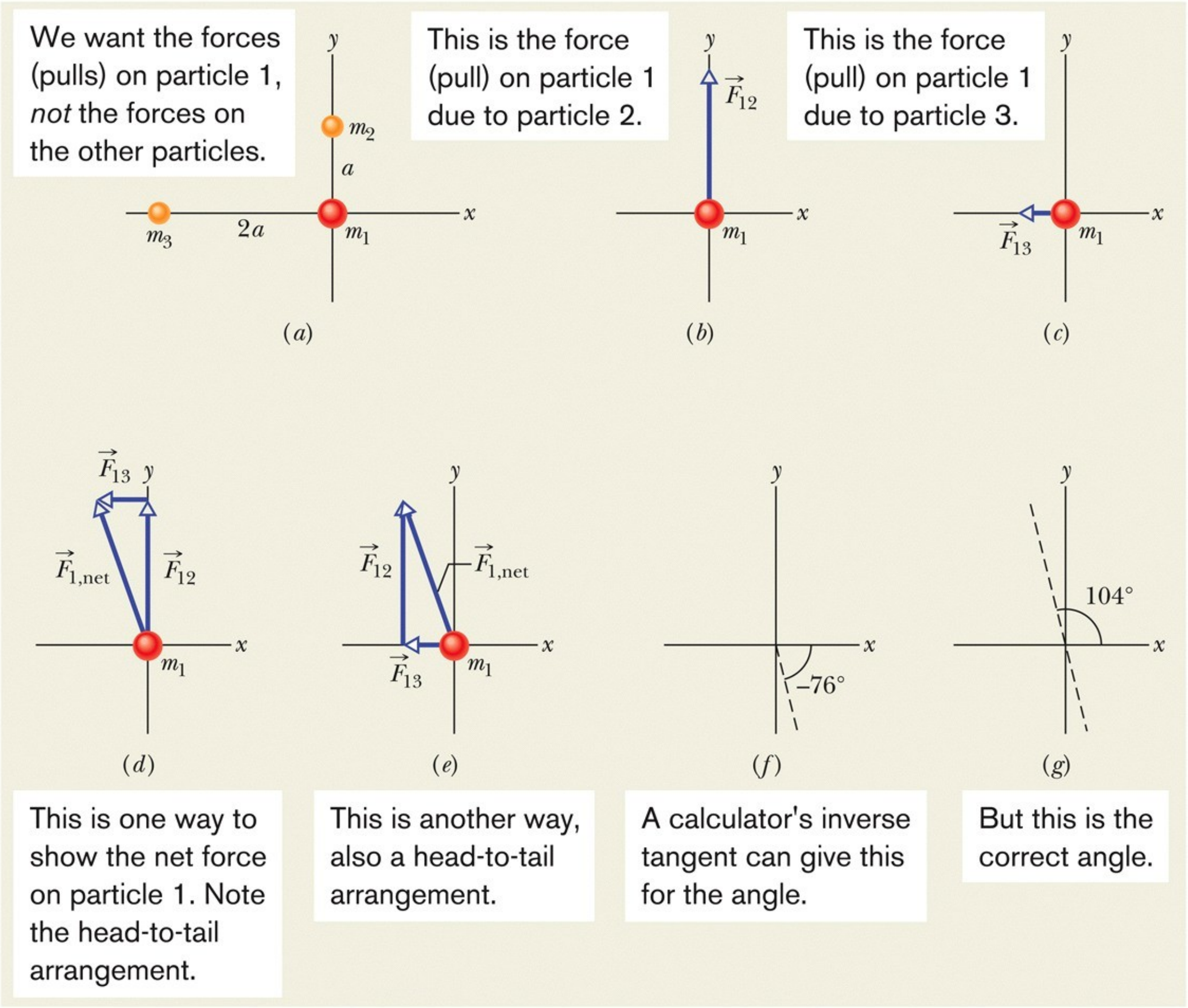
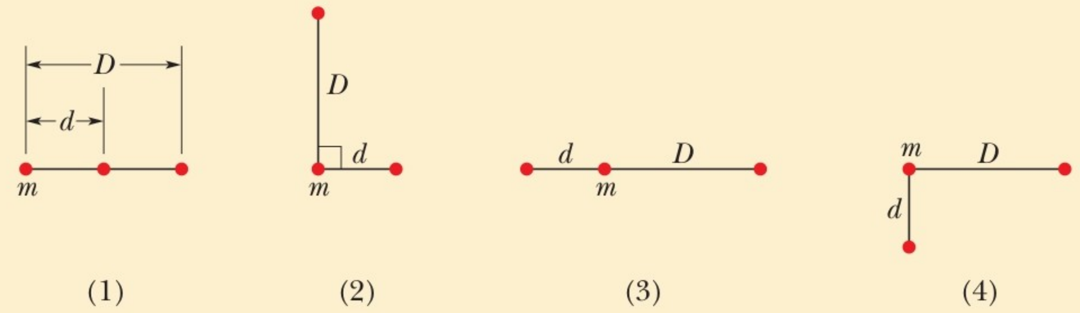


