LECTURE 7

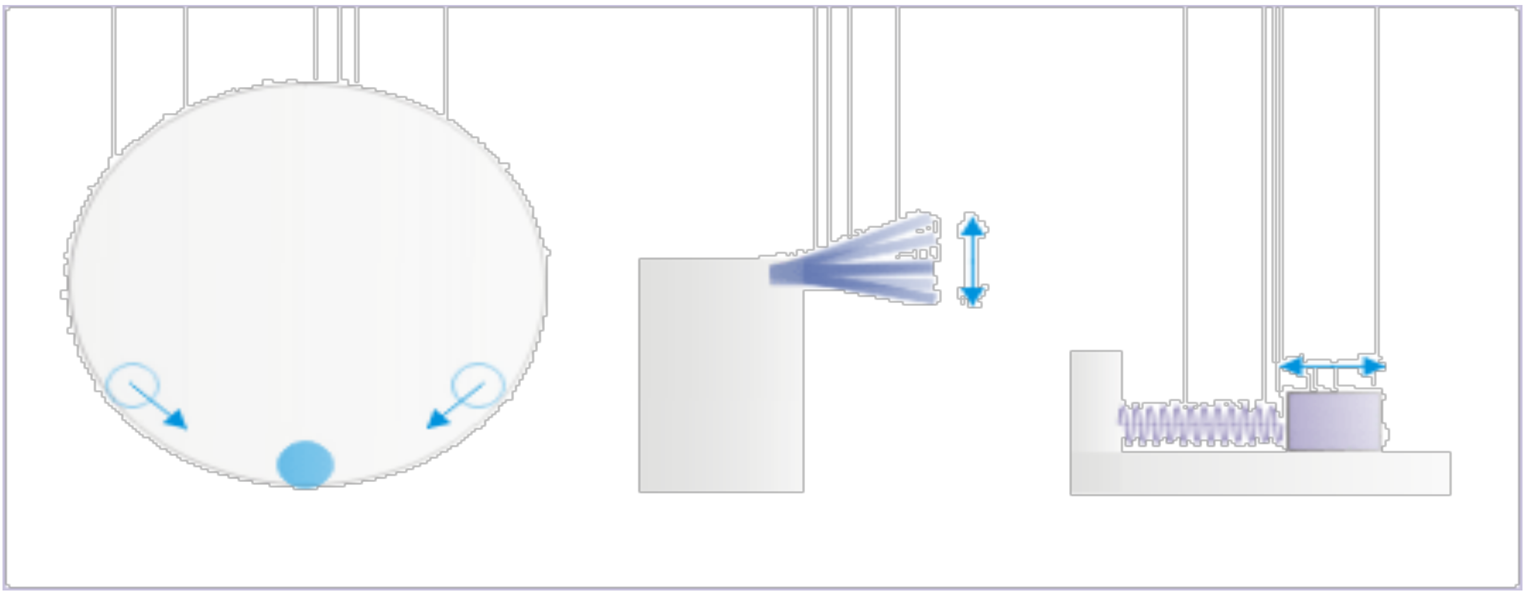
Oscillatory Motion and Wave Motion

TOPICS

✓	Oscillatory Motion	Simple Harmonic Motion
✓		Motion of an
✓		Object Attached to a Spring
✓		The Pendulum
✓	Wave Motion	Waves
✓		Transverse and Longitudinal Waves
✓		Sinusoidal Wave Function

DEFINITION : *OSCILLATION*

Oscillation is a periodic, to and from, bounded motion about a reference, usually the position of equilibrium.



Examples of oscillation

Figure 1: **The object undergoes "to and from" periodic motion.**

SIMPLE HARMONIC MOTION

- Simple harmonic motion is a special case of oscillatory motion that occurs when the restoring force is proportional to the displacement.
- As a model for simple harmonic motion, consider a block of mass *m* attached to the end of a spring, with the block free to move on a horizontal, frictionless surface.
- the spring exerts on the block a force that is proportional to the position, and given by Hooke's law : $\mathbf{F = - k\,x}$
- We call this force F: a restoring force because it is always directed toward the equilibrium position, and therefore *opposite* the displacement from equilibrium.
- The minus sign means the spring always pulls back to the equilibrium position.

The equation of the harmonic oscillator

The spring force is **$F = - k x$**

Since: $F = ma$ & $a = d^2x/dt^2$

So the equation of the harmonic oscillator is:

$$a = -\left(\frac{k}{m}\right) \mathbf{x}$$

or :