Figure 13-4

13-2 Gravitation and the Principle of Superposition

Checkpoint 2

The figure shows four arrangements of three particles of equal masses. (a) Rank the arrangements according to the magnitude of the net gravitational force on the particle labeled *m*, greatest first. (b) In arrangement 2, is the direction of the net force closer to the line of length *d* or to the line of length *D*?



Answer: (a) 1, 2 & 4, 3   (b) line of length *d*

13-3 Gravitation Near Earth's Surface

Learning Objectives

13.06 Distinguish between the free-fall acceleration and the gravitational acceleration.

13.07 Calculate the gravitational acceleration near but outside a uniform, spherical astronomical body.

13.08 Distinguish between measured weight and the magnitude of the gravitational force.

13-3 Gravitation Near Earth's Surface

- Combine $F = GMm/r^2$ and $F = ma_g$:

$$a_g = \frac{GM}{r^2}.$$

Eq. (13-11)

- This gives the magnitude of the gravitational acceleration at a given distance from the center of the Earth
- Table 13-1 shows the value for a_g for various altitudes above the Earth's surface

Table 13-1 Variation of a_g with Altitude

Altitude (km)	a_g (m/s^2)	Altitude Example
0	9.83	Mean Earth surface
8.8	9.80	Mt. Everest
36.6	9.71	Highest crewed balloon
400	8.70	Space shuttle orbit
35 700	0.225	Communications satellite

Table 13-1

13-3 Gravitation Near Earth's Surface

- The calculated a_g will differ slightly from the measured g at any location
- Therefore the calculated gravitational force on an object will not match its weight for the same 3 reasons:

1. Earth's mass is not uniformly distributed, Fig. 13-5
2. Earth is not a sphere
3. Earth rotates

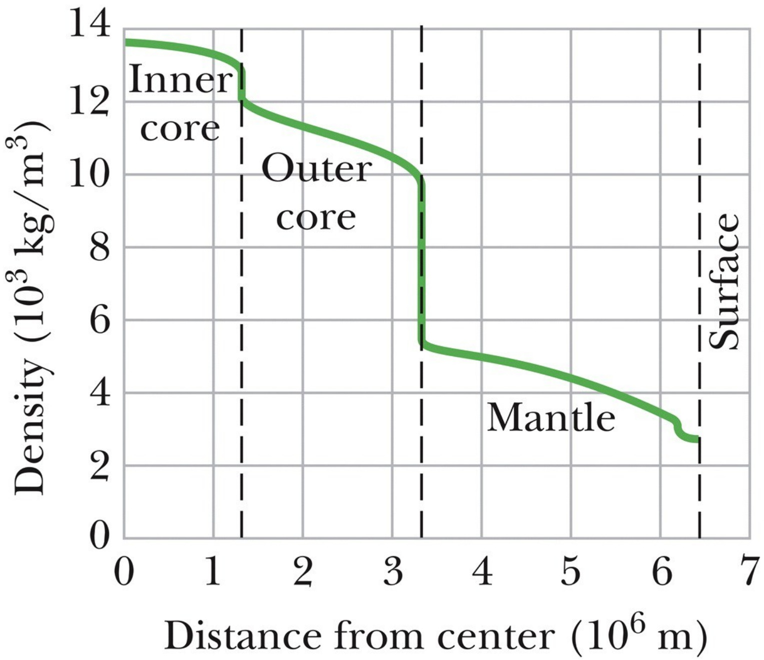


Figure 13-5

13-3 Gravitation Near Earth's Surface

Example Difference in gravitational force and weight due to rotation at the equator:

$$F_N - ma_g = m(-\omega^2 R).$$ **Eq. (13-12)**

$$g = a_g - \omega^2 R,$$ **Eq. (13-14)**

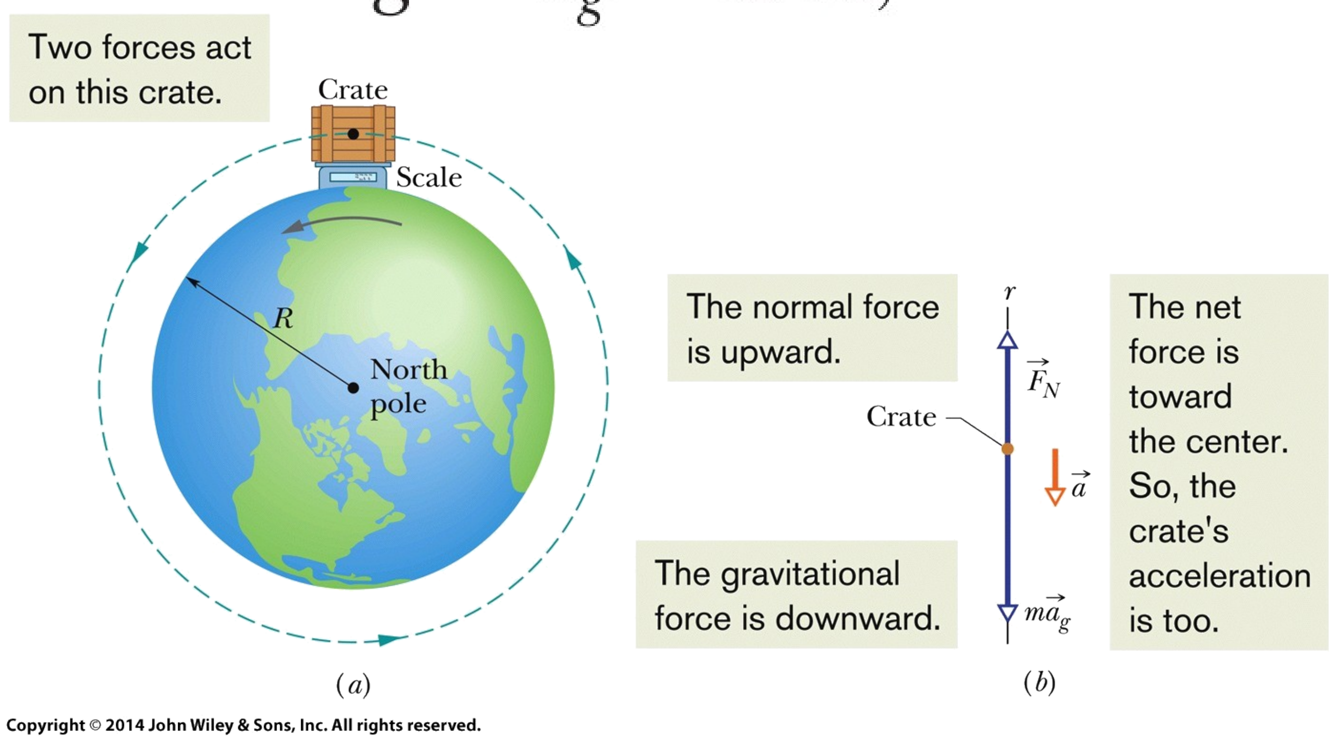


Figure 13-6

13-4 Gravitation Inside Earth

Learning Objectives

13.09 Identify that a uniform shell of material exerts no net gravitational force on a particle located inside it.

13.10 Calculate the gravitational force that is exerted on a particle at a given radius inside a nonrotating uniform sphere of matter.