$$x = A \cos \left(\omega t + \emptyset \right)$$

Simple Harmonic Motion <u>Components</u>

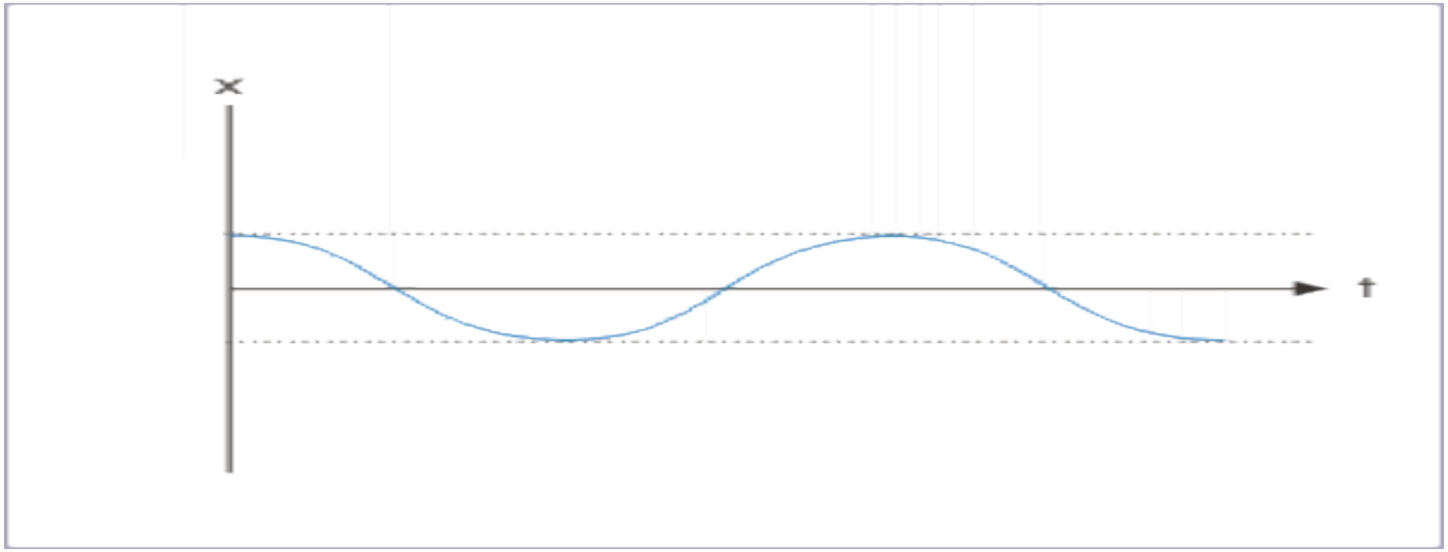
$$\boldsymbol{x} = \boldsymbol{A} \cos\left(\boldsymbol{\omega t} + \emptyset \right)$$

1- Position (x)

Position versus time for an object in simple harmonic motion as following:

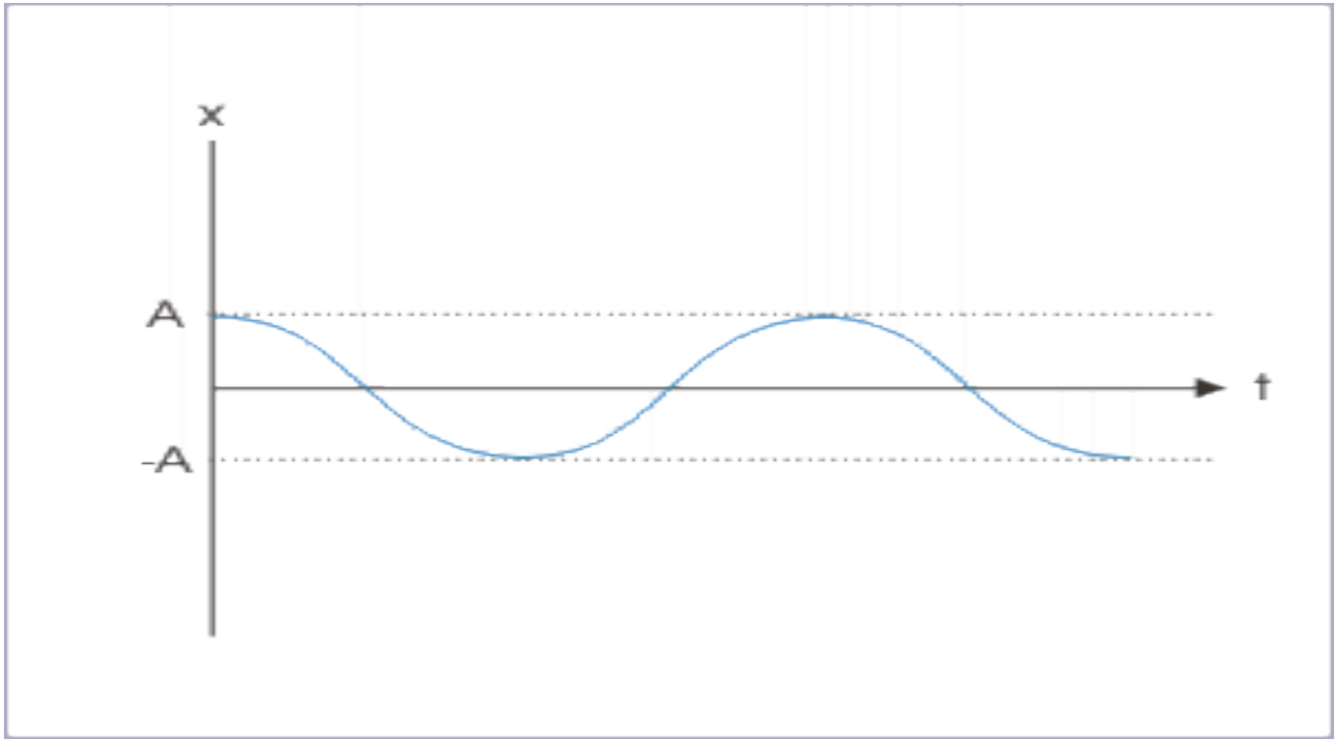
$$x = A\cos(\omega t + \emptyset)$$

where A , ω , and $\emptyset$ are constants.



2- AMPLITUDE (A)

(A) amplitude of motion is : The maximum value of the position of the particle in either the (+x) or (–x) direction.

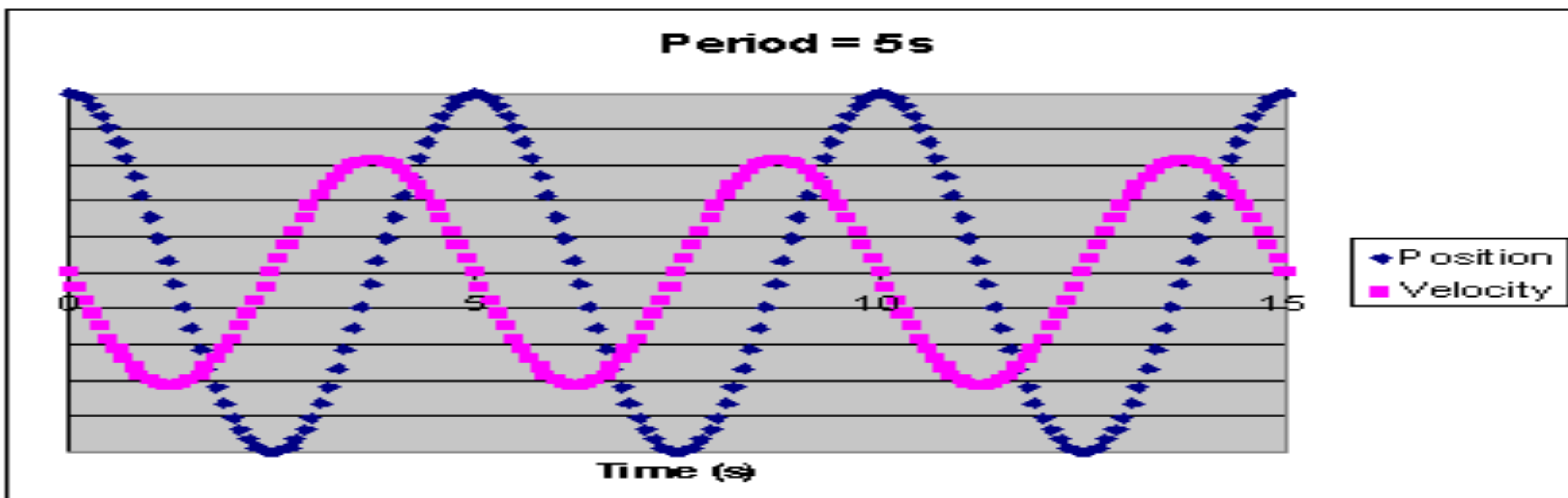


3- PHASE CONSTANT (∅)

The phase constant ∅ (or initial phase angle) is determined uniquely by the position and velocity of the particle. If the particle is at its maximum position $x = A$ at t=0, the phase constant is ∅= 0 .

The quantity (ωt + ∅) is called the phase of the motion.

Note that the function $x(t)$ is periodic and its value is the same each time ωt increases by 2π radians.



4- PERIOD (T)