13-4 Gravitation Near Earth's Surface

- The shell theorem also means that:

> A uniform shell of matter exerts no net gravitational force on a particle located inside it.

- Forces between elements do not disappear, but their vector sum is 0
- Let's find the gravitational force inside a uniform-density Earth
- a solid sphere, not a shell:

$$F = \frac{GmM_{\text{ins}}}{r^2}.$$

Eq. (13-17)

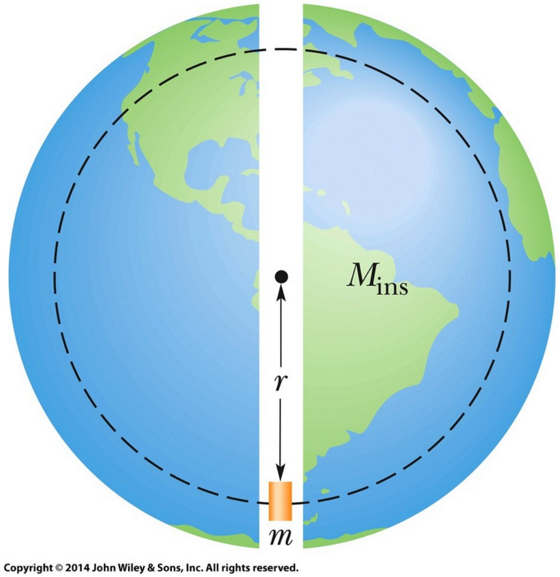


Figure 13-7

13-4 Gravitation Near Earth's Surface

- The constant density is:

$$\rho = \frac{M_{\text{ins}}}{\frac{4}{3}\pi r^3} = \frac{M}{\frac{4}{3}\pi R^3}.$$

- Substitute in to Eq. 13-17:

$$F = \frac{GmM}{R^3} r. \quad \text{Eq. (13-19)}$$

- If we write this as a vector equation, substituting K for the constants:

$$\vec{F} = -K\vec{r}, \quad \text{Eq. (13-20)}$$

- Object dropped through Earth oscillates (Hooke's law)

13-5 Gravitational Potential Energy

Learning Objectives

13.11 Calculate the gravitational potential energy of a system of particles (or uniform spheres that can be treated as particles).

13.12 Identify that if a particle moves from an initial to a final point while experiencing a gravitational force, the work done by that force (and thus the change in gravitational potential energy) is independent of which path is taken.

13.13 Using the gravitational force on a particle near an astronomical body (or some second body that is fixed in place), calculate the work done by the force when the body moves.

13.14 Apply the conservation of mechanical energy (including gravitational potential energy) to a particle moving relative to an astronomical body (or some second body that is fixed in place).

13-5 Gravitational Potential Energy

13.15 Explain the energy requirements for a particle to escape from an astronomical body (assumed to be a uniform sphere).

13.16 Calculate the escape speed of a particle in leaving an astronomical body.

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13-5 Gravitational Potential Energy

- Note that gravitational potential energy is a property of a pair of particles
- We cannot divide it up to say how much of it “belongs” to each particle in the pair
- We often speak as of the “gravitational potential energy of a baseball” in the ball-Earth system
- We get away with this because the energy change appears almost entirely as kinetic energy of the ball
- This is only true for systems where one object is much less massive than the other

13-5 Gravitational Potential Energy

- Gravitational potential energy for a two-particle system is written:

$$U = -\frac{GMm}{r} \quad \text{(gravitational potential energy).}$$

Eq. (13-21)

- Note this value is negative and approaches 0 for $r \to \infty$
- The gravitational potential energy of a system is the sum of potential energies for all

Eq. (13-22)

$$U = -\left(\frac{Gm_1m_2}{r_{12}} + \frac{Gm_1m_3}{r_{13}} + \frac{Gm_2m_3}{r_{23}}\right).$$