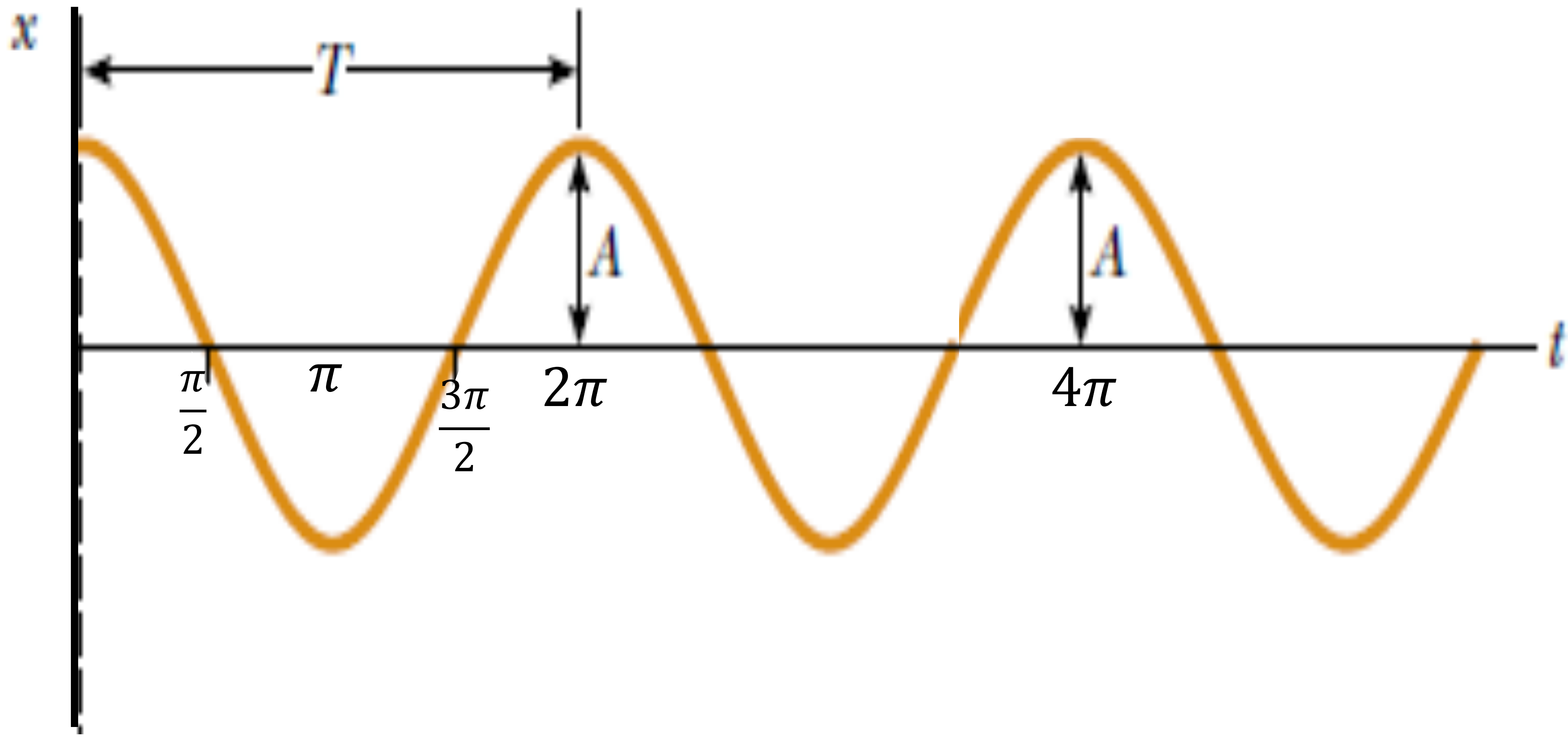
$$T = \frac{2\pi}{\omega}$$

The **period T** of the motion is: **the time interval required for the particle to go through *one full cycle* of its motion.**

ω is called the angular frequency $\omega = \sqrt{\frac{k}{m}}$

and has units of rad/s.

NOTE: The angular frequency is a measure of how rapidly the oscillations are occurring—the more oscillations per unit time, the higher is the value of ω.



x
T
A
A
t
π/2
π
3π/2
2π
4π

5- FREQUENCY(f)

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

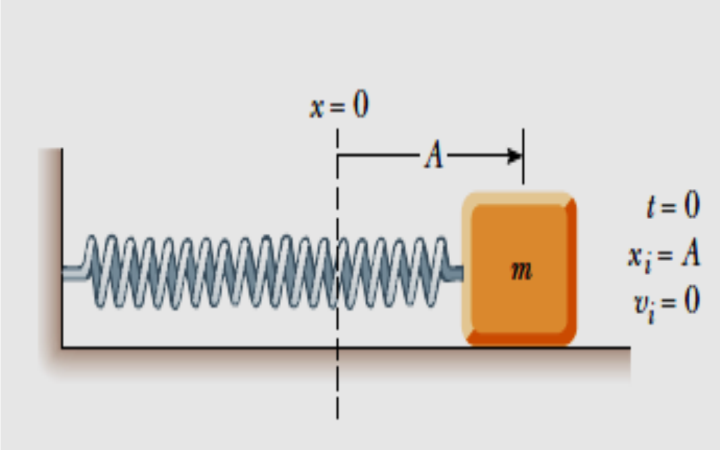
The **frequency** ***f*** **is: the number of oscillations that the particle undergoes per unit time interval,**

The units of *f* ***are cycles per second, or hertz (Hz).***

Example: A Block–Spring System

A 200-g block connected to a light spring for which the force constant is 5.00 N/m is free to oscillate on a horizontal, frictionless surface. The block is displaced 5.00 cm from equilibrium and released from rest, as in Figure.

(A) Find the period of its motion.



Example: A Block–Spring System

We know that the angular frequency of a block–spring system is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{5.00 \text{ N/m}}{200 \times 10^{-3} \text{ kg}}} = 5.00 \text{ rad/s}$$

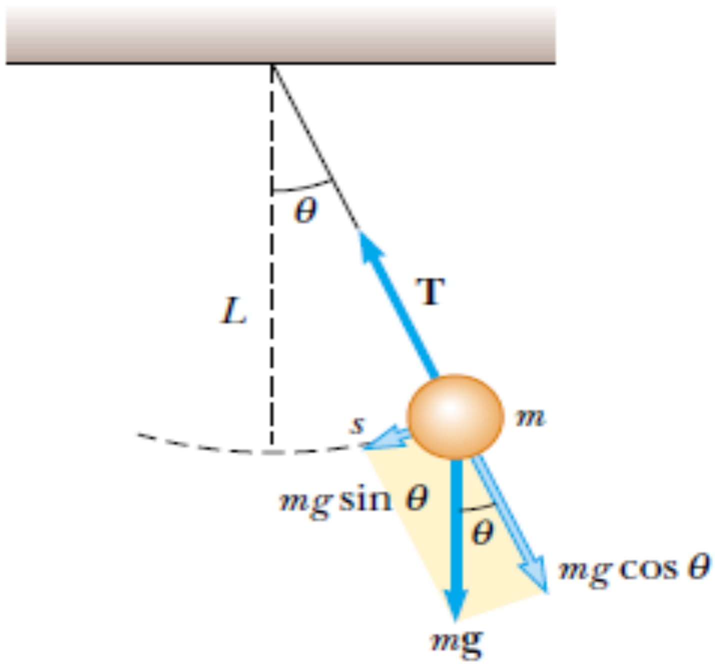
and the period is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{5.00 \text{ rad/s}} = \boxed{1.26 \text{ s}}$$

THE PENDULUM

- Something hanging from a fixed point which, when pulled back and released, is free to swing down by force of gravity and then out and up because of its inertia.
- Inertia: means that bodies in motion, will stay in motion; bodies at rest, will stay at rest, unless acted on by an outside force.

The simple pendulum



angular frequency of a simple pendulum

$$\omega = \sqrt{\frac{g}{l}}$$

PERIOD OF A SIMPLE PENDULUM

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

The period and frequency of a simple pendulum depend only on *the length* of the string and *the acceleration due to gravity*.

WHY is a pendulum scientifically IMPORTANT?

1- Time keeping.

2-it can be used to measure g (the acceleration due to gravity) which is important in determining the **shape** of the earth and the distribution of materials within it .

EXAMPLE: A CONNECTION BETWEEN LENGTH AND TIME

Huygens (1629-1695), the greatest clock master in history, suggested that an international unit of length could be defined as the length of a simple pendulum having a period of exactly 1 second. How much shorter would our length be if his suggestion had been followed?

Example: A connection Between Length and Time Period of a simple pendulum

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

$$L = \frac{T^2 g}{4\pi^2} = \frac{(1.00\ \text{s})^2(9.80\ \text{m/s}^2)}{4\pi^2} = \boxed{0.248\ \text{m}}$$

Thus, the meter's length would be slightly less than one fourth of its current length. Note that the number of significant digits depends only on how precisely we know g because the time has been defined to be exactly 1 s.

Wave Motion

Wave, is a periodic disturbance traveling through a medium.

Propagation of a Disturbance

All mechanical waves require:

(1)some source of disturbance,

(2)a medium that can be disturbed, and

(3)some physical mechanism through which elements of the medium can influence each other.

- All waves carry energy.

Types of Waves

1- Transverse wave:

A traveling wave or pulse that causes the elements of the disturbed medium to move perpendicular to the direction of propagation.

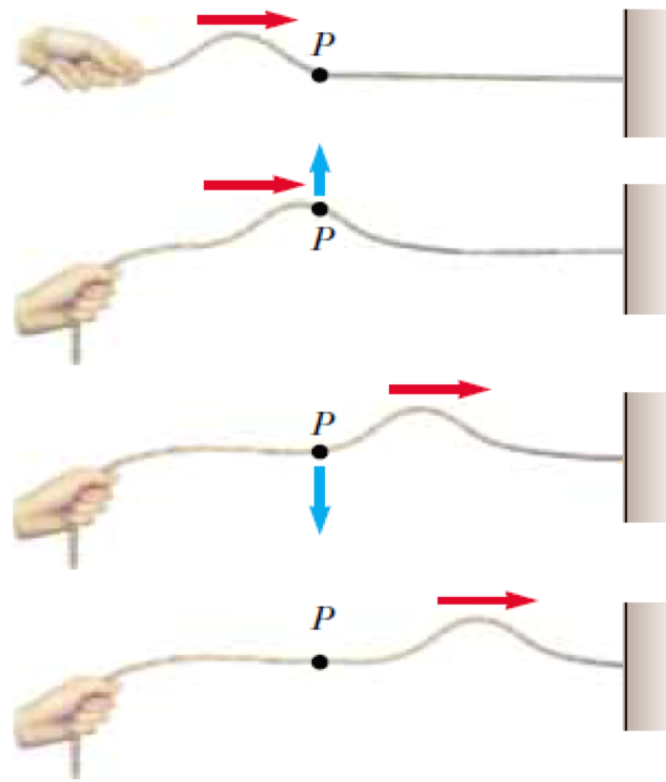
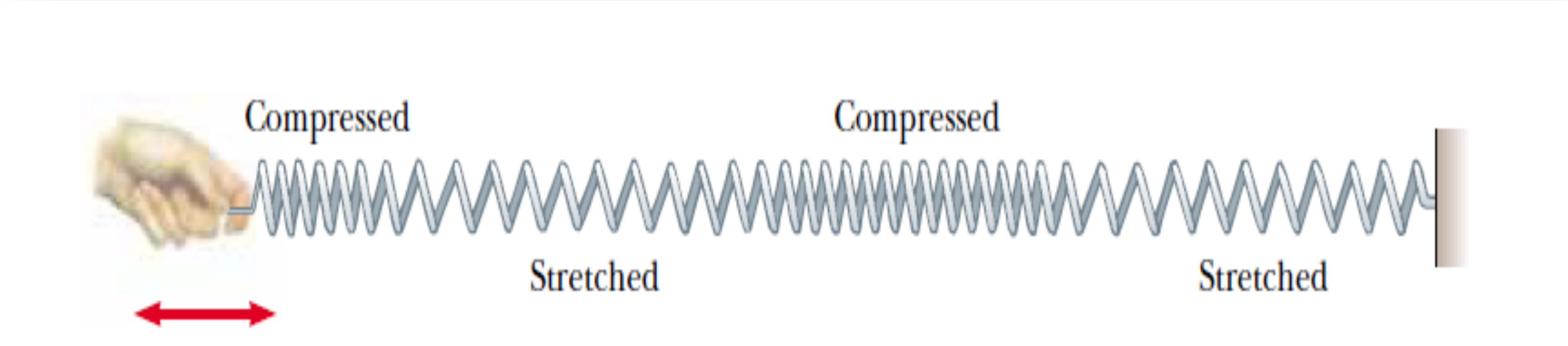