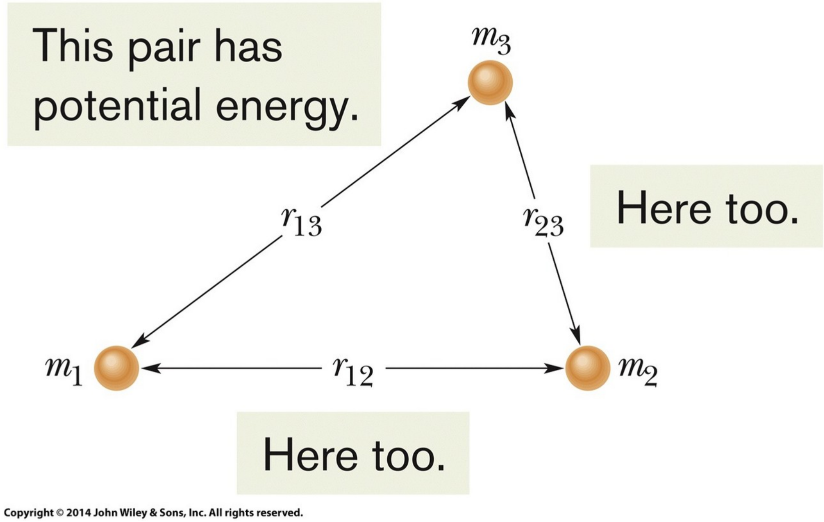

Figure 13-8

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13-5 Gravitational Potential Energy

- The gravitational force is conservative
- The work done by this force does not depend on the path followed by the particles, only the difference in the initial and final positions of the particles
- Since the work done is independent of path, so is the gravitational potential energy change

$$\Delta U = U_f - U_i = -W.$$ **Eq. (13-26)**

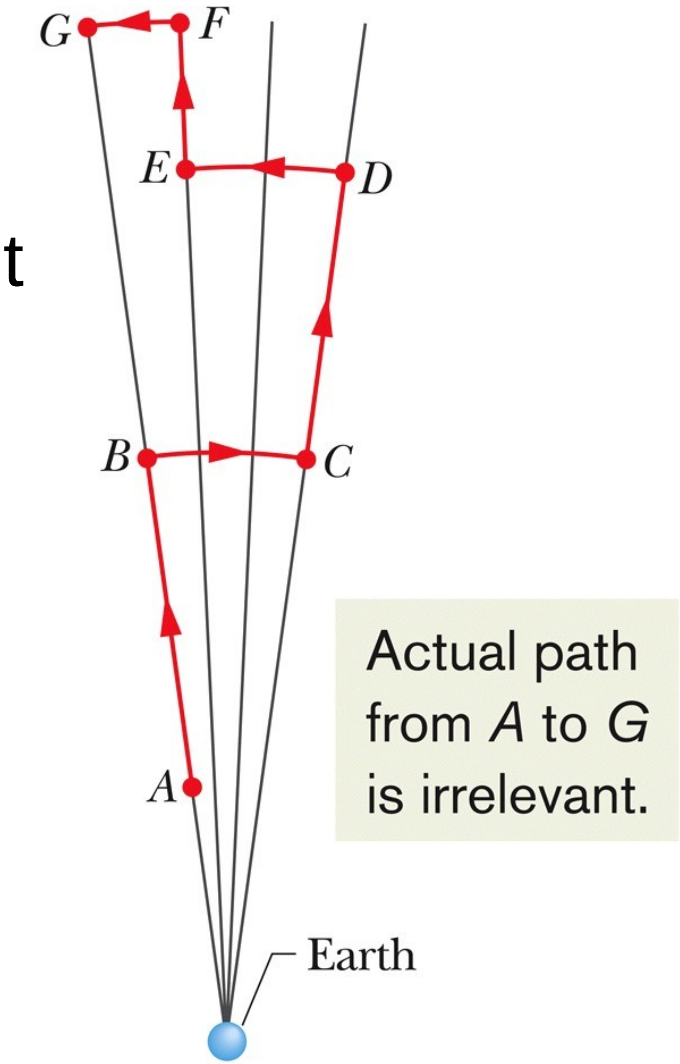


Figure 13-10

13-5 GPE and Scape Speed

- Newton's law of gravitation can be derived from the potential energy formula by taking the derivative
- For a projectile to escape the gravitational pull of a body, it must come to rest only at infinity, if at all
- At rest at infinity: $K = 0$ and $U = 0$ (because $r \to \infty$)
- So $K + U$ must be at least 0 at the surface of the body:

$$K + U = \tfrac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right) = 0.$$

$$v = \sqrt{\frac{2GM}{R}}.$$

Eq. (13-28)

- Rockets launch eastward to take advantage of Earth's rotational speed, to reach v more easily

13-5 Gravitational Potential Energy

Table 13-2 Some Escape Speeds

Body	Mass (kg)	Radius (m)	Escape Speed (km/s)
Ceres[a]	1.17×10^{21}	3.8×10^{5}	0.64
Earth's moon[a]	7.36×10^{22}	1.74×10^{6}	2.38
Earth	5.98×10^{24}	6.37×10^{6}	11.2
Jupiter	1.90×10^{27}	7.15×10^{7}	59.5
Sun	1.99×10^{30}	6.96×10^{8}	618
Sirius B[b]	2×10^{30}	1×10^{7}	5200
Neutron star[c]	2×10^{30}	1×10^{4}	2×10^{5}

Table 13-2

Checkpoint 3

You move a ball of mass m away from a sphere of mass M. (a) Does the gravitational potential energy of the system of ball and sphere increase or decrease? (b) Is positive work or negative work done by the gravitational force between the ball and the sphere?

Answer: (a) increases (b) negative work

13-6 Planets and Satellites: Kepler's Laws

Learning Objectives

13.17 Identify Kepler's three laws.

13.18 Identify which of Kepler's laws is equivalent to the law of conservation of angular momentum.

13.19 On a sketch of an elliptical orbit, identify the semimajor axis, the eccentricity, the perihelion, the aphelion, and the focal points.

13.20 For an elliptical orbit, apply the relationships between the semimajor axis, the eccentricity, the perihelion, and the aphelion.

13.21 For an orbiting natural or artificial satellite, apply Kepler's relationship between the orbital period and radius and the mass of the astronomical body being orbited.

13-6 Planets and Satellites: Kepler's Laws

- The motion of planets in the solar system was a puzzle for astronomers, especially curious motions such as in Figure 13-11
- Johannes Kepler (1571-1630) derived laws of motion using Tycho Brahe's (1546-1601) measurements