Figure 16.2 A transverse pulse traveling on a stretched rope. The direction of motion of any element *P* of the rope (blue arrows) is perpendicular to the direction of propagation (red arrows).

2- Longitudinal wave



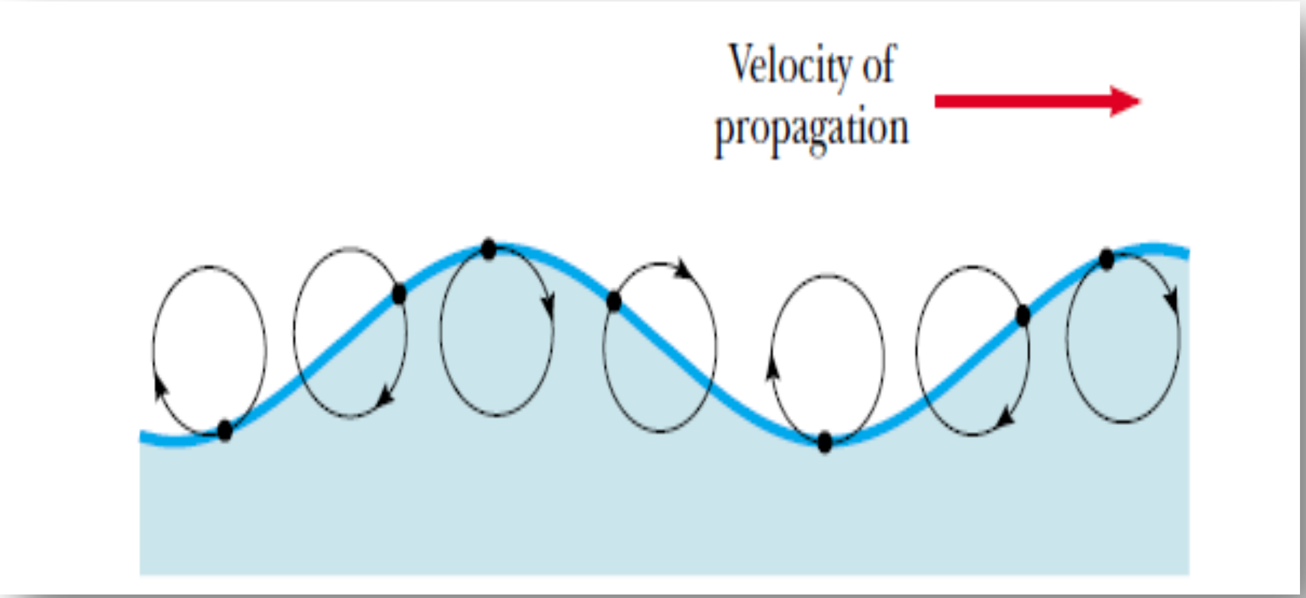
Notice that the direction of the displacement of the coils is parallel to the direction of propagation of the compressed region.

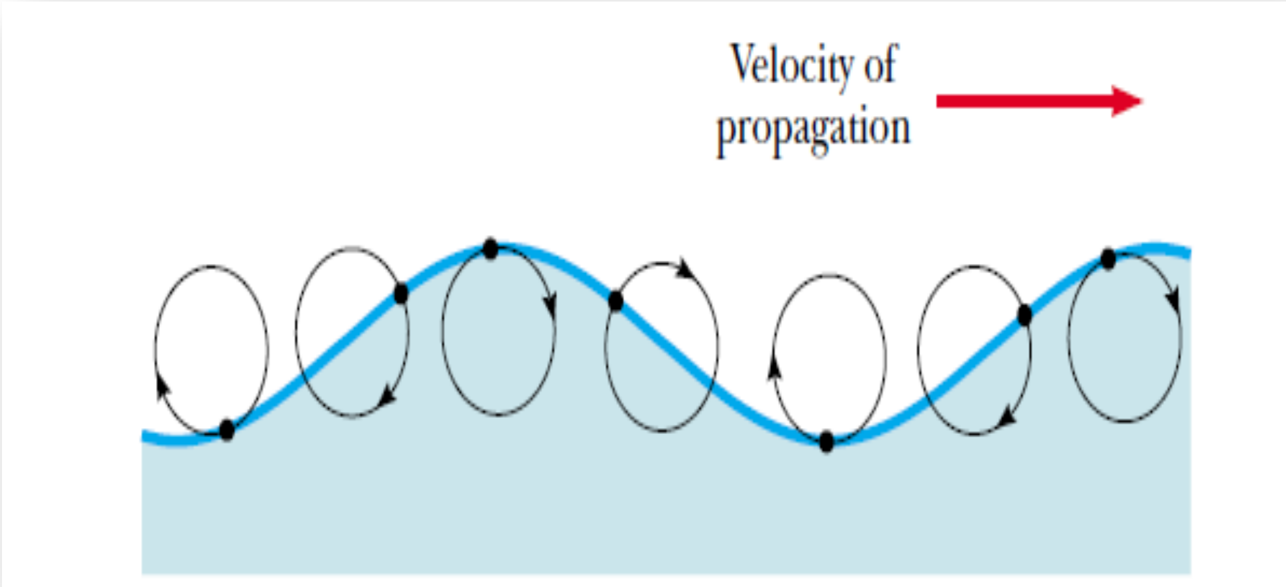
A traveling wave or pulse that causes the elements of the medium to move parallel to the direction of propagation is called a **Longitudinal wave**.

► Sound waves, are an example

of longitudinal waves

Some waves in nature exhibit a combination of transverse and longitudinal displacements. Surface water waves are a good example. When a water wave travels on the surface of deep water, elements of water at the surface move in nearly circular paths, each element is displaced both horizontally and vertically from its equilibrium position. as shown in the figure :

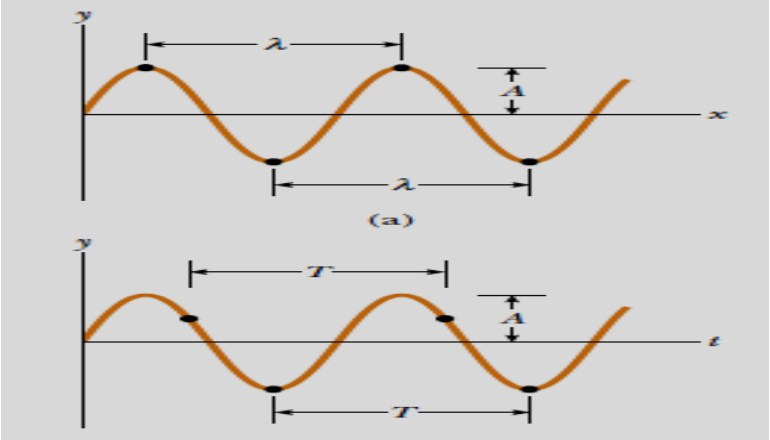




The transverse displacements seen in the figure represent the variations in vertical position of the water elements. The longitudinal displacement can be explained as follows: as the wave passes over the water's surface, water elements at the highest points move in the direction of propagation of the wave, whereas elements at the lowest points move in the direction opposite the propagation.

SINUSOIDAL WAVES

The wave represented by this curve is called a sinusoidal wave because the curve is the same as that of the function sinθ plotted against θ.



The point at which the displacement of the element from its normal position is highest is called the **crest** of the wave.

The distance from one crest to the next is called the **wavelength λ** or, the wavelength is the minimum distance between any two identical points (such as the crests) on adjacent waves.

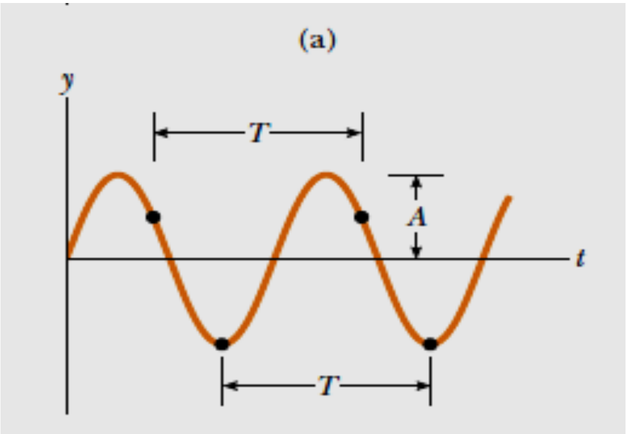
The time interval required for two identical points (such as the crests) of adjacent waves to pass by a point , is called **period *T*** of the waves

The number of crests (or troughs, or any other point on the wave) that pass a given point in a unit time interval, is called **frequency *f***, of a periodic wave is

$$f = \frac{1}{T}$$

Frequency unit is: second^{-1} , or hertz (Hz).