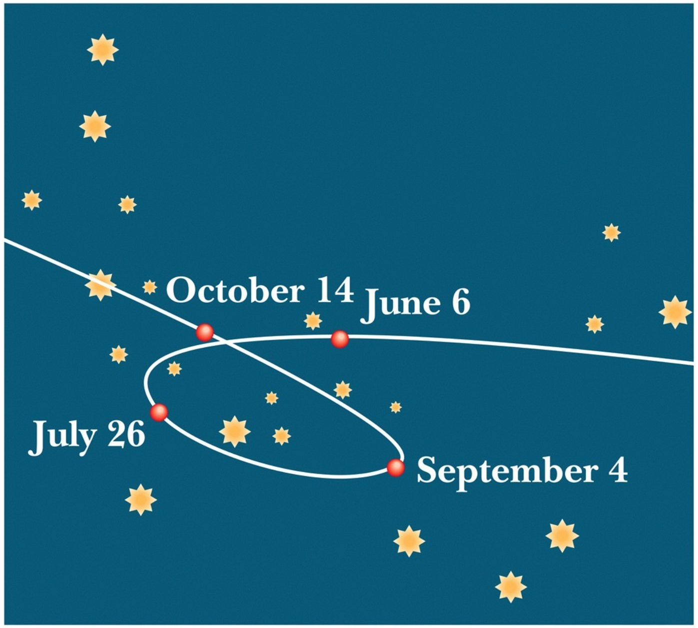


Figure 13-11

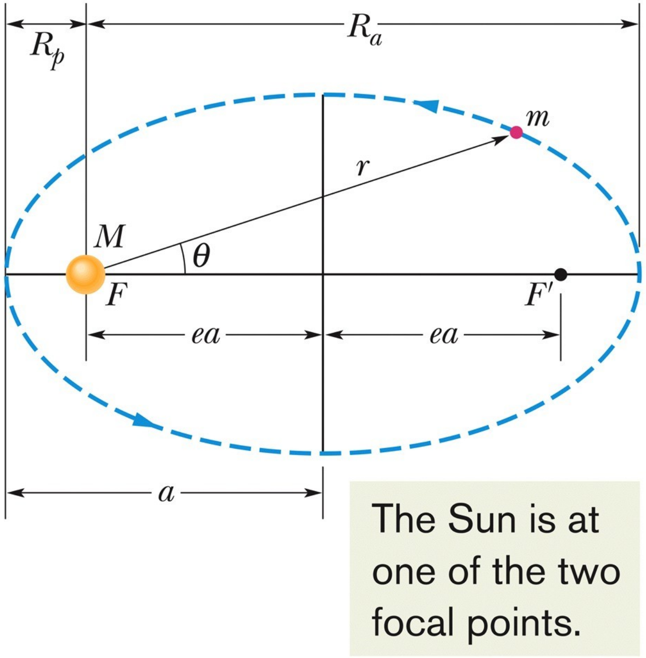


Figure 13-12

13-6 Planets and Satellites: Kepler's Laws

1. THE LAW OF ORBITS: All planets move in elliptical orbits, with the Sun at one focus.

- The orbit is defined by its **semimajor axis** *a* and its **eccentricity** *e*
- An eccentricity of zero corresponds to a circle
- Eccentricity of Earth's orbit is 0.0167

2. THE LAW OF AREAS: A line that connects a planet to the Sun sweeps out equal areas in the plane of the planet's orbit in equal time intervals; that is, the rate dA/dt at which it sweeps out area A is constant.

- Equivalent to the law of conservation of angular momentum

13-6 Planets and Satellites: Kepler's Laws

- Equivalent to the law of conservation of angular momentum

$$\Delta A \approx \tfrac{1}{2} r^2 \Delta\theta.$$

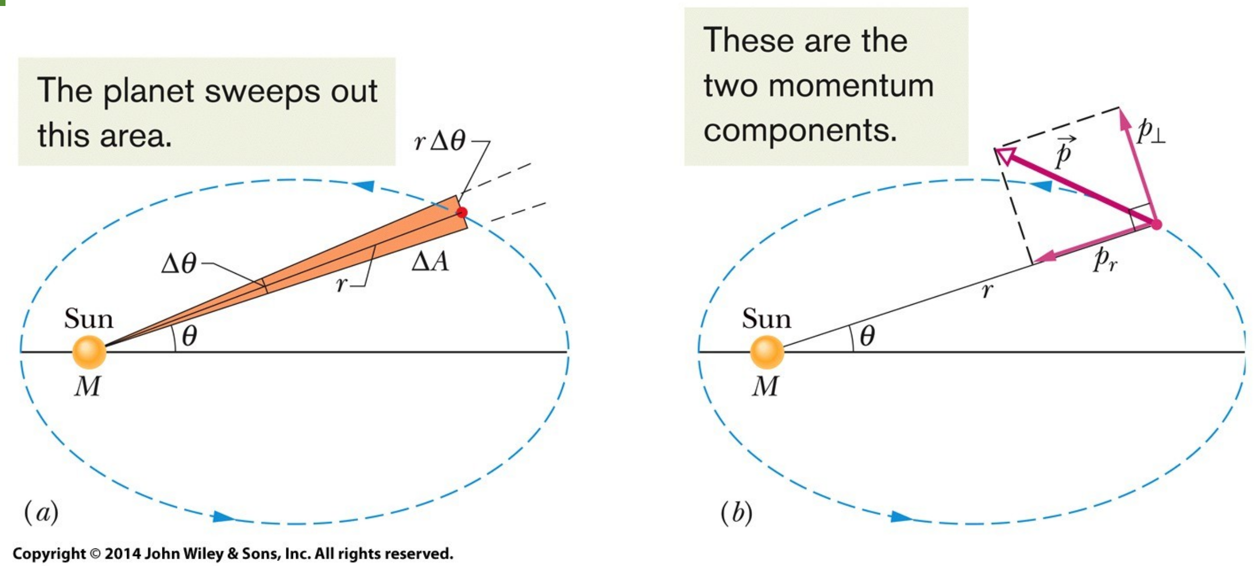


Figure 13-13

$$\frac{dA}{dt} = \tfrac{1}{2} r^2 \frac{d\theta}{dt} = \tfrac{1}{2} r^2 \omega,$$

$$L = rp_\perp = (r)(mv_\perp) = (r)(m\omega r) = mr^2\omega,$$

$$\frac{dA}{dt} = \frac{L}{2m}.$$

If dA/dt is constant, as Kepler said it is, then Eq. 13-32 means that L must also be constant—angular momentum is conserved. Kepler's second law is indeed equivalent to the law of conservation of angular momentum.

3. THE LAW OF PERIODS: The square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.

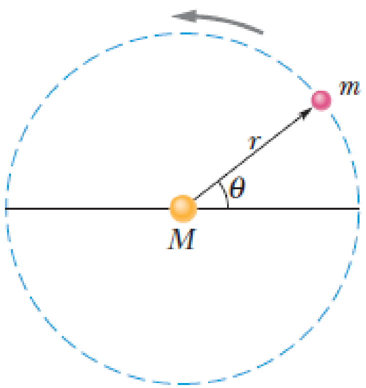
- The law of periods can be written mathematically as:

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \quad \text{(law of periods).}$$

Eq. (13-34)

To see this, consider the circular orbit of Fig., with radius *r* (the radius of a circle is equivalent to the semimajor axis of an ellipse). Applying Newton's second law (*F* *ma*) to the orbiting planet in Fig.

$$\frac{GMm}{r^2} = (m)(\omega^2 r).$$



replace ω with 2/*T*, where *T* is the period of the motion

- Holds for elliptical orbits if we replace *r* with *a*

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13-6 Planets and Satellites: Kepler's Laws

Table 13-3 Kepler's Law of Periods for the Solar System

Planet	Semimajor Axis a (10^{10} m)	Period T (y)	T^2/a^3 (10^{-34} y^2/m^3)
Mercury	5.79	0.241	2.99
Venus	10.8	0.615	3.00
Earth	15.0	1.00	2.96
Mars	22.8	1.88	2.98
Jupiter	77.8	11.9	3.01
Saturn	143	29.5	2.98
Uranus	287	84.0	2.98
Neptune	450	165	2.99
Pluto	590	248	2.99

Table 13-3

Checkpoint 4

Satellite 1 is in a certain circular orbit around a planet, while satellite 2 is in a larger circular orbit. Which satellite has (a) the longer period and (b) the greater speed?

Answer: (a) satellite 2 (b) satellite 1

13 Summary

The Law of Gravitation

$$F = G\frac{m_1 m_2}{r^2}$$

Eq. (13-1)

$$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$$

Eq. (13-2)

Gravitational Behavior of Uniform Spherical Shells

- The net force on an *external* object: calculate as if all the mass were concentrated at the center of the shell

Superposition

$$\vec{F}_{1,\mathrm{net}} = \sum_{i=2}^{n} \vec{F}_{1i}.$$

Eq. (13-5)

Gravitational Acceleration

$$a_g = \frac{GM}{r^2}.$$

Eq. (13-11)

13 Summary

Free-Fall Acceleration and Weight

- Earth's mass is not uniformly distributed, the planet is not spherical, and it rotates: the calculated and measured values of acceleration differ

Gravitation within a Spherical Shell

- A uniform shell exerts no net force on a particle inside
- Inside a solid sphere:

$$F = \frac{GmM}{R^3} r. \quad \text{Eq. (13-19)}$$

Gravitational Potential Energy

$$U = -\frac{GMm}{r} \quad \text{Eq. (13-21)}$$

Potential Energy of a System

$$U = -\left(\frac{Gm_1m_2}{r_{12}} + \frac{Gm_1m_3}{r_{13}} + \frac{Gm_2m_3}{r_{23}}\right).$$

Eq. (13-22)

13 Summary

Escape Speed

$$v = \sqrt{\frac{2GM}{R}}.$$

Eq. (13-28)

Kepler's Laws

- The law of orbits: ellipses
- The law of areas: equal areas in equal times
- The law of periods:

$$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$

Eq. (13-34)

Kepler's Laws

- Gravitation and acceleration are equivalent
- **General theory of relativity** explains gravity in terms of curved space