The maximum displacement from equilibrium of an element of the medium is called the **amplitude *A*** of the wave.



General expression for a sinusoidal Wave function:

$$y = A\sin(kx - \omega t + \phi)$$

Angular wave number :

$$k = \frac{2\pi}{\lambda}$$

The angular frequency :

$$\omega = \frac{2\pi}{T}$$

The speed of a wave:

$$v = \lambda f$$

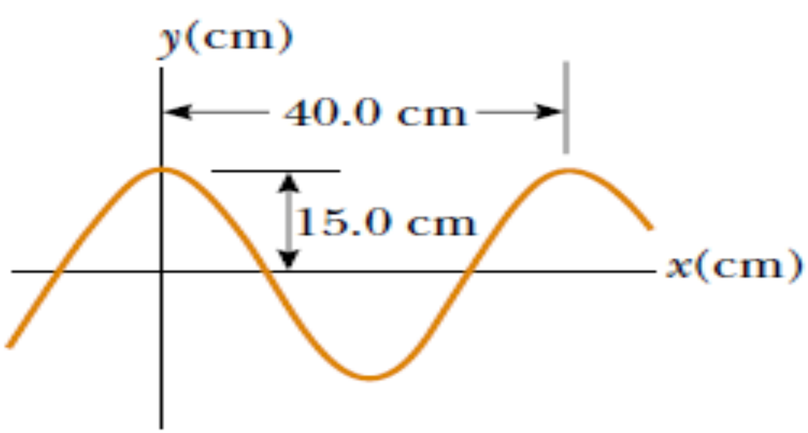
$\emptyset$ *is* the phase constant.

EXAMPLE : A TRAVELING SINUSOIDAL WAVE

A sinusoidal wave traveling in the positive x direction has an amplitude of 15.0 cm, a wavelength of 40.0 cm, and a frequency of 8.00 Hz. The vertical position of an element of the medium at $t = 0$ and $x = 0$ is also 15.0 cm.

(A) Find the wave number k, period T, angular frequency ω, and speed v of the wave.

(B) Determine the phase constant ∅, and write a general expression for the wave function.



A:

Solution

$$k = \frac{2\pi}{\lambda} = \frac{2\pi \text{ rad}}{40.0 \text{ cm}} = \boxed{0.157 \text{ rad/cm}}$$

$$T = \frac{1}{f} = \frac{1}{8.00 \text{ s}^{-1}} = \boxed{0.125 \text{ s}}$$

$$\omega = 2\pi f = 2\pi(8.00 \text{ s}^{-1}) = \boxed{50.3 \text{ rad/s}}$$

$$v = \lambda f = (40.0 \text{ cm})(8.00 \text{ s}^{-1}) = \boxed{320 \text{ cm/s}}$$

B:

Solution Because $A = 15.0$ cm and because $y = 15.0$ cm at $x = 0$ and $t = 0$, substitution into Equation 16.13 gives

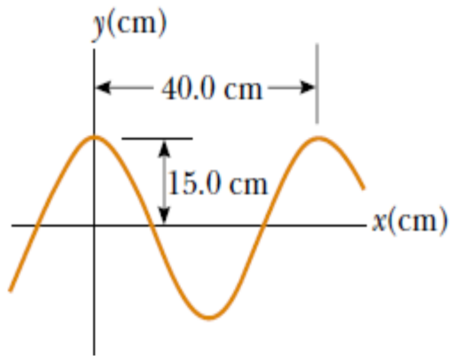
$$15.0 = (15.0) \sin \phi \qquad \text{or} \qquad \sin \phi = 1$$

We may take the principal value $\phi = \pi/2$ rad (or 90°). Hence, the wave function is of the form

$$y = A \sin\left(kx - \omega t + \frac{\pi}{2}\right) = A \cos(kx - \omega t)$$

By inspection, we can see that the wave function must have this form, noting that the cosine function has the same shape as the sine function displaced by 90°. Substituting the values for A, k, and ω into this expression, we obtain

$$y = \boxed{(15.0 \text{ cm}) \cos(0.157x - 50.3t)}$$



HOMEWORK

- At what point during the oscillation of a spring is the force F on the mass m greatest?
- What is the period of oscillation of a mass of 40 kg on a spring with constant k = 10 N/m?
- A 4 kg mass attached to a spring is observed to oscillate with a period of 2 seconds. What is the period of oscillation if a 6 kg mass is attached to the spring?
- A transverse wave is observed to be moving along a lengthy rope. Adjacent crests are positioned 2.4 m apart. Exactly six crests are observed to move past a given point along the medium in 9.1 seconds. Determine the wavelength, frequency and speed of these waves